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Constitutive relations for dynamic soil behaviour

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ABSTRACT : After a general presentation of constitutive relations which are able to describe dynamic/cyclic soil behaviour, we discuss a key point in modelling plastic strains for cyclic or non-proportional loading : that is the so-called "incremental constitutive non-linearity", i.e. the fact that the relation between incremental non-viscous strain and incremental stress is non-linear. With respect to this basic question we will give an overview of all the existing models. Finally by using a briefly presented constitutive relation we consider three kinds of typical applications related to dynamic problems :

- cyclic liquefaction of sands,
- non-proportional loading with closed loops,
- non-proportional loading with multiple sharp bends.

1 - INTRODUCTION

The constitutive relation is now generally considered as a key point in a finite elements computation, developed to model the response of a structure or an engineering work submitted to certain loading. This constitutive relation allows to describe the mechanical behaviour of the various materials or media. It is usually recognized that the linear visco-elasticity - even if it has been used for many analytical developments - is not a convenient framework to describe quantitatively the response of soils to dynamic or cyclic loading as soon as the strain levels are more than, let's say, 10^{-4} (or 10^{-2} %) (Hicher 1985).

The purpose of this paper is therefore to focuse on constitutive models, which are of elasto-visco-plastic type and able to describe the mechanical response of soils outside their linear visco-elastic domain. In such situations we observe experimentally instantaneous rate-independent irreversible strains, that means plastic strains. We will see farther how we can describe these plastic strains.

The constitutive aspects which characterized dynamic problems are both :

1 - Dynamic loading implies generally a high loading rate. Therefore viscous materials will exhibit rate dependent effects as, for example, an increase of their strength. In order to describe such effects it is necessary to define a viscous constitutive term in the constitutive model.

2 - Generally the soil domain is submitted to a high number of repeated loading, what implies that multiple changes in the local incremental loading direction are appearing. In order to describe the soils response to such changes we need to define a rate-independent constitutive tensor which must vary in a convenient manner with the incremental loading direction.

Both these points will be taken into account in the next paragraph.

2 - FORMULATION OF CONSTITUTIVE RELATIONS

Constitutive relations are based on the principle of determinism. This principle, in the field of rheology, can have two different formulations : "in the large" and "in the small".

2.1 - Principle of determinism in the large

Considering a mechanically homogeneous sample, this principle implies that a given loading path induces a determined and unique response path. It is important to notice that any relation doesn't exist between the current stress state and strain state but rather between stress path and strain path.
Indeed it is obvious from an experimental point of view that, as soon as there are some viscous or plastic strains, an infinite number of strain states can be put in relation with a given stress state.
Mathematically such a relation implies the existence of a tensorial functional between strain history and current stress state.
In small strains we can write :

$$\underset{\sim}{\sigma}(t) = \underset{-\infty < \tau \leq t}{\mathcal{F}} [\underset{\sim}{\varepsilon}(\tau)] \qquad (1)$$

In relation (1) the functional $\mathcal{F}$ has two main properties :

1. That is a non-linear functional. If not, we obtain the limited framework of the linear visco-elasticity.
2. That is a non-differentiable functional. If not, Owen and Williams (1969) have demonstrated that for non-viscous materials the assumption of differentiability implies the absence of any internal dissipation.

As it is difficult to manage a non-linear and non-differentiable functional, that is more efficient to consider an other expression of the principle of determinism.

2.2 - Principle of determinism in the small
Let us call :

incremental strain : $d\underset{\sim}{\varepsilon} = \underset{\sim}{D}\, dt$, where $\underset{\sim}{D}$ is the strain rate tensor,

incremental stress : $d\underset{\sim}{\sigma} = \overset{\nabla}{\underset{\sim}{\sigma}}\, dt$, where $\overset{\nabla}{\underset{\sim}{\sigma}}$ is an objective derivative of the Cauchy stress tensor.

The principle of determinism implies that a "small" loading applied during time increment dt induces a determined and unique "small" response.
Mathematically we obtain a tensorial function between $d\underset{\sim}{\varepsilon}$, $d\underset{\sim}{\sigma}$ and dt :

$$\underset{\sim}{F}(d\underset{\sim}{\varepsilon}, d\underset{\sim}{\sigma}, dt) = \underset{\sim}{0} \qquad (2)$$

The function defined by relation (2) has also two noticeable properties :

1. That is a non-linear function. If not, we are not able to describe any plastic deformations, because plasticity needs a non-linear relation between $d\underset{\sim}{\varepsilon}$ and $d\underset{\sim}{\sigma}$ (see further paragraph 3).
2. That is an anisotropic function, since $\underset{\sim}{F}$ depends on some state variables (for example, the stress tensor) and memory parameters which characterize the previous strain history. For that reason if we rotate the incremental stress, the incremental strain will not rotate in the same way.

Now we assume that it is possible to decompose the incremental strain into an instantaneous part and a deferred one :

$$d\underset{\sim}{\varepsilon} = (d\underset{\sim}{\varepsilon})_{instantaneous} + (d\underset{\sim}{\varepsilon})_{deferred} \qquad (3)$$

The instantaneous strain has an elasto-plastic nature by definition and is rate-independent. Thus it depends only on the incremental stress :

$$(d\underset{\sim}{\varepsilon})_{instantaneous} = \underset{\sim}{G}(d\underset{\sim}{\sigma}) \qquad (4)$$

The differed part is viscous, rate-dependent and is a linear function of dt :

$$(d\underset{\sim}{\varepsilon})_{deferred} = \underset{\sim}{C}\, dt \qquad (5)$$

Finally we obtain the general formulation :

$$d\underset{\sim}{\varepsilon} = \underset{\sim}{G}(d\underset{\sim}{\sigma}) + \underset{\sim}{C}\, dt \qquad (6)$$

In paragraph 3 we will consider successively the viscous tensor $\underset{\sim}{C}$ and the elasto-plastic function $\underset{\sim}{G}$.

3 - VISCO-ELASTO-PLASTIC RELATIONS

3.1 - The viscous tensor

The interpretation of the viscous tensor $\underset{\sim}{C}$ in relation (6) can be obtained by considering creep loading, for which the stress tensor keep a constant value. Thus, in small deformations, for creep loading :

$d\underset{\sim}{\sigma} = \underset{\sim}{0}$

As $\underset{\sim}{G}(d\underset{\sim}{\sigma})$ is rate-independent, we have :

$\underset{\sim}{G}(\underset{\sim}{0}) = \underset{\sim}{0}$

Therefore :

$$\underset{\sim}{C} = \left(\frac{d\underset{\sim}{\varepsilon}}{dt}\right)_{\underset{\sim}{\sigma} = constant} \qquad (7)$$

In small deformations, the viscous tensor $\underset{\sim}{C}$ is exactly the creep rate of the material.

3.2 - The elasto-plastic tensor

In relation (6) the elasto-plastic part of the deformation is given by : $\underset{\sim}{G}(\underset{\sim}{d\sigma})$.

In order to state more precisely the form of the tensorial function $\underset{\sim}{G}$, we will consider first of all its three basic properties :

1. $\underset{\sim}{G}$ is an homogeneous function of degree one, because of the rate-independency condition. Indeed the behaviour of an elasto-plastic material is independent of the loading rate. That means that if we multiply the stress rate by any positive scalar λ, the strain rate is multiplied by the same scalar.

Thus :

$$\forall \lambda \in R^* : \lambda \, \underset{\sim}{d\varepsilon} \equiv \underset{\sim}{G}(\lambda \, \underset{\sim}{d\sigma}) \qquad (8)$$

It comes :

$$\forall \lambda \in R^* : G(\lambda \, \underset{\sim}{d\sigma}) \equiv \lambda \, \underset{\sim}{G}(\underset{\sim}{d\sigma}) \qquad (9)$$

Identity (9) demonstrates that $\underset{\sim}{G}$ is an homogeneous function of degree one (in a restricted sense to the positive values of λ).

2. $\underset{\sim}{G}$ is an anisotropic function of $\underset{\sim}{d\sigma}$.

As $\underset{\sim}{G}$ depends on other arguments as state variables and memory parameters, it is not an isotropic function of $\underset{\sim}{d\sigma}$. This anisotropy describes the mechanical anisotropy, which is generally observed for deformed geomaterials.

3. $\underset{\sim}{G}$ is a non-linear function of $\underset{\sim}{d\sigma}$, because of plastic irreversibilities. In such cases we know that, if $\underset{\sim}{d\sigma}$ induces $\underset{\sim}{d\varepsilon}$, - $\underset{\sim}{d\sigma}$ will not induce - $\underset{\sim}{d\varepsilon}$.

Thus :

$\underset{\sim}{G}(-\underset{\sim}{d\sigma})$ is not equal to : - $\underset{\sim}{G}(\underset{\sim}{d\sigma})$, what proves the non-linearity of $\underset{\sim}{G}$.

Now we come back to the homogeneity property of $\underset{\sim}{G}$ and use the Euler's Identity for homogeneous functions.

It comes :

$$\underset{\sim}{d\varepsilon} \equiv \frac{\partial \underset{\sim}{G}}{\partial(\underset{\sim}{d\sigma})} \underset{\sim}{d\sigma}$$

where the partial derivatives $\partial \underset{\sim}{G}/\partial(\underset{\sim}{d\sigma})$ are homogeneous functions of degree zero. Thus they depend only on the direction of $\underset{\sim}{d\sigma}$ and not on its intensity (i.e. its norm).

Finally the general expression of all elasto-plastic (rate-independent) constitutive relations is given by :

$$\underset{\sim}{d\varepsilon} = \underset{\sim}{M}(\underset{\sim}{u}) \, \underset{\sim}{d\sigma} \qquad (10)$$

where :

$\underset{\sim}{u} = \underset{\sim}{d\sigma}/\|\underset{\sim}{d\sigma}\|$ characterizes the direction of $\underset{\sim}{d\sigma}$, with $\|\underset{\sim}{d\sigma}\| = \sqrt{d\sigma_{ij} \, d\sigma_{ij}}$

and the elasto-plastic tensor $\underset{\sim}{M}$ is the gradient tensor of the function $\underset{\sim}{G}$.

The fact that $\underset{\sim}{G}$ is a non-linear function of $\underset{\sim}{d\sigma}$ - or in few words the incremental non-linearity - implies the directional dependency of the gradient tensor $\underset{\sim}{M}$ with $\underset{\sim}{u}$.

The incremental non-linearity is directly linked to the existence of plastic irreversibilities. In dynamic loading, where we have a high number of changes in the incremental loading direction, it is particularly important to use a convenient and quantitatively realistic description of $\underset{\sim}{M}$ with respect to $\underset{\sim}{u}$ or, in other words, of the incremental non-linearity.

4 - THE INCREMENTAL CONSTITUTIVE NON-LINEARITY

Considering general relation (10) it is possible to distinguish four different groups of rate-independent constitutive relations with respect to the kind of chosen directional dependency.

1. $\underset{\sim}{M}$ is independent on the incremental stress direction.

Thus :

$$\forall \, \underset{\sim}{d\sigma} : \underset{\sim}{M}(u) \equiv \underset{\sim}{M} \qquad (11)$$

Relation (11) characterises all the elastic laws. No irreversible strains can be described. This elasticity can be linear or non-linear (with a dependency of $\underset{\sim}{M}$ with the stress tensor), isotropic or anisotropic. In "hyperelasticity" no internal dissipation can appear, while in "hypoelasticity" some can occur.

2. $\underset{\sim}{M}$ can take two values

The "simplest" way to describe plastic irreversibilities is to consider two different determinations for $\underset{\sim}{M}$ with respect to a loading-unloading criterion.

Thus :

$$\begin{cases} \underset{\sim}{A}(\underset{\sim}{\sigma}) \cdot \underset{\sim}{d\sigma} < 0 : \underset{\sim}{M}(\underset{\sim}{u}) \equiv \underset{\sim}{M}^e \\ \underset{\sim}{A}(\underset{\sim}{\sigma}) \cdot \underset{\sim}{d\sigma} > 0 : \underset{\sim}{M}(\underset{\sim}{u}) \equiv \underset{\sim}{M}^{ep} \end{cases} \qquad (12)$$

where $\underset{\sim}{M}^e$ is the elastic tensor associated to the unloading domain and $\underset{\sim}{M}^{ep}$ the elasto-plastic one associated to the loading domain.

In the six-dimensional $d\underset{\sim}{\sigma}$ space, both these domains are separated by an hyper-plane whose equation is given by :

$$\underset{\sim}{A}(\underset{\sim}{\sigma}) \cdot d\underset{\sim}{\sigma} \equiv 0 \ .$$

We obtain here all the elasto-plastic relations with one unique yield surface.

This yield surface or elastic limit can be confounded with the flow rule or plastic potential ("associated elasto-plasticity") or not ("non-associated elasto-plasticity"). Yield surface and flow rule are varying with the previous strain history by means of hardening parameters. If these surfaces varies homothetically, we have an isotropic hardening ; if they can trânslate, that is a kinematic hardening.

3. $\underset{\sim}{M}$ can take four or more different values.

The general writing is given by :

$$\begin{cases} \underset{\sim}{A}_1(\underset{\sim}{\sigma}) \cdot d\underset{\sim}{\sigma} < 0 \\ \quad \vdots \\ \underset{\sim}{A}_n(\underset{\sim}{\sigma}) \cdot d\underset{\sim}{\sigma} > 0 \end{cases} \quad \underset{\sim}{M}(\underset{\sim}{u}) \equiv \underset{\sim}{M}^1, \ldots, \underset{\sim}{M}^{2^n} \qquad (13)$$

We have here incrementally piece-wise linear constitutive relations. All the elasto-plastic relations with several yield surfaces and plastic potentials are of this type. Following the different loading unloading conditions which are verified or not by the current incremental stress direction, $\underset{\sim}{M}$ takes different determinations (for n loading - unloading criterions, we obtain 2^n different constitutive tensors $\underset{\sim}{M}$). All these determinations are not independent but linked by a continuity condition which must be verified at each border between two adjacent domains. In elasto-plastic theory this continuity condition comes from the "consistency equation".

These notions can be generalised without introducing neither yield surface nor flow rule by the concept of "tensorial zone" (Darve and Labanieh 1982), defined as any domain in $d\underset{\sim}{\sigma}$ space where the constitutive relation is tensorially linear.

4. $\underset{\sim}{M}$ varies continuously with the incremental stress direction

That can be considered as a generalization of the previous incrementally piece-wise linear relations. The so-called "endochronic" models (Valanis 1971) are of this type with an incremental non-linearity characterized by the scalar $\|d\underset{\sim}{\sigma}\|$ (or in a dual form, $\|d\underset{\sim}{\varepsilon}\|$)

$$d\varepsilon_{ij} = C_{ijkl}\, d\sigma_{kl} + D_{ij}\, \|d\underset{\sim}{\sigma}\| \qquad (14)$$

An other model has been proposed by Kolymbas (1984), and more recently by Desrues and Chambon (1991). The hypoplasticity developed by Dafalias (1986) and issued from the elasto-plastic bounding surface theory has an incrementally non-linear nature.

All the examples which will be presented in the next paragraphs are all issued from two constitutive relations :

1. The incremental "octo-linear" relation (Darve 1974) which is piece-wise linear with eight different determinations for the constitutive tensor.

Its expression in stress-strain principal axes (unique case considered for the applications in this paper) is given by :

$$\begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_3 \end{bmatrix} = \frac{1}{2}(\underset{\sim}{N}^+ + \underset{\sim}{N}^-) \begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \end{bmatrix} + \frac{1}{2}(\underset{\sim}{N}^+ - \underset{\sim}{N}^-) \begin{bmatrix} |d\sigma_1| \\ |d\sigma_2| \\ |d\sigma_3| \end{bmatrix} \qquad (15)$$

where $\underset{\sim}{N}^+$ and $\underset{\sim}{N}^-$ are two (3 x 3) matrices depending on state variables and memory parameters.

2. The incremental non-linear relation of second order whose general form is the following :

$$d\varepsilon_{ij} = C_{ijkl}\, d\sigma_{kl} + \frac{1}{\|d\underset{\sim}{\sigma}\|} D_{ijklpq}\, d\sigma_{kl}\, d\sigma_{pq} \qquad (16)$$

In stress-strain principal axes the previous form degenerates with some added assumptions into the following expression :

$$\begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\varepsilon_3 \end{bmatrix} = \frac{1}{2}(\underset{\sim}{N}^+ + \underset{\sim}{N}^-) \begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \end{bmatrix} + \frac{1}{2\|d\underset{\sim}{\sigma}\|}(\underset{\sim}{N}^+ - \underset{\sim}{N}^-) \begin{bmatrix} (d\sigma_1)^2 \\ (d\sigma_2)^2 \\ (d\sigma_3)^2 \end{bmatrix} \qquad (17)$$

with $\quad \|d\underset{\sim}{\sigma}\| = \sqrt{(d\sigma_1)^2 + (d\sigma_2)^2 + (d\sigma_3)^2}$

As it is not the purpose of this paper to detail any particular constitutive relation, the reader will find exhaustive presentations of this model in Darve and Dendani (1989) and Darve (1990).

In conclusion the interest to use incrementally non-linear relations lays on two different aspects. A practical interest is the fact that it is not necessary to define neither elastic domain nor plastic potential. It is well known how it is difficult to characterize experimentally both these surfaces, which vary in a complex manner with the strain history.

More fundamentally we need incrementally non-linear models in all the situations where the dependency of the constitutive tensor with the incremental stress direction is excited. That occurs in two kinds of problems :

1. Instabilities as :

- shear bands formation by bifurcation of the strain mode from a diffuse one to a strictly localized one (Darve 1984) ;
- loss of constitutive unicity by nullity of the jacobian determinant of $\underset{\sim}{G}$ function (Darve and Chau 1986)
- nullity of the work of second order.

2. Severely non-proportional paths as such encountered in dynamic or cyclic situations.

Now we will concentrate on this second point by considering three classes of such loading.

5 - LIQUEFACTION OF SANDS

The liquefaction is a phenomenon of constitutive type defined by annuling the effective (intergranular) stresses for a certain class of monotonous or cyclic loading paths. Usually the liquefaction is studied both from experimental and theoretical points of view for "undrained" paths. In such a case the volume of the soil sample is kept constant by preventing the water of a saturated sample to go out or into the sample. With strain controlled testing machines, which allow to respect the no-volume condition, that is possible to obtain liquefaction on dry samples. That is also possible to reach a liquefied state by other loading, for example with a dense sand by a dilatant proportional strain path or with a very loose sand by a contractant proportional strain path (Meghachou 1992).

For a given sand and a given class of loading (for example, undrained path), it was showed experimentally that monotonous liquefaction could be reached at low initial density and cyclic liquefaction at higher initial densities (the number of needed cycles for liquefying increases as this initial density). The liquefaction potential of a given sand is directly linked to its contractancy potential for drained (monotonous or cyclic) shear.

These now generally accepted results are illustrated on fig. 1, where the liquefaction of Monterey loose sand is modelled by the incremental octo-linear relation (eq. (15)). At the left of this figure we see a continuous plastic deformation without rupture in case 1, a classical plastic failure in case 2 for a higher value of the initial void ratio and finally a monotonous liquefaction in case 3 for the highest value of this void ratio. At the right of fig. 1 a cyclic liquefaction is modelled for a value of the initial void ratio equal to that in case 2.

6 - CIRCULAR LOADING PATHS

To study such paths is interesting for basic reasons in relation with dynamic problems. Indeed let us consider any kind of repeated loading for which the same level of stress is reached successively many times. At a given point inside the geomaterial domain the stress (but not necessarily the strain !) will also go through the same value many times. That means that if we plot the stress path inside the six-dimensional stress space we will obtain a closed loop.

For such closed loops the direction of the incremental stress (tangent to the loop) is constantly changing. Thus a circular loading path will exhibit the same kind of difficulty in order to model quantitatively the strain response than any closed loop.

Such a circular path in a deviatoric stress plane (the mean pressure is therefore kept constant and the three principal stresses are varying in such a way that the stress state follows a circle in the three-dimensional principal stress space) was used as a benchmark test at the International Workshop on Constitutive Equations for Granular Non Cohesive Soils, held at Cleveland in July 1987 (Saada and Bianchini 1989). The results of the predictions, which have been obtained with our incremental non-linear model (eq. (17)), are compared with the experimental measures on fig. 2. The agreement can be noticed taking into account the fact that it is predictions class A without knowing previously the experimental results. At the top of the

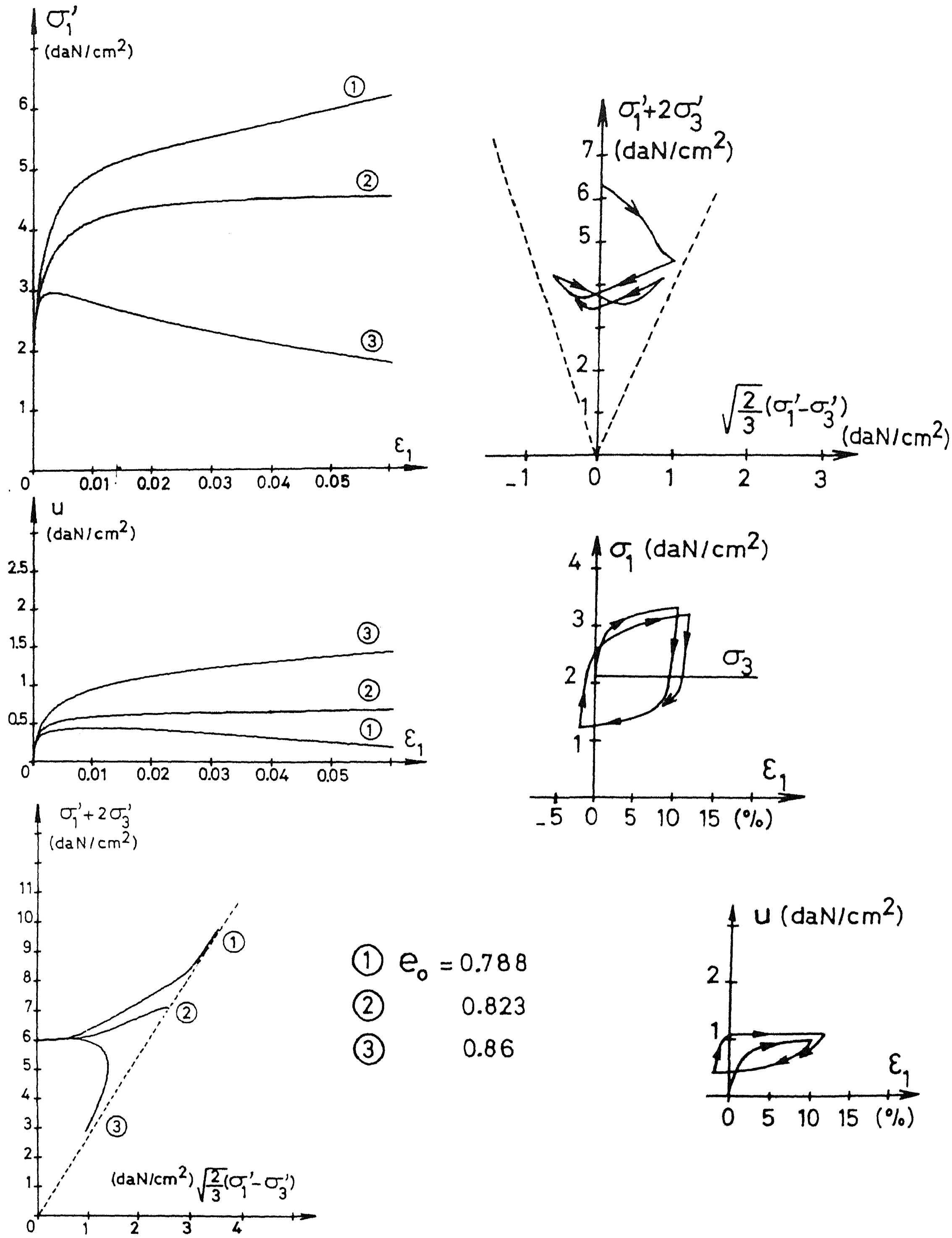
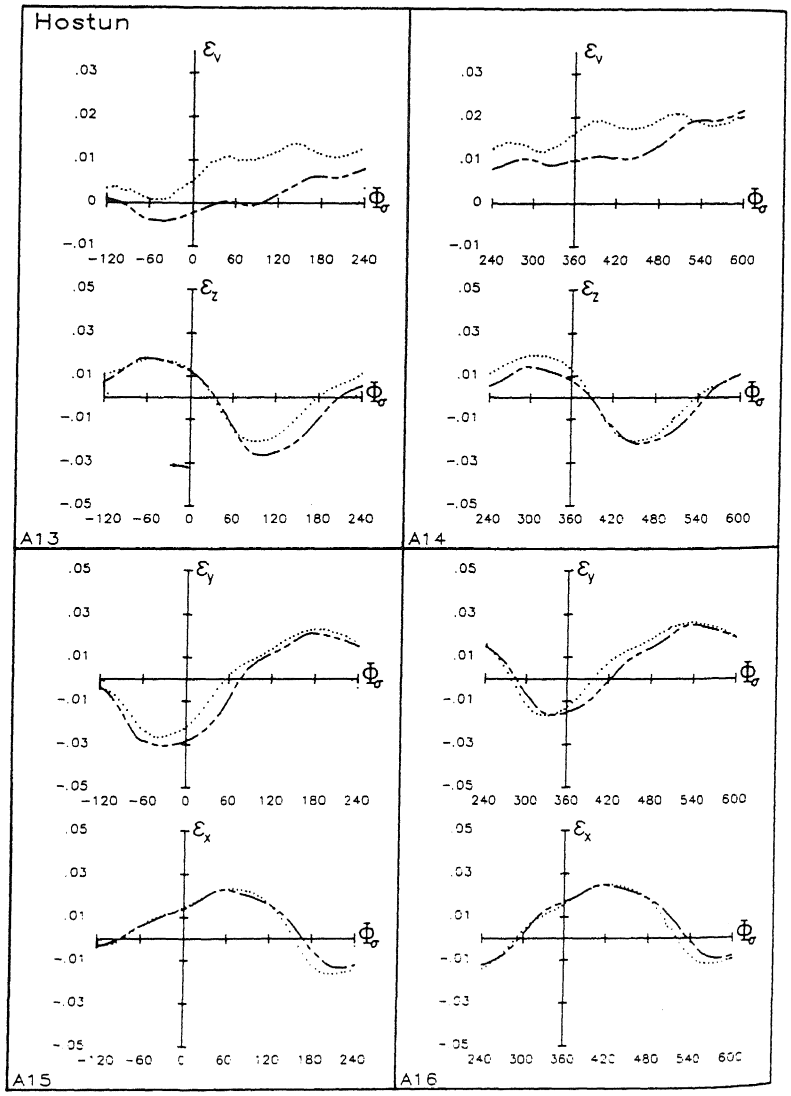


Figure 1 : Modelling the liquefaction of Monterey loose sand for monotonous and cyclic undrained triaxial loading. e_0 is the initial void ratio and u the pore water pressure, $\underset{\sim}{\sigma}$ is the total stress and $\underset{\sim}{\sigma}'$ the effective stress ($\underset{\sim}{\sigma} = \underset{\sim}{\sigma}' + u\,\underset{\sim}{I}$)



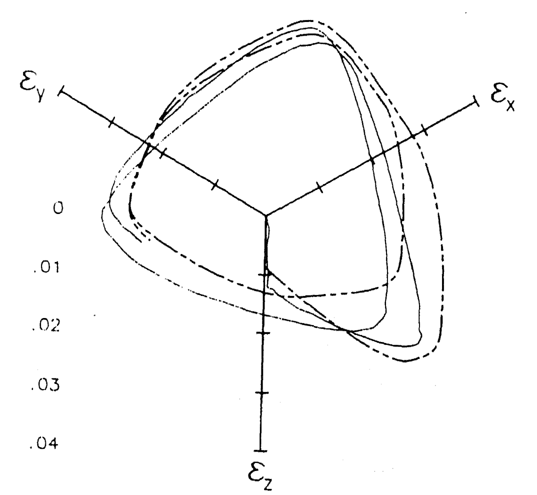


Figure 2 : Modelling the response for a circular loading path followed twice in a stress deviatoric plane. The predictions are given by dashed line : —— - - —— (Darve and Dendani 1989).

figure the principal strains variations and volume variations are plotted, at the bottom that is the response projected on a strain deviatoric plane.

The same kind of circular stress path was investigated, in order to compare octo-linear and non-linear models. Fig. 3 shows an appreciable difference between both these models. Let us recall here that both these relations have exactly the same constitutive constants with same values. They differ only by their structure in the sense that the octo-linear model is piece-wise linear with eight determinations for the constitutive tensor while the non-linear model is incrementally non-linear with a continuous variation of the constitutive tensor with the incremental stress direction. That illustrates the fact that for such circular path the kind of dependency of M with u has a drastic influence on the response.

7 - LOADING PATHS WITH MULTIPLE SHARP BENDS

The objectives in studying such paths are both :

1. to verify a sort of consistency condition by comparing responses obtained for a rectilinear proportional loading and for a path close to it and formed by a high number of small cycles surrounding the proportional path,

2. to discuss the validity of the principle of superposition for incremental loading.

This principle is not generally valid, since the incremental non-linearity implies that :

$\underset{\sim}{G}(d\underset{\sim}{\sigma}^1 + d\underset{\sim}{\sigma}^2)$ is not equal to : $\underset{\sim}{G}(d\underset{\sim}{\sigma}^1) + \underset{\sim}{G}(d\underset{\sim}{\sigma}^2)$.

However we know that stress or strain controlled testing machines allow to follow in an approximated manner only a prescribed path practically with successive "small" steps. Based on the fact that the principle of superposition is not verified a question arouses : is a systematic error induced by such a procedure or not ?

Using the incremental non-linear relation E. Flavigny and M. Meghachou have compared responses obtained for both these kinds on paths : the direct proportional one and some near ones with sharp bends (Darve, Flavigny and Meghachou, 1992).

Figures 4 and 5 show such results for a drained triaxial path compared with a path decomposed into isotropic loading and constant mean pressure loading. Fig. 4 is concerned by a dense sand and fig. 5 by a loose one. The influence of the length of the bends is visible on the figures. Two main results can be exhibited :

1. When the length of bends tends to zero value, the response curves are converging,

2. This converged response does not coincide with the response given by the direct rectilinear path, but both are very near.

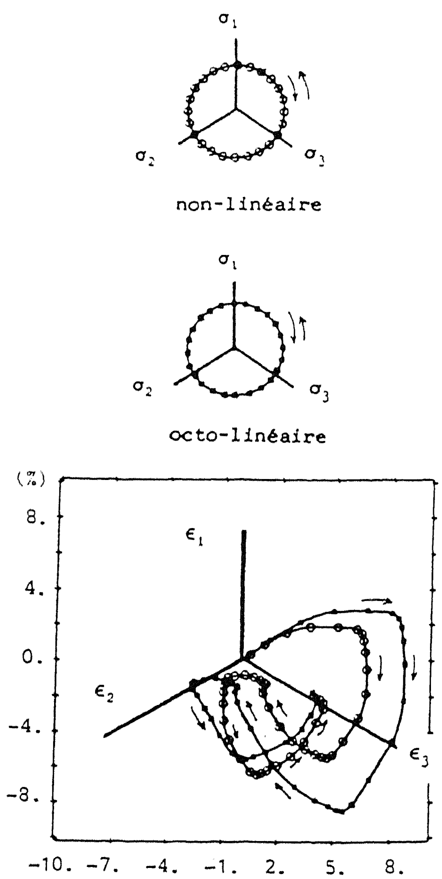


Figure 3 : Comparison between results given by the octo-linear model (black points) and the non-linear one (white points) for a circular stress path in a deviatoric stress plane, followed twice in opposite sense.
(Chau 1989).

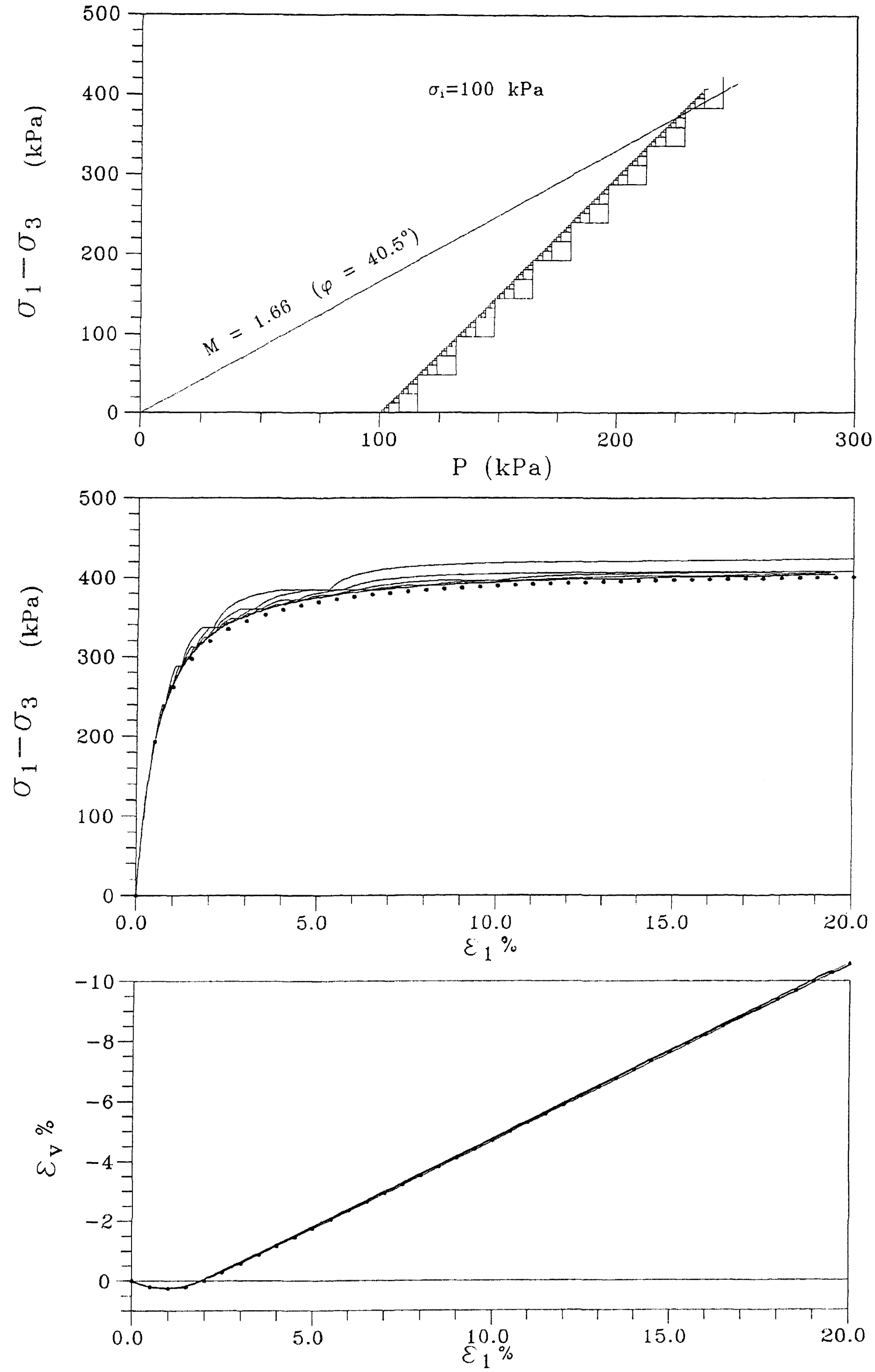


Figure 4 : Comparison between results given by the incremental non-linear model for a rectilinear drained triaxial path and step paths with different lengths of the step. Case of a dense sand.
(Meghachou 1992).

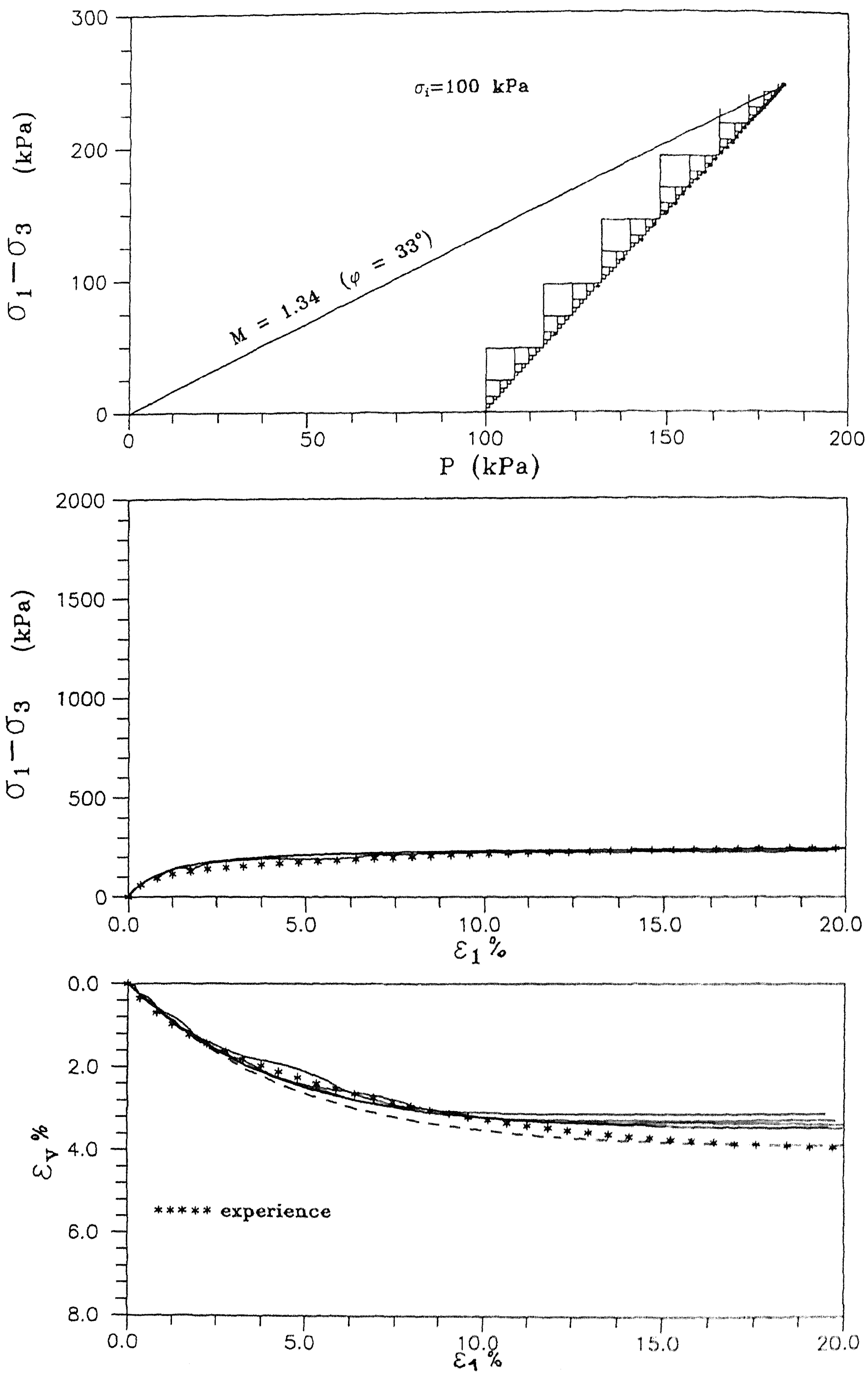


Figure 5 : Comparison between results given by the incremental non-linear model for a rectilinear drained triaxial path and step paths with different lengths of the step. Case of a loose sand.
(Meghachou 1992).

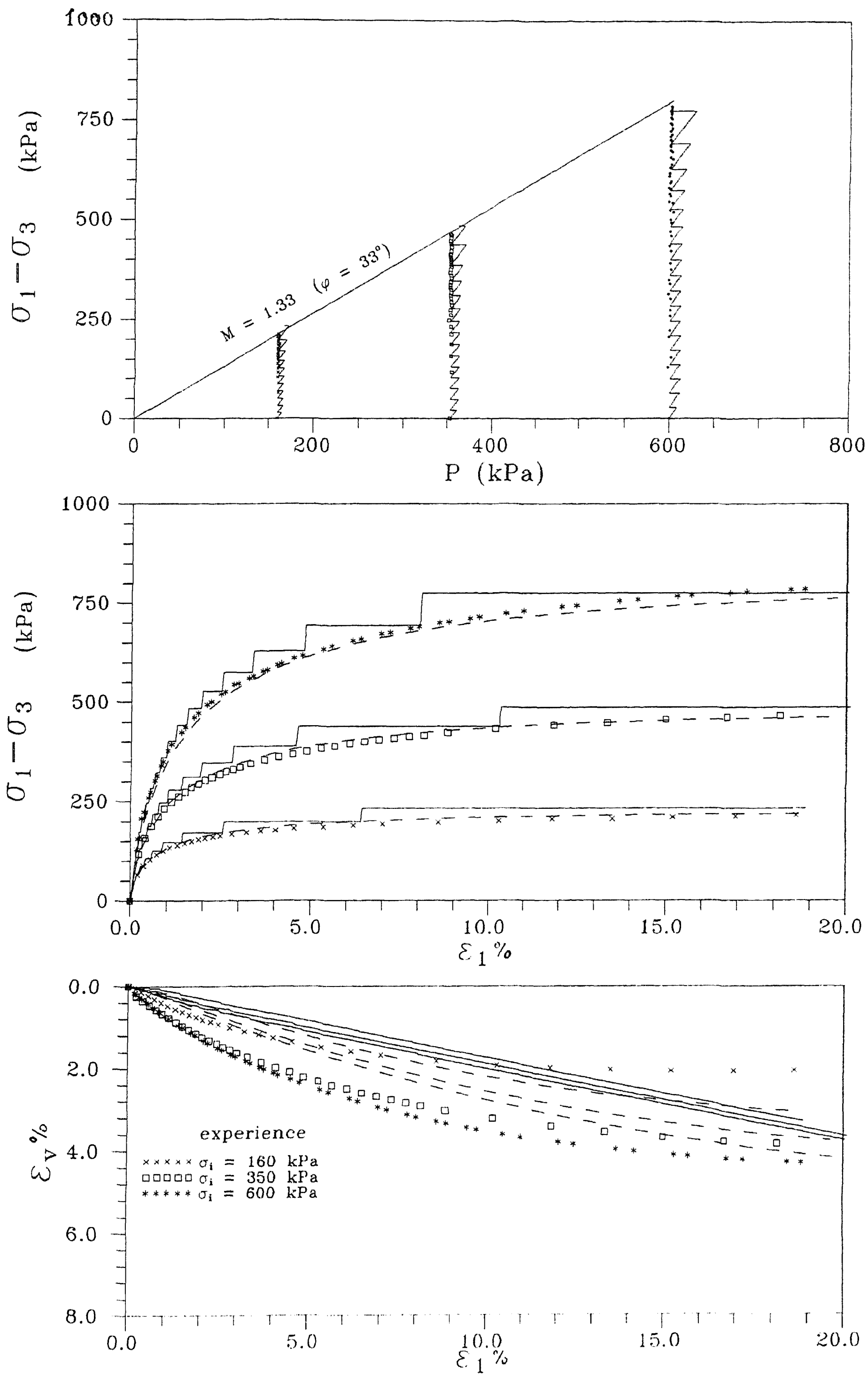


Figure 6 : Comparison between results given by the incremental non-linear model for rectilinear constant mean pressure (160 kPa, 350 kPa, 600 kPa) paths (diagrams represented by dashed lines) and for sharp bends paths (diagrams represented by continuous lines). Points are experimental results.
(Meghachou 1992).

Fig. 6 considers an other case with a rectilinear constant mean pressure path compared with sharp bends paths constituted by drained triaxial loading and isotropic unloading.

Here also the figure shows that the responses are different but near.

In conclusion an incremental non-linear relation can model a high number of small cycles without obtaining unrealistic results. The principle of superposition for incremental loading, even if it is not verified strictly speaking, remains usable in order to confirm experimental results obtained by certain testing machines.

8 - CONCLUSION

The linear visco-elasticity, which is conventionally utilised in dynamics, has a domain of validity restricted to the very small deformations (less than $10^{-2}\%$). For larger strains we need to take into account plastic irreversibilities inside the framework of a visco-elasto-plastic model. The basic difficulty lays on the fact that for describing plastic deformations the incremental constitutive relation must be non-linear : that is the essential question of incremental non-linearity. We have discussed the various ways available to describe this non-linearity.

Moreover plastic strains are largely dependent on the previous strain history. Some memory parameters are needed. This memory is not so rapidly erased as in viscosity. Indeed two kinds of memory parameters must be defined : discrete ones and continuous ones (Darve and Dendani 1989, Darve 1990). The question of the description of the memory has not been considered in this paper.

Considering the most advanced constitutive models we have showed how they can describe monotonous and cyclic liquefaction of sands. Circular loading is an interesting approximation of closed paths, as they are encountered in dynamic problems. Conventional elasto-plastic relations are usually not able to describe such paths (Saada and Bianchini 1989) essentially because of the very large variation of the incremental stress direction. Incrementally non-linear relations with adequate memory parameters seem to be more powerful as we have seen on an example.

Usually in dynamic loading we are confronted to a very high number of changes of the incremental stress direction. Thus it is important to validate models with respect to their stability for a high number of small perturbations. We have considered such cases and discussed in the same framework the validity of the experimental results issued from testing machines utilizing a feed-back loop device, by confirming them largely.

While utilizing linear visco-elasticity analytical developments are possible for conventional boundary conditions, the incremental non-linear models as the more classical elasto-plastic ones need to be incorporated into a finite elements code in order to be applied to practical boundary problems. Such codes which can solve cyclic and dynamic problems in engineering practice (Pastor Zienkiewicz and Chan 1990, Aubry and Modaressi 1990, Meimon, Lassoudière and Kodaissi 1987, Prevost 1981,...) are now available even if many questions in relation with dynamic modelling stay still open : some of them have been approached in this paper !

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