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Modelling problems in earthquake resistant design: Uncertainties and needs

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ABSTRACT: While regular buildings may be designed according to present codes, to resist moderate earthquakes without significant structural damage and large earthquakes without collapse, our ability to predict with any degree of accuracy the actual amount of damage that a specific structure would suffer when subjected to a given earthquake, which causes it to behave inelastically, is still rather limited. Present models and analysis procedures allow to predict that a structure will remain in the elastic range or that it will definitely collapse, but the amount of inelastic behavior and damage for structures behaving between these two extremes is very hard to quantify. This paper discusses the present procedures used for seismic analysis of buildings or important, special, structures such as nuclear power plants, some of the limitations inherent in these procedures, the main sources of uncertainties or errors, and the research which could improve the present situation.

1. INTRODUCTION

Professor George Housner, in a wonderful lecture on the history of earthquake engineering that he has been giving under the auspices of the Earthquake Engineering Research Institute, reminds us that in 1907, Professor Charles Derleth of the University of California at Berkeley, wrote in the transactions of ASCE in the discussion of the 1906 San Francisco earthquake: "Many engineers appear to have the idea that earthquake stresses in framed structures can be calculated so that rational designs to resist earthquake destruction can be made, just as one may allow for dead and live loads, or wind and impact stresses. An attempt to calculate earthquake stresses is futile. Such calculations could lead to no practical conclusions of value."

These statements may seem short-sighted or even preposterous after listening for a week to the research on earthquake engineering that has been conducted during the last four years around the world. For there is no doubt that there has been a tremendous amount of progress since 1906, or just in the last twenty years, in our understanding of the nature of earthquakes and earthquake mechanisms, the effects of geology, topography and local soil properties on the characteristics of the seismic motions that can be expected at a specific site, and the behavior of structures and soils when subjected to seismic waves. Most of the traditional research in the structural field has dealt moreover with the development of analytical models and methodologies, in what may appear as a complete disregard of Professor Derleth's warning, or with the improvement of structural details through experimental work. Research on the more creative, conceptual, phase of the design process, exploring alternative structural configurations which may be better suited to resist the loads resulting from earthquake excitations, as well as new mechanisms of energy dissipation, base isolation or, in general, active and passive control systems, is only relatively recent. In spite of this as a result of these research efforts, buildings can be designed today, according to present codes, to resist earthquakes much more safely than at the time of publication of the first seismic code, over half a century ago. Although this increase in safety may also be credited in part to improvements in the supervision and quality control of the materials used and the construction process, there is no denying that the numerous changes which have been introduced continuously in the code requirements, particularly in relation to reinforcement of concrete members, connections and structural detailing, have also played a significant role in adding to the safety. It is fair to say in fact that regular buildings can be designed at present to perform satisfactorily under the potential earthquakes they may experience during their lifetime, according to the code's philosophy. The key words in this statement which must be left conveniently vague are "regular" and "satisfactorily".

The objective of an engineer involved in the design of a specific building is to come up with a structure which satisfies the functional requirements both aesthetically and economically and which can perform its intended use safely under all potential loads and environmental actions. From his point of view the general qualitative knowledge that the structure will perform reasonably well, according to the code, if subjected to an earthquake, may be sufficient, without the need for a quantitative estimate of the damage that can be expected over its lifetime or that a specific earthquake motion would produce. The

planner or government official interested in estimating the potential economic losses that a city or region may expect due to earthquakes needs statistical data on damage recorded in different types of buildings rather than detailed information on the behavior of one particular structure. On the other hand researchers would like to improve their knowledge and understanding of the behavior of soils and structures subjected to seismic excitation to be able to predict quantitatively with some degree of accuracy the damage that a particular earthquake record would cause to a specific building. This level of knowledge may also be necessary to design unusual or very important structures which have very little or no history of past performance under strong motion earthquakes, or to justify adequately the substantial expense of retrofitting existing buildings which may be considered marginal.

The question that must be asked then, in relation to Derleth's comments, is whether the models and analysis procedures suggested by seismic codes and used in engineering practice for seismic design are capable of predicting structural damage with any reasonable degree of accuracy. Or, when dealing specifically with code procedures, whether linear elastic analyses, as those performed to determine the stresses caused by normal dead and live loads (service loads by opposition to extreme loads), static or dynamic, can provide results of any real value in the calculation of damage for structures which are going to behave inelastically. For the codes accept implicitly in their design philosophy that buildings will undergo large inelastic deformations when subjected to a strong earthquake, yet require only linear elastic analyses, assuming that there is a simple and direct relationship between the level of forces that would result from these analyses, relative to the member capacities, and the amount of damage that may be expected (through the ductility requirements). Professor Derleth states actually later on in the same article: "What is needed in earthquake countries is design emphasis, not on the elastic components, but on the yielding frame or unit".

In spite of all the progress achieved through research the answer to the above question is still negative. Code procedures may be satisfactory, as stated earlier, for the design of buildings for which the experience obtained from the observations during many actual earthquakes tell us that the behavior will be adequate when properly built and detailed, and dimensioned for the levels of forces required by the codes. They cannot provide, however, accurate quantitative measures of damage. Even the nonlinear analyses procedures which are commonly used for research purposes and which are seeing increased application in practice for the design of important structures, are primarily intended for two-dimensional, planar, frames, and include a number of approximations and simplifications. The nature and magnitude of the errors introduced by these approximations are not always well understood.

The purpose of this paper is to review some of the present models and procedures used for seismic structural analyses and to identify sources of errors and uncertainties as well as areas in need of further study in each one of the main phases of the analysis: the definition of the seismic input, the modification of the earthquake characteristics to account for geological and local soil effects, the development of the structural model, its modification to account for soil structure interaction effects (flexibility of the foundation), the methods of analysis and the procedures for damage assessment. In the discussion of present approaches a distinction must be made between regulations for very special and important structures, such as nuclear power plants, seismic codes for typical buildings and methods used in research.

2. DEFINITION OF SEISMIC INPUT

2.1 Present Procedures

Seismic regulations for the design of nuclear power plants have always required the consideration of two levels of earthquake for the design. Although their names have changed through the years, the lower level motion represents an earthquake with a return period similar to the life of the structure, while the higher level is associated with the maximum credible earthquake, with a much larger return period. The reactor is supposed to continue its normal operation during the smaller motion and should be able to be safely shut down during the second. The characteristics of these two motions are determined from detailed seismological and geological studies of the zone surrounding the site, but in practice the seismic hazard studies may provide only the value of the peak ground acceleration for the safe shutdown earthquake. The peak ground acceleration of the design basis or operating basis earthquake is often taken as one half of the above and minimum values are imposed, somewhat arbitrarily, in areas without known seismicity. The definition of the seismic input is completed through the use of a set of standard smoothed elastic design spectra which are the same for both motions and are simply scaled by the value of the peak ground acceleration.

Analyses must be conducted independently for the two levels of earthquake. At one time the dynamic responses were calculated directly from the specified response spectra using modal spectral analysis and combining the modal maxima through one of the many available approximations. In this case the analyses could be performed only once since the results for the two motions would be the same except for a scaling factor (corresponding to the peak ground acceleration). More recently analyses have generally been conducted through a direct solution in the time or frequency domain generating sets of synthetic (or artificial) earthquakes to match the target response spectra. This approach was made necessary not only to increase the accuracy but also by the desire to account for soil effects (soil amplification and soil structure interaction). Because of the different levels of the motions the soil properties used will be different for the two earthquakes. In this case the results will no longer be proportional and both analyses must be carried out. For each earthquake the structure is assumed to remain linearly elastic although the load and resistance factors (or safety factors in more conventional terms) are different. It is not uncommon to find that each one of the motions controls then different parts of the design. Requiring the structure of a nuclear power plant to remain elastic or very nearly

elastic even under the maximum credible earthquake appears to be a prudent and reasonable requirement given the importance of the facility and the state of the art in the computation of damage in the inelastic range. There are, unfortunately, strong pressures to allow for inelastic behavior under the maximum earthquake. For each motion, one must consider 3 components, 2 horizontal and 1 vertical, which must be uncorrelated. The two horizontal components have the same characteristics. The vertical component has in most cases a reduction in the peak ground acceleration and sometimes changes in the shape of the response spectra (different frequency content). The analyses are often performed separately for each component and the results combined approximately as suggested in the codes (using in general the square root of the sum of the squares).

The concept of more than one level of earthquake has also been adopted as the basic philosophy in the seismic regulations of building codes. Two or three levels of earthquakes are implicitly considered. Buildings are assumed to withstand a small earthquake that could be expected to occur several times during its life without any damage; a moderate earthquake without structural damage, and a strong earthquake without collapse and loss of life. Only the moderate and large earthquake are mentioned when considering two instead of three levels. In the case of normal buildings, however, detailed seismic hazard analyses are not required and the precise meaning of the words small, moderate and strong is not provided. Although one would normally assume that the small earthquake is as defined above, the moderate earthquake is again one with a return period similar to the life of the structure and the strong earthquake would have a much longer return period, neither the structure's life not the return periods are specified. More importantly, and by opposition to the case of a nuclear power plant, only one level of excitation is required for design, without any indication as to which one of the earthquakes it corresponds, although one must assume that it is the largest. The implicit assumption would be that designing for a single level of forces, according to code provisions, all requirements will be automatically satisfied, i.e. if the structure can resist the largest earthquake without collapse, it will withstand the moderate one without structural damage. While this might be true, it has never been demonstrated. The design earthquake is characterized by the value of the base shear which is proportional to the total mass (or weight) of the building, an effective ground acceleration, a variable number of factors representing the type of structure, its importance and in some cases the soil conditions, and a seismic coefficient, function of the estimated natural period of the structure. The seismic design coefficient is derived from a smooth elastic design spectrum for 5% damping, introducing some changes and simplifications and applying reduction factors to account approximately for inelastic behavior. Analyses are performed for the same excitation in two orthogonal horizontal directions; a vertical component is not specified, and is not considered for analysis.

As important as the way in which the earthquake is defined is the location where it is specified. Seismic regulations for nuclear power plants have specified, or allow to specify, the design motions at the free surface of the soil deposit, at the outcropping of rock, at bedrock (bottom of the soil deposit), at the base of the structure, at the foundation level accounting for the excavation, or at the foundation level in the free field. The preferred, or required, location has changed through the years in a haphazard way and sometimes against all logic. The differences between the motions that would be recorded at these different locations are due to soil amplification (if the motion is specified at the foundation level in the free field, outcropping of rock or bedrock) and soil structure interaction effects (kinematic interaction if the motion is specified at the foundation level accounting for the excavation and both kinematic and inertial interaction when applied at the base of the structure). While these differences have been extensively discussed in the literature it appears that there is still confusion between the specification of the motion at the foundation level in the free field or accounting for the geometry of the excavation in the case of an embedded foundation. The transfer function from the free surface of the soil deposit to the depth of the foundation in the free field (ratio of the amplitudes of motion between the foundation level and the free surface as function of frequency) is only a function of the types of seismic waves considered and the soil properties between the two locations (assuming a horizontally stratified soil deposit as is normally the case). The transfer function from the free surface to the base of the foundation before the structure is built is a function on the other hand not only of the types of waves and the soil properties, but also of the geometry and stiffness of the foundation. It represents the kinematic interaction. Figure 1 shows the response spectra of compatible motions at the three levels (free surface, foundation level free field and foundation level with excavation) for an actual reactor building. The spectrum of the motion in the free field shows a very large reduction at the natural period of the soil layer between that level and the free surface whereas the effect of kinematic interaction is a relatively more uniform reduction in amplitude (particularly for the smaller periods).

Seismic codes for regular buildings have traditionally neglected all soil structure interaction effects specifying the motion directly at the base of the structure. Recently (since the publication of the ATC-3 recommendations in 1978) inertial interaction effects can be accounted for by considering a change in the effective natural period and an increase in damping but kinematic interaction is still ignored. This implies that for practical purposes the design earthquake is specified at the base of the structure or at the foundation level accounting for the geometry of the excavation.

2.2 Limitations

The inability of a single parameter, whether the peak ground acceleration, the effective acceleration, or any other variable, to characterize properly an earthquake and its potential effects on structures has been recognized for a very long time. Yet the use of a single parameter has withstood the attacks of time and

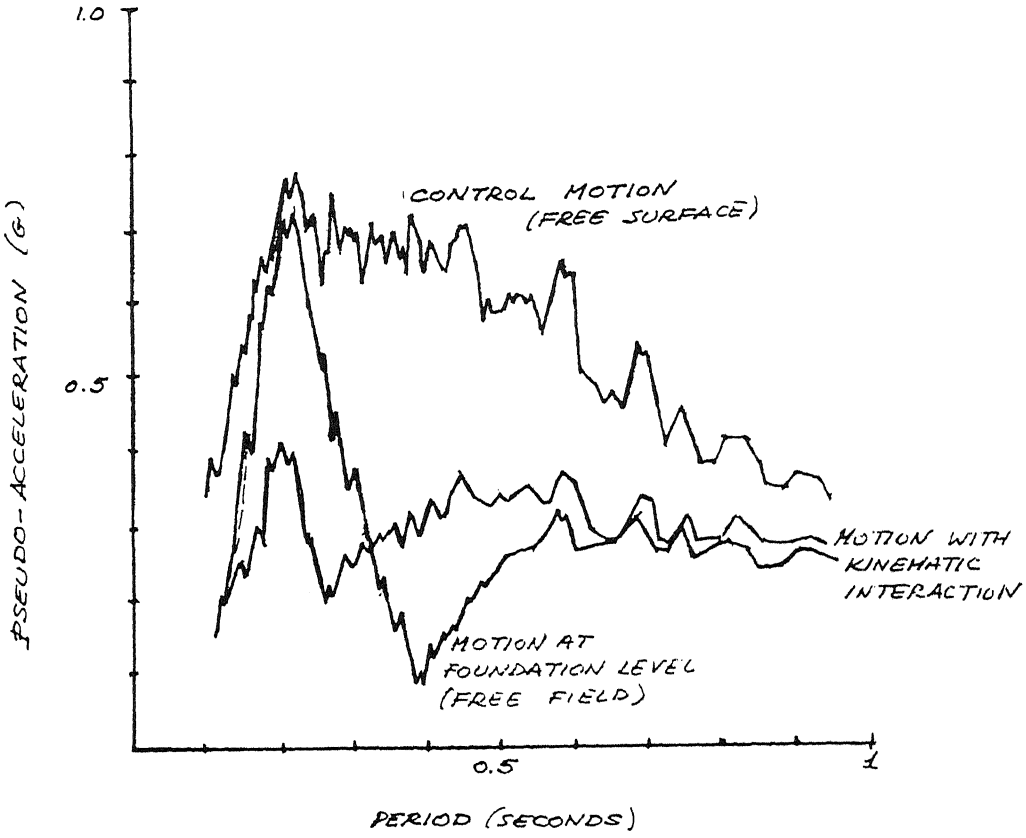


Fig. 1 Spectra of Compatible Motions at Foundation Level

the research community and continues to be accepted in seismic design. It seemed in 1967 that a more rational approach was emerging with the publication of rules to construct design spectra from the values of the peak ground acceleration, velocity and displacement suggested by Newmark. Unfortunately this trend was short lived and soon the research effort was again diverted to the search of the magic single parameter that could provide the complete information, on the assumption that the values of the response spectrum over a small range of frequencies are sufficient for design. This would be true if structures were single degree of freedom systems and were to behave elastically under the earthquake.

The characterization of the design earthquake by a peak or effective ground acceleration and standard sets of response spectra or seismic design coefficients does not account for the effect of source mechanism, magnitude, distance and overall geological and topographical conditions on the frequency content of the motions nor the duration of the strong motion (which is of great importance in the assessment of damage). It should be noticed that while the use of the three parameters suggested by Newmark provides information on the frequency content it still fails to include duration.

The definition of the earthquake by response or design spectra implies also the use of a modal spectral analysis with the resulting approximations for the combination of the modal maxima and its questionable validity in the inelastic range. The alternative is to generate synthetic accelerograms to match the target spectra using mathematical techniques with little physical basis. While these artificial earthquakes may be acceptable for linear analyses they tend to contain far more energy than real motions and their use for nonlinear analyses to compute damage would be inappropriate.

The potential importance of the vertical component of the earthquake on the seismic response of a structure has been a recurring, cyclic theme in the literature. It is generally accepted that vertical effects are not very important, and certainly much less so than those due to the horizontal components, when considering the overall structural behavior. The arguments are that the vertical ground accelerations are normally smaller than the horizontal ones (which is true at some distance from the causative fault), that the

buildings are very stiff in vertical vibration, and that they are designed for important gravity loads with good margins of safety. In nuclear reactor buildings, the vertical earthquake component has been found important in the response of relatively flexible floor systems, of interest for the design of equipment supported at those levels. Papaleontiou (1992) conducted a series of studies on a 4x3, a 10x3, a 16x3, and a 20x3 frame (number of stories by number of bays). He found that in the linear range their fundamental vertical frequency, accounting for the masses of the slabs and of the live load between frames, were of the order of 4 to 6 Hz. They could be higher for actual buildings when including walls or nonstructural components with large axial stiffness. There are a number of earthquakes for which the vertical component of motion can have a considerable amount of its energy in this frequency range. This is the case for instance of the Capitola record of the Loma Prieta earthquake. The results of Papaleontiou's studies, using distributed masses for all members with a continuous solution in the frequency domain shows that the effects of the vertical earthquake component may become non-negligible in some cases and particularly for the shorter, less slender buildings and their top columns. Figure 2 shows the dynamic axial forces in the top outside column of the 4x3 frame considering both components of motion or only one. It can be seen that the main contribution comes from the vertical component. Figure 3 shows the dynamic axial forces in the outside bottom column. The contribution of the vertical motion is still important and larger at times than that of the horizontal component but not as predominant as for the top column. It should be noticed that these results do not include the static axial forces due to gravity loads (dead weight and live loads). The static forces are of the same order as the dynamic ones. Figure 4 shows the dynamic bending moment at the top of the top column. Although the contribution of the horizontal motion is the largest that of the vertical component (vertical vibrations of the beams) is not negligible. The contribution of the vertical earthquake to the moments in the bottom columns was negligible in all cases.

2.3 Uncertainties and possible improvements

The largest uncertainties involved in the seismic design of structures are associated with the prediction of the potential earthquakes that may affect a specific site during a fixed period of time and their characteristics (duration, frequency content, maximum acceleration, types of waves). There are uncertainties in the possible existence of unknown faults, inactive for many years but potentially active, the level of activity along known faults, the attenuation laws for acceleration, velocity and displacement accounting for the topography and geological conditions between the fault and the site, and the types of waves, which arrive at the site, their amplitude and frequency content. Much progress has been achieved in these areas during the last years through seismologic research but much more remains to be done. Of particular significance is the work done on the simulation of earthquakes using physical models, following the propagation of the rupture along a fault and the paths of the stress waves which are generated. This approach when combined with the statistical analysis of actual records and random vibration models, for validation and tuning purposes, can provide a substantial amount of new knowledge and understanding. It is interesting to notice that Housner had already proposed a simple physical model in the 50's but this line of research was not pursued and researchers preferred to concentrate in the development of stochastic models without any physical basis. An attempt by Rascon and Cornell to revive interest in physical models in 1968 was also isolated and it was only in the late 70's that work along these lines was pursued on a broader front.

The many uncertainties associated with the definition of the seismic input suggest immediately the need to use a probabilistic formulation. There has been a considerable amount of research performed during the last 25 or 30 years on the development of probabilistic methodologies for the prediction of the seismic hazard as well as for full seismic risk analyses. These methodologies provide us with a powerful tool to estimate the effect of the uncertainties on the reliability of our calculations and to assess the relative importance of the various uncertainties (much as traditional sensitivity analyses would). It must be realized on the other hand that the results of reliability studies are not exact and provide mostly estimates. The application of the theory of probabilities cannot eliminate or reduce uncertainties but just provide very valuable information on their significance. Uncertainties can only be reduced through a better understanding and modelling of the actual physical processes.

Present procedures would be improved by defining the design motions by more than a single parameter, including duration, frequency content and ideally types of waves. When performing sophisticated seismicity studies the result should not be a single value of the peak ground acceleration but rather a set of possible motions generated from physical models or records of actual earthquakes. The use of more than one motion is particularly important when there are various possible sources which can lead to earthquakes with different magnitudes, frequency contents, duration, etc., and it is not clear which one might control the different aspects of the design. Motions should be specified preferably at an outcropping of rock (real or hypothetical) performing then the needed analyses to account for the specific soil conditions at the site as well as kinematic interaction effects if so desired. It would also be preferable to specify the earthquake characteristics in probabilistic terms; the use of power spectra instead of the standard response spectra would be for instance an improvement within the present format if one is going to generate artificial earthquakes.

It is clearly not possible to conduct for each regular building the same type of detailed seismological studies required for nuclear power plants. It would be possible, however, when conducting seismic zonation or microzonation studies for a region or a metropolitan area to specify the characteristics of the design motions (and preferably more than one) with some of the improvements mentioned above or at least with more details to account for the local seismicity than can be provided in general codes.

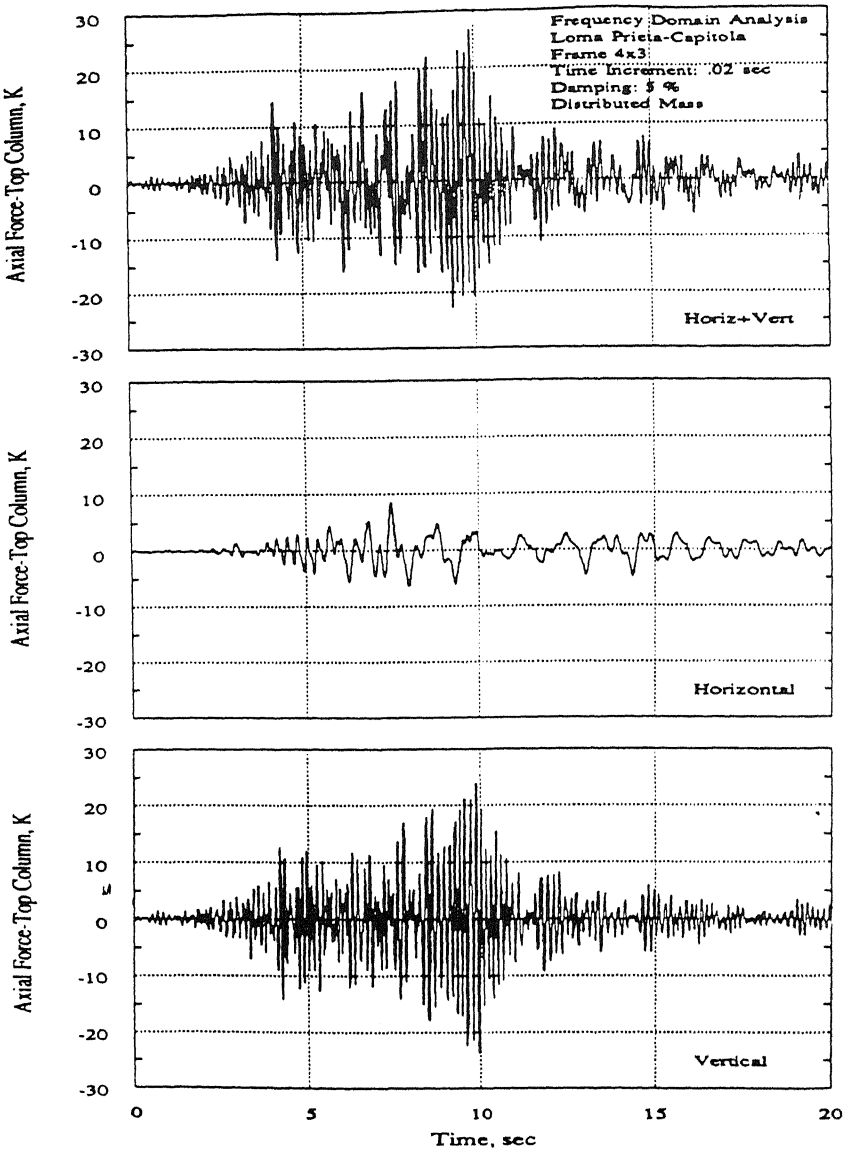


Fig. 2 Effect of Earthquake Component on Axial Force in Top Column - Frame 4x3

3. EFFECT OF SOIL CONDITIONS

3.1 Present Procedures

The effect of local soil conditions on the characteristics of the seismic motions that can be expected at a particular site has been a subject of considerable interest and research for over 30 years. It has also been at times a controversial subject. It is now recognized, however, that soil properties are one of the many factors that can and will influence the frequency content and the amplitude of an earthquake motion. The most clear example of this effect has always been the valley of Mexico City where the earthquakes have a long duration and a very predominant period, corresponding to the natural period of the soil at each site in the lake zone. There is, however, an increasing body of evidence of important soil effects at many other sites.

Seismic regulations for nuclear power plants use a set of standard response spectra which represent basically an envelope of the spectra for motions recorded at a large number of sites with very different soil conditions, although they would not include such extreme cases as the lake zone of Mexico City. When using these spectra for sites with average soil conditions, one must therefore assume that the soil amplification effects are already included and the

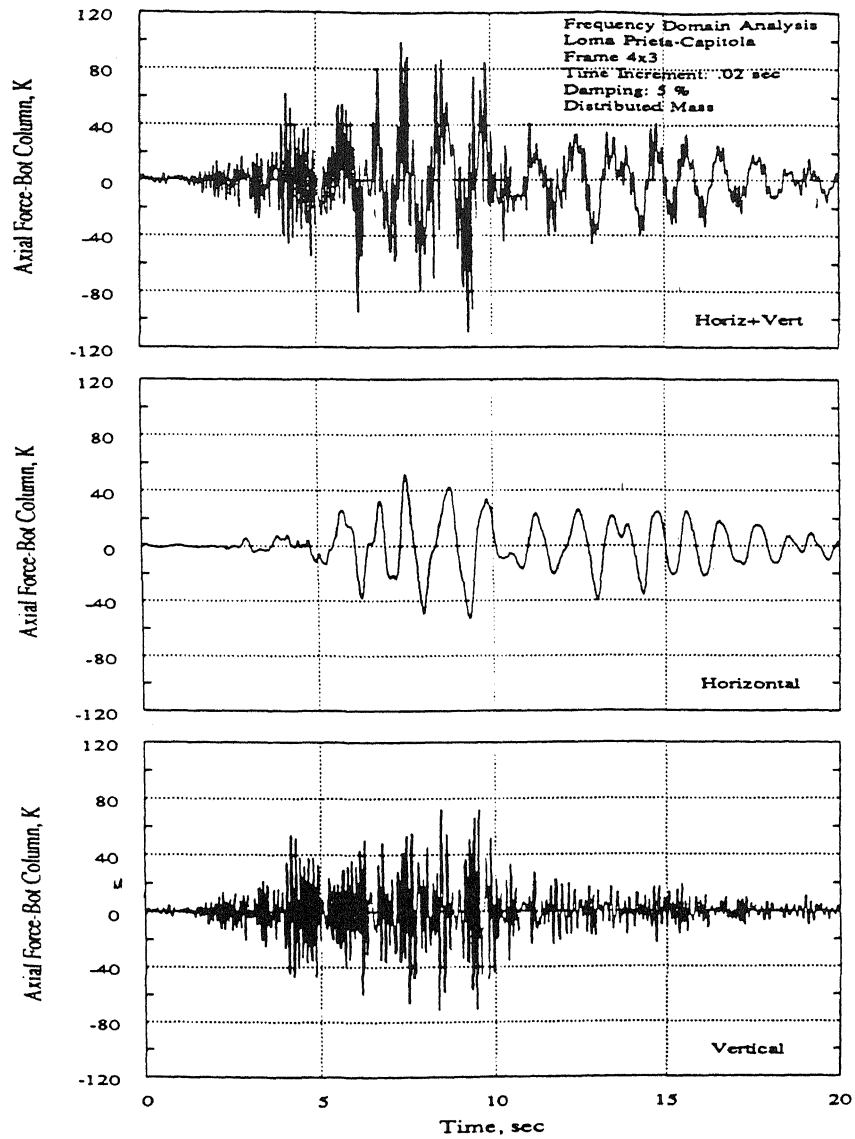


Fig. 3 Effect of Earthquake Component on Axial Force in Bottom Column - Frame 4x3

motion must be considered to be specified at the free surface of the soil deposit. Even so, it is normally necessary to conduct inverse amplification studies, determining compatible motions at various depths in the free field given the motion at the free surface in order to apply these motions at the base of a finite element model of the soil for some soil structure interaction analyses, or just to find soil properties consistent with the expected levels of strain when attempting to account approximately for nonlinear soil behavior. This is often referred to as a deconvolution. There are many cases, nevertheless, where soil amplification or convolution analyses are performed in order to account for the effect of local soil conditions.

If these studies are carried out starting with motions corresponding to the standard spectra, amplification effects may be included twice; for these purposes one should use motions with the characteristics appropriate for rock outcropping. Convolution and deconvolution analyses are normally carried out using one dimensional models (horizontally stratified soil deposits where properties may change with depth but remain constant in horizontal planes) and assuming vertically propagating shear waves for horizontal motions and P waves for the vertical components. Moreover, the analyses are in most cases linear and nonlinear soil effects are simulated performing iterations, changing the soil moduli and damping

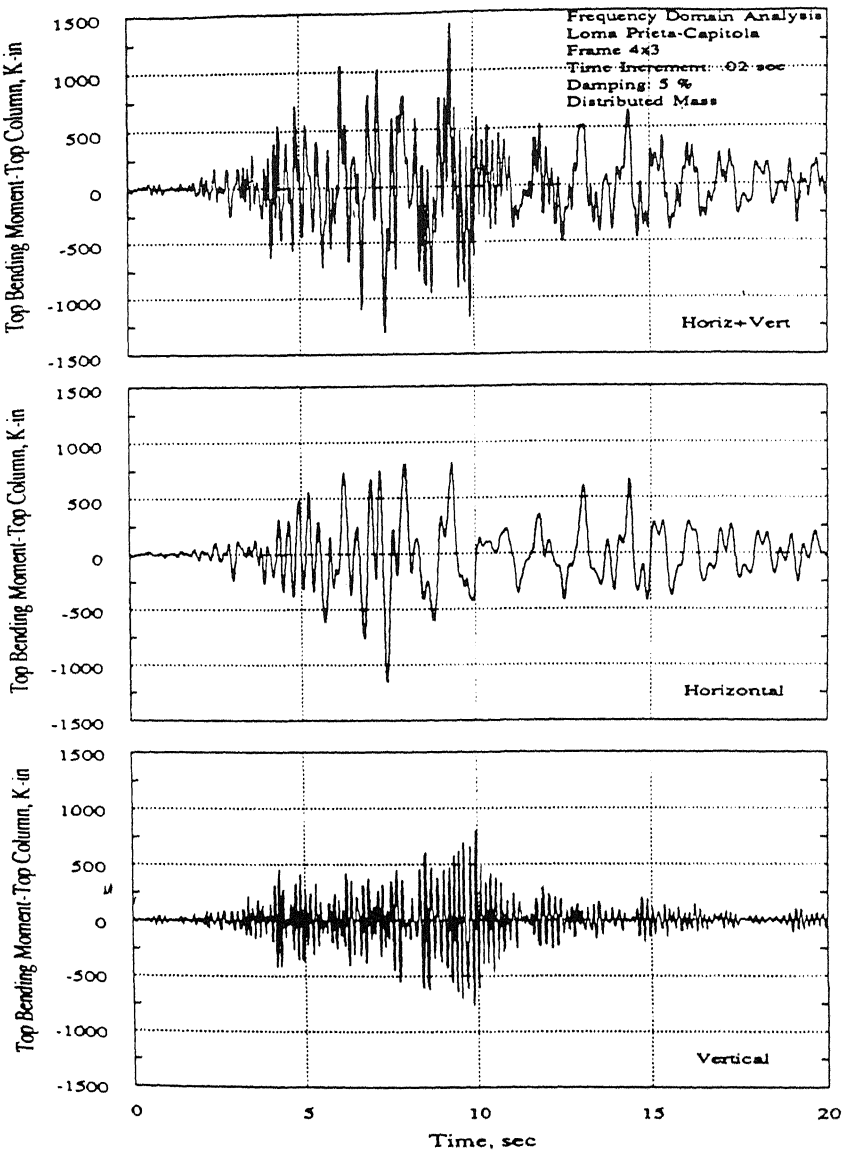


Fig. 4 Effect of Earthquake Component on Top Bending Moment in Top Column - Frame 4x3

values until they are consistent with the computed levels of strain (defined by a single value of a characteristic strain for each soil layer).

Modern seismic codes for buildings try to incorporate soil effects by specifying more than one seismic design coefficient (equivalent to the response spectrum with some simplifications). The ATC-3 recommendations suggested a spectral shape for rock, a second one for firm soils and two additional ones for deep and soft deposits, depending on the level of accelerations. While there are various important factors which are ignored in this approach, it represents a clear improvement over previous codes and it attempts to reproduce correctly some basic trends.

3.2 Limitations

While the procedure recommended in the ATC-3 is a trend in the right direction, it ignores the effect of magnitude and distance on the amplitude and frequency content of the incoming waves which will then be modulated by the soil deposits. The spectral shapes recommended were based on a number of parametric studies and primarily on the observation of

a number of actual records classified according to soil types. A careful inspection of these records shows that in some cases they corresponded all to a single earthquake and similar epicentral distances. More recent studies conducted by Mohraz and presented at this Conference illustrate clearly the effect of magnitude and distance on the motions recorded on each soil type.

Amplification or deconvolution studies are based on one dimensional solutions for vertically propagating waves, although the analytical solutions for other types of waves at arbitrary angles of incidence have been known for a long time. Figure 5 shows typical amplification functions (magnitude of the transfer functions from outcropping of rock to the free surface of a soil layer) for SH waves propagating at various angles of incidence in the rock (90º, representing vertical incidence, 60º and 30º). In all cases, the waves propagate nearly vertically in the soil. The shapes of the amplification functions are relatively similar, the main difference being in the magnitude of the peaks (at the natural frequencies of the soil layer). It should be noticed that given one of these curves and assuming it corresponded to actual recorded data it would always be possible to get a match of the first peak considering vertically propagating waves by changing the value of the soil damping although the other peaks would not match. This points out the difficulty in trying to obtain values of internal soil damping from records of motions obtained at different depths, without knowing the angles of the waves, and the inadequacy of proofs of validity of the assumption of vertically incidence in the rock without knowing the soil damping. Figures 6,7, and 8 show the amplification curves corresponding to combinations of SV and P waves at different angles of incidence in the rock and for different relative amplitudes (amplitude of the incoming SV wave 3 times, 2 times and equal to that of the P wave). For the first 2 cases the curves are still relatively similar to those for SH waves although some coupling between the waves can be observed at the natural frequencies of the layer in dilatation or compression (the first one in between the first and second natural frequencies in shear). The angle of incidence affects again more the first peak than the following ones so that in general the predominance of the first shear frequency of the layer is less pronounced. Experimental transfer functions exhibit often this type of behavior. When the amplitudes of the SV and the P waves are equal (a case which may not be realistic in most instances) the maximum amplification may be associated with higher modes and coupling effects become very clear.

A second point of concern is the use of frequency independent linear hysteretic damping. While the internal damping in soils due to inelastic behavior is frequency independent it is however a function of strain amplitude and the strains associated with different frequency components may be quite different. Normal amplification studies using constant linear hysteretic damping filter out high frequencies as the waves travel upward through the soil. For deep soil deposits the motions computed at the free surface may not have any energy for frequencies above 8 or 10 Hz. This of course is not consistent with recorded data. When the same model is used to conduct deconvolution analyses the high frequency content will increase steadily with depth. Eventually the amplitudes of the motion may become extremely large and the solution become unstable. Figure 9 shows for instance the variation of the amplitude with depth for a half space with 5% hysteretic damping, whereas figure 10 shows the corresponding results for 10% damping. Clearly these results are illogical. They are caused by the use of the constant linear hysteretic damping as a means to simulate nonlinear soil behavior. When

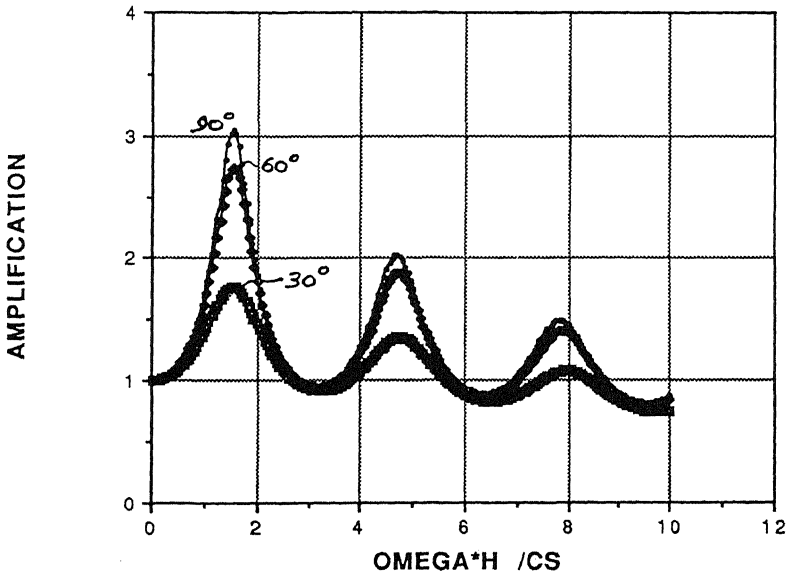


Fig. 5 Effect of Angle of Incidence on Amplification of SH Waves

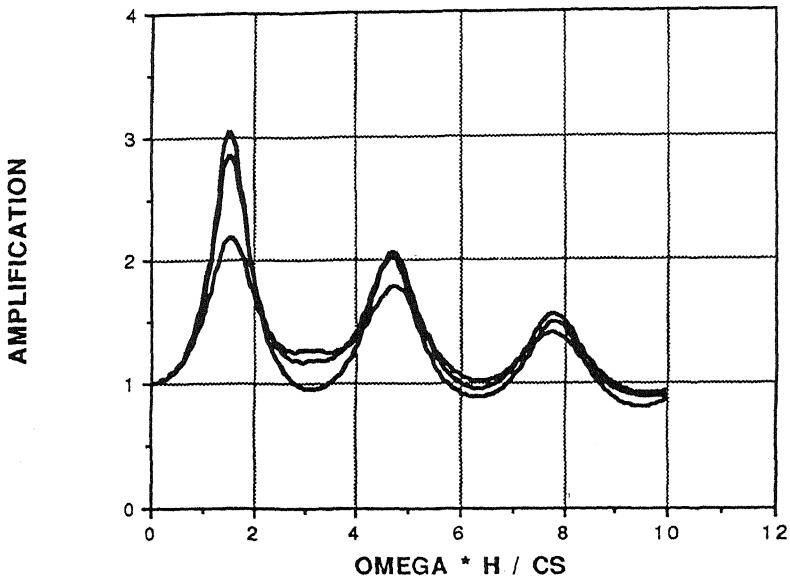


Fig. 6 Effect of Angle of Incidence on Amplification of SV and P Waves ($A_{SV}=3A_P$)

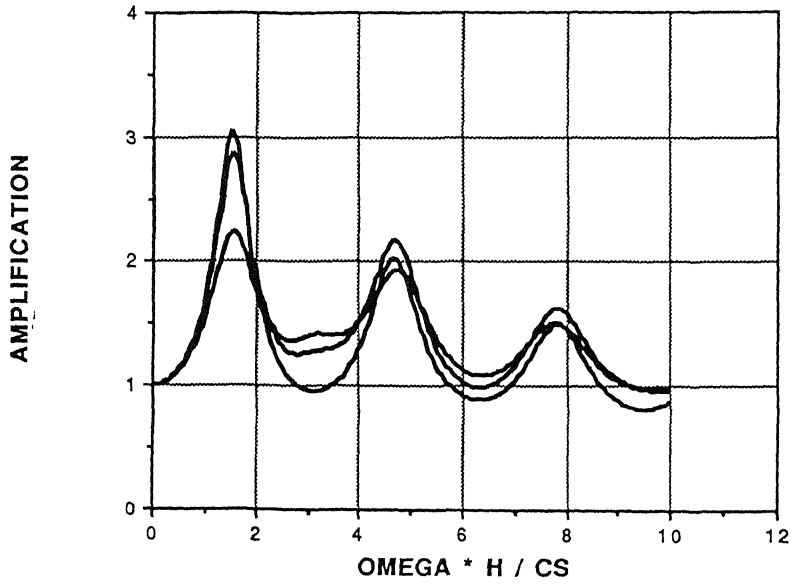


Fig. 7 Effect of Angle of Incidence on Amplifications of SV and P Waves ($A_{SV}=2A_P$)

using this procedure it is necessary at times to suppress any components with frequencies above 8 or 10 Hz from the surface motions, an adjustment which is illogical and inconsistent with other requirements in seismic regulations. These problems with high frequencies do not occur when performing actual nonlinear analyses.

When dealing with soil deposits which are not too deep the iterative linear analyses conducted in practice to simulate nonlinear soil behavior produce results which are qualitatively reasonable. Constantopoulos (1973) found that the peak accelerations at the free surface tended to be overpredicted, in comparison to true nonlinear analyses with consistent soil models, by as much as 20%, which is not too much. Relative displacements and strains, on the other hand, could be underestimated by 50%. Use of the procedure to determine strains

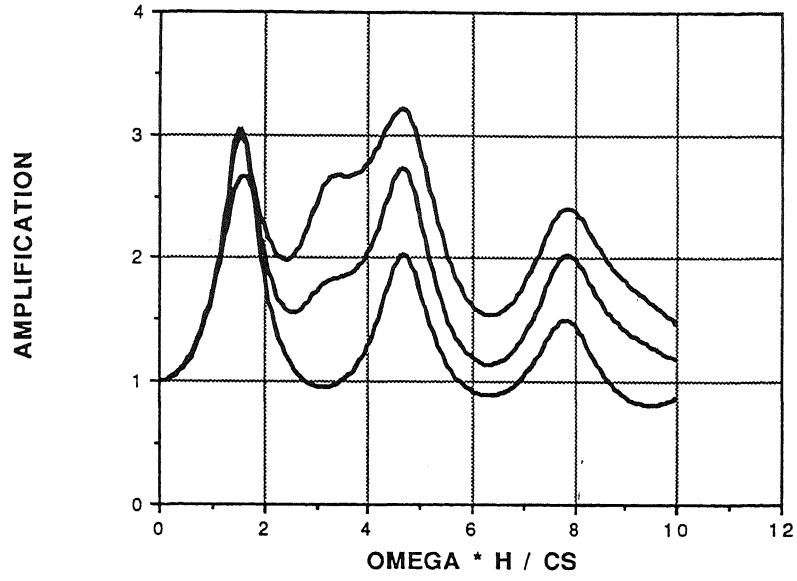


Fig. 8 Effect of Angle of Incidence on Amplifications of SV and P Waves ($A_{SV}=A_P$)

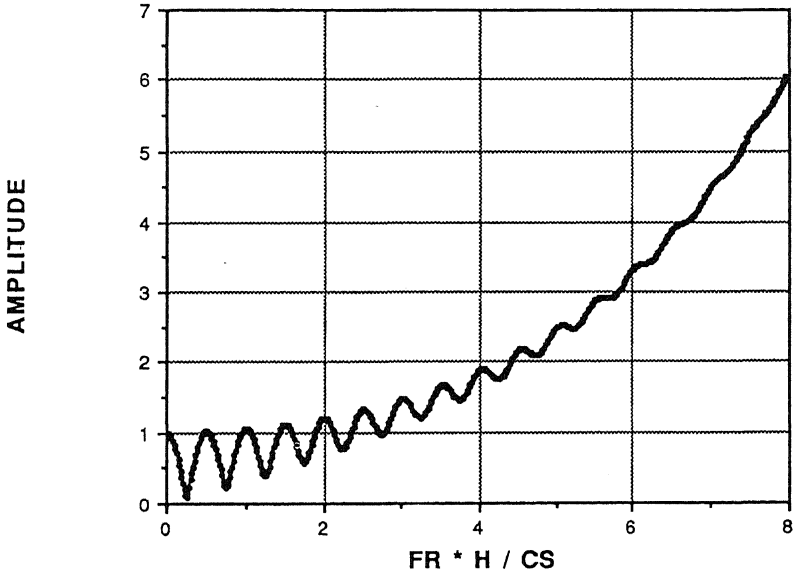


Fig. 9 Variation of Amplitude with Depth. Deconvolution. D=0.05

cannot be recommended and the inaccuracies would increase with increasing level of excitation (and nonlinearities) or increasing depth of the deposit. It is important to realize in this context that linear analyses, as conducted in the iterative approach or as associated with more sophisticated and laborious equivalent linearization techniques, may be able to predict approximately the value of one response quantity but can never reproduce accurately all the features of a nonlinear response. Figure 11 shows the amplification functions for a soil layer using linear analyses with the soil properties that may correspond to two different levels of excitation (peak accelerations of 0.05 and 0.35 g). There is clearly a shift in the fundamental period of the stratum (an elongation) as the intensity of the motion increases, as well as a reduction in the peak amplification due to the increase in internal damping, which is what one would expect qualitatively. A

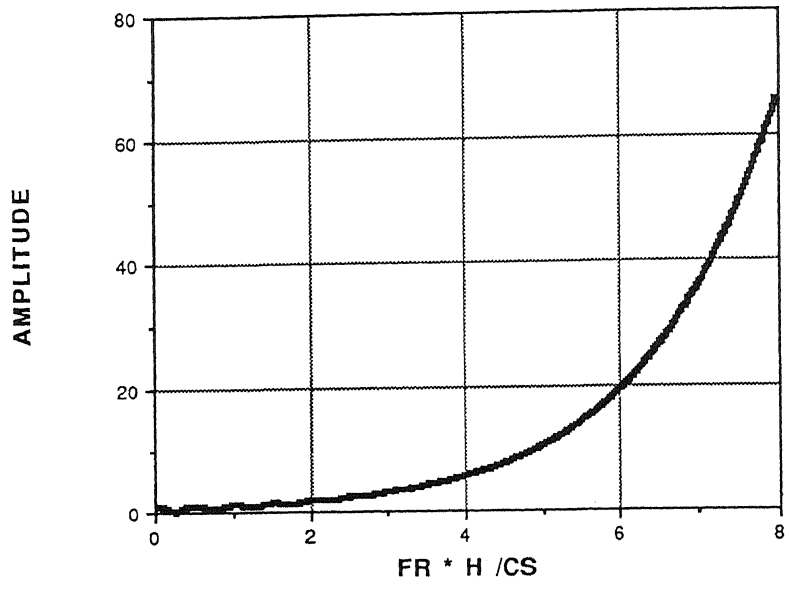


Fig. 10 Variation of Amplitude with Depth. Deconvolution. D=0.10

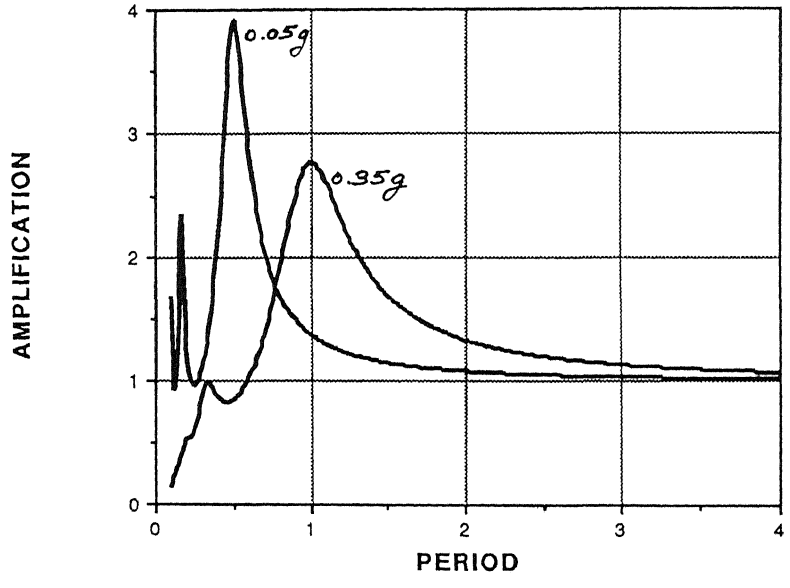


Fig. 11 Amplification Functions for Different Earthquake Levels

nonlinear system does not have, however, a constant natural frequency during the complete cycle of vibration, and therefore the motions resulting from a true nonlinear analysis should have a broader frequency content than those of the linear model. The importance of the soil nonlinearity and even the existence of nonlinear behavior have been the subject of great controversy for several years with a number of people, and seismologists in particular, negating them.

The fact that soils exhibit nonlinear force deformation characteristics with a much smaller linear range than other civil engineering materials is a physical property that can be easily verified through laboratory tests and cannot be denied. It is true on the other hand that the nonlinearity of clays, and particularly very plastic clays, was badly exaggerated in the past (the curves providing variation of modulus versus shear strain for clays showed nonlinear behavior starting at levels of

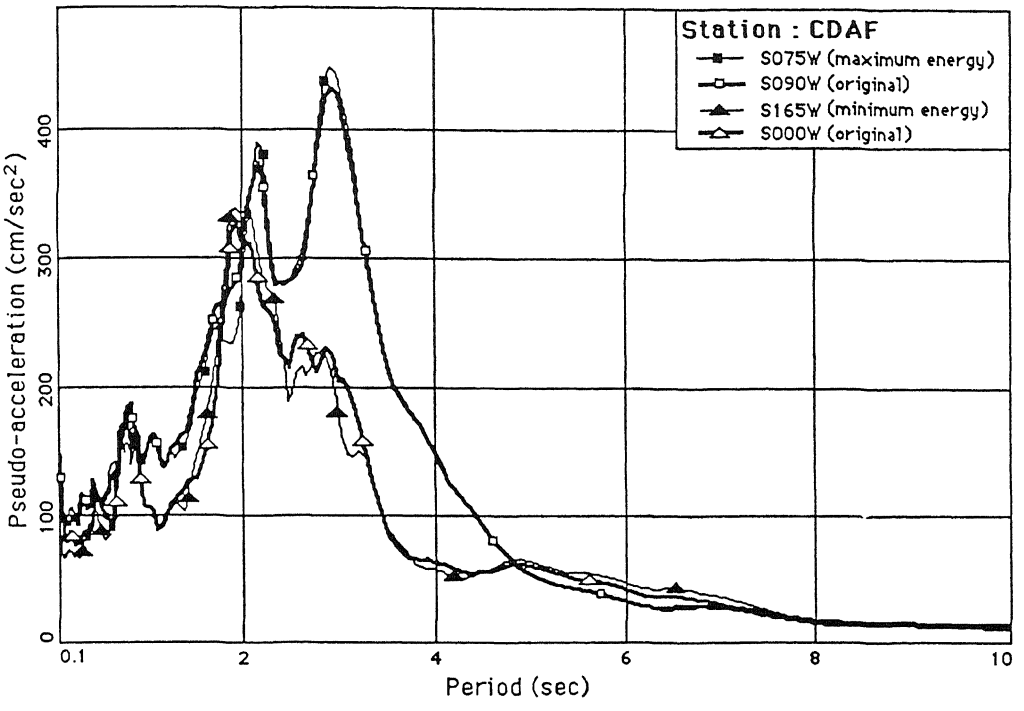


Fig. 12 Response Spectra - 1985 Mexico City Earthquake - Station CDAF

strain one order of magnitude smaller than the actual one). And it is also true that the effects of nonlinear soil behavior will not be identifiable if one averages the amplification values over windows with a width of 1 second and if changes in the value of the amplification by a factor of 2 or less are not considered significant. Yet changes in the frequency at which the peak amplification occurs or in the value of the peak by 50 or 100% are significant from a design point of view.

It should be noticed finally that the use of different spectral shapes or soil amplification studies will only account for the effects of the soil on the characteristics of the seismic motions assuming that it maintains its integrity. The possibility of large soil displacements, liquefaction, slides and foundation failures, or excessive settlements is not considered in the preceding discussion but must be investigated.

3.3 Uncertainties and possible improvements

The main uncertainties involved in this part of the seismic analyses are related to the seismic input, the topography of the site and the soil properties. As discussed above the amplification functions are affected by the types of waves, their relative amplitudes as a function of frequency and their angles of incidence. In fact, while the motions in two orthogonal directions at a given site are somewhat similar, they are far from identical in their frequency content, and in some cases they can exhibit significant variations. Figure 12 shows the response spectra for the 2 components of motion recorded at the CDAF in Mexico City during the 1985 earthquake as well as the spectra corresponding to the motions in the two directions of maximum and minimum energy. These last two are very similar to the other two. It can be clearly seen that the S090 W component exhibits a peak in the response spectrum at a period of 3 seconds which is not apparent in the S000 W record.

Soil deposits are rarely horizontally layered as assumed in the amplification analyses. During recent years there has been an important amount of research work on the study of amplification effects by 2 dimensional geometries (rectangular valleys, deep triangular valleys or triangular hills) and in a few cases three dimensional topographies and valuable results have been obtained. There is little known on the other hand on the effect of slanting soil layers on a deposit which is nearly horizontally stratified or irregular interfaces between layers. While the methodology to solve these problems is known the number of parameters involved in the definition of arbitrary geometries is very large making general studies, by opposition to the solution for a specific case, difficult. Perhaps more importantly the possibility of knowing accurately the soil properties over an extended area is remote.

The soil properties in the field are the second largest source of uncertainties. To conduct proper amplification (or soil structure interaction) analyses it is necessary to know the properties in situ for very low levels of strain (what would be the linear elastic range for construction materials) and the variation of these properties with levels of strain. These properties are normally determined through laboratory tests. The values of the shear modulus at very low levels of strain computed in the laboratory are usually different from those of the material in situ and the differences can be substantial (factors of 2 or more). There are a number of correction factors which can be applied to the laboratory tests to extrapolate the results to the field. It is better, however, to determine the properties at low levels of strain directly at the site using geophysical methods which are now well established and reliable. The curves providing the variation of shear modulus with level of shear strain, computed in the laboratory, must then be expressed in dimensionless form using the ratio of the modulus at any strain to that at low strains.

Considering all of the above uncertainties, the use of amplification studies for average type soils is hard to justify and should be reserved perhaps to cases where soils are clearly out of the ordinary. Even so there are a number of improvements which can be introduced in the present procedures. Some of the uncertainties like those associated with the soil properties and their variation both in the vertical and horizontal directions can be eliminated or at least reduced by making careful determinations in situ and in the laboratory. It would be obviously desirable to be able to estimate some nonlinear properties (for larger level of strains) also in the field to validate the extrapolation of the laboratory tests and this may be possible in the near future. It would also be a substantial improvement to perform true nonlinear analyses when there are going to be substantial nonlinearities rather than relying on the iterative linear solutions. For one dimensional conditions this can be easily accomplished. For two or three dimensional states of stress it would be necessary, on the other hand, to define appropriately nonlinear constitutive models. Depending on the particular model used the number of parameters which must be determined to fully characterize soil behavior may be very large. For sites with clearly two or three dimensional geometries (deep valleys or hills) some of the new techniques recently developed should be applied either for research purposes, to conduct a larger number of parametric studies and to derive approximate formulae or procedures, or for specific studies in the case of an important structure or a sufficiently large number of similar structures.

Other uncertainties, like those associated with the types of waves which constitute the earthquake are harder to solve, but they may be reduced as additional knowledge is obtained.

4 STRUCTURAL MODELS

4.1 Present procedures - Stiffness

Although there has been a considerable amount of research on single degree of freedom systems which has served as a basis for a large part of the code provisions, and while the computation of the dynamic effects is often based purely on these studies, the determination of the forces in the different components of the structure is normally performed using plane frames, frames coupled with shear walls (where the walls are modelled as deep columns) or coupled shear walls. Even computer codes intended for the study of complete buildings assume that each frame or wall in the structure behaves as a plane element and that the slabs are rigid diaphragms in their own plane. True three dimensional effects are normally neglected. Nuclear reactor buildings have extremely complicated structures for which it is hard to establish equivalent plane frames. The models used for the seismic analyses of nuclear power plants can change widely in sophistication from relatively simple stick models (close-coupled systems) with lumped masses, to complex and detailed finite element models.

Establishing the structural model requires the assembly of a stiffness matrix in all cases and mass and damping matrices if a dynamic analysis is to be performed. The derivation of the stiffness matrix for a frame or combinations of frames and walls is straightforward yet the computer programs developed for seismic structural analyses have a number of variations depending on the details or additional effects which may be included. Among these, for instance, the finite joint size, joint flexibility, effect of axial forces on the lateral stiffness of each member (P-Delta effects), effect of shear deformation, etc.. Joint size, or member depth, effects are important when dealing with very deep members such as walls considered as columns and coupled to frames or other walls, or deep spandrel beams. While there have been a number of procedures suggested in the literature to account for the depth of columns (or walls) by considering the beams as members with 3 segments, each with a different depth, the selection of the dimensions of the external segments, within the columns, is arbitrary and the model may lead to numerical errors or even instabilities when the stiffness of the side segments is too large. A much simpler solution is to consider that the free span of the beam is the distance between the column faces, rather than between their centroidal axes, and to consider a rigid link connection between the column face and the joint at the axis. A similar model can be used to account for deep girders considering now vertical links and changing the length of the columns. The net result of these changes is to increase the stiffness of the frame. Member forces will also change since they are computed at the faces of the columns or girders rather than at the joints (intersection of the centroidal axes of the members). Joints are not, however, infinitely rigid and will deform under the internal forces coming from the connecting members. This will tend to make the structure more flexible, counteracting to a variable extent the previous effect. Finally the beams, or columns, on opposite sides of a joint may not have the same dimensions and their centroidal axes may not coincide. The fact that the member axes will not intersect at one point can be taken into account by introducing eccentricities between the selected joint and the member ends. The modification of the conventional formulation where all members are assumed to be straight lines (very small depth) and

they all intersect at one point to include the three effects above mentioned is relatively simple, yet many computer programs will include none or only one of these effects.

The effect of axial forces on the lateral stiffness of the members (or the effect of the change in geometry due to the deformations on the equilibrium of the forces) can be taken into account by a variety of methods. For very flexible structures which may experience very large displacements (in particular, large rotations), and, as a result, substantial changes in their geometry, true geometrically nonlinear analyses can be performed using an iterative procedure to converge to the exact solution. For most building frames this complexity is unnecessary. Use of stability functions, which depend on the axial force, to compute the actual stiffness coefficients of each member is a very simple and accurate solution. For frames with more than one or two stories, one can alternatively use the P-Delta approach, either computing the stiffness coefficients with a simplified form of the stability functions or adding to the right hand side of the equations "equivalent" force terms resulting from the unbalance of the internal forces in the deformed position. The main effect of the axial forces is to decrease the lateral stiffness of the frame since the net forces, when including gravity loads, will be compressive for the majority of the columns, if not all. Under static loads this will result in increased lateral displacements and member forces when there are horizontal loads. Under dynamic loads on the other hand, the reduction in stiffness will result in an increase in the natural period (decrease in the natural frequency). Under a real, specific, earthquake an increase in the natural period may lead to an increase or a decrease in displacements or forces. When dealing with a smooth design spectrum it will result for most buildings in an increase in displacements proportional to the increase in the period (thus inversely proportional to the square root of the stiffness rather than to the stiffness itself) but a decrease in dynamic forces. P-Delta effects under seismic excitation are therefore more complex than under static lateral loads and should not be predicted from static considerations. The modifications needed to include these effects in the analysis are again relatively minor and thus should be included in all computer programs. Some programs will include P-Delta effects only for the static gravity loads, neglecting the effect of the dynamic axial forces. While this may provide a reasonable approximation for global measures of response it will not be able to reproduce properly the distribution of forces among the different columns.

There appears to be a widespread impression that P-Delta effects are extremely important in the seismic response and potential failure of structures. This impression is the result of studies on one bay - one story frames where the yielding is forced to occur in the columns (top and bottom simultaneously). These frames are modelled as single degree of freedom systems with an elastic - perfectly plastic force deformation relationship without vertical forces and a descending second branch (negative tangent stiffness) when P-Delta effects are included. For large values of the axial load the second slope may be very large (negative). These systems will experience a pronounced plastic drift. As a result, P-Delta effects have been blamed at times for permanent plastic deformations that would occur anyway in a nonlinear system even without these effects. Dynamic analyses of realistic frames with logical values of the axial forces in the columns and small amounts of strain hardening show on the other hand increases in displacements and ductilities of the order of 10 to 20% at most, and even reductions in some cases. Figures 13 and 14 show results obtained by Aziz (1976) for a 10 story steel frame studied by Anderson (1969) and subjected to the NS component of the 1940 El Centro earthquake. These results and the magnitude of the P-Delta effects are representative of those obtained in a variety of studies. Boubenider (1988) obtained similar or smaller effects for buildings typical of the construction in Mexico City subjected to the 1985 earthquake with and without soil structure interaction effects. Thus, while P-Delta effects should be included in the analyses, their potential importance should not be exaggerated. Procedures which have been suggested to estimate P-Delta effects from linear static analysis by assuming that the structure is in the position corresponding to the maximum dynamic displacements are likely to yield unrealistic results.

The effect of shear deformations on the stiffness of the different elements can be important when dealing with short and deep members as in low rise shear wall buildings. There is no reason, however, not to incorporate it automatically for all members in structural analysis programs since it represents again a very simple modification and does not imply any increase in the cost of the computations.

4.2 Present procedures - Nonlinear models

While seismic codes do not require nonlinear analyses (whether static or dynamic) these are used extensively for research purposes and increasingly for the design of important buildings. Research studies have used extensively single degree of freedom systems with variable force deformation relationships (elastic - perfectly plastic, bilinear, trilinear, models with cyclic stiffness degradation, etc.) to arrive at the simple rules used to construct inelastic response spectra. The applicability of these rules when dealing with multidegree of freedom systems has sometimes been investigated by using close-coupled, lumped mass and spring, shear beam type models in which the masses are concentrated at the floor levels and the springs connecting them, representing floor stiffnesses, have the force deformation characteristics corresponding to interstory displacements. This model would be appropriate for frames with infinitely stiff, unyielding, girders but its accuracy deteriorates severely as yielding occurs in the girders, as the designer expects. As a result the ductilities predicted by this model may not reproduce those that would be expected in actual buildings.

The models most commonly used for research and practice are again plane frame models where inelastic behavior is concentrated at specified points (sections), which are normally the member ends but could also be intermediate sections. They are sometimes referred to as point hinge models. Two main alternatives are used in this idealization. The

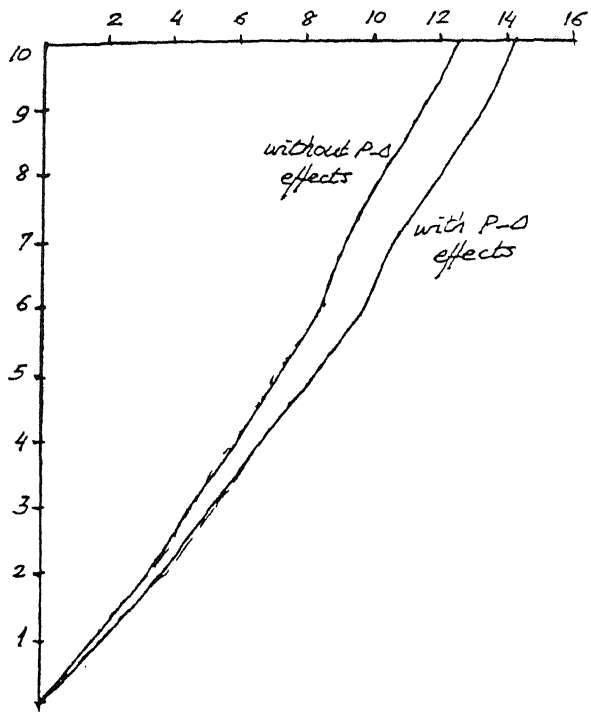


Fig. 13 P-Δ Effects on Maximum Displacements - 10-Story Frame

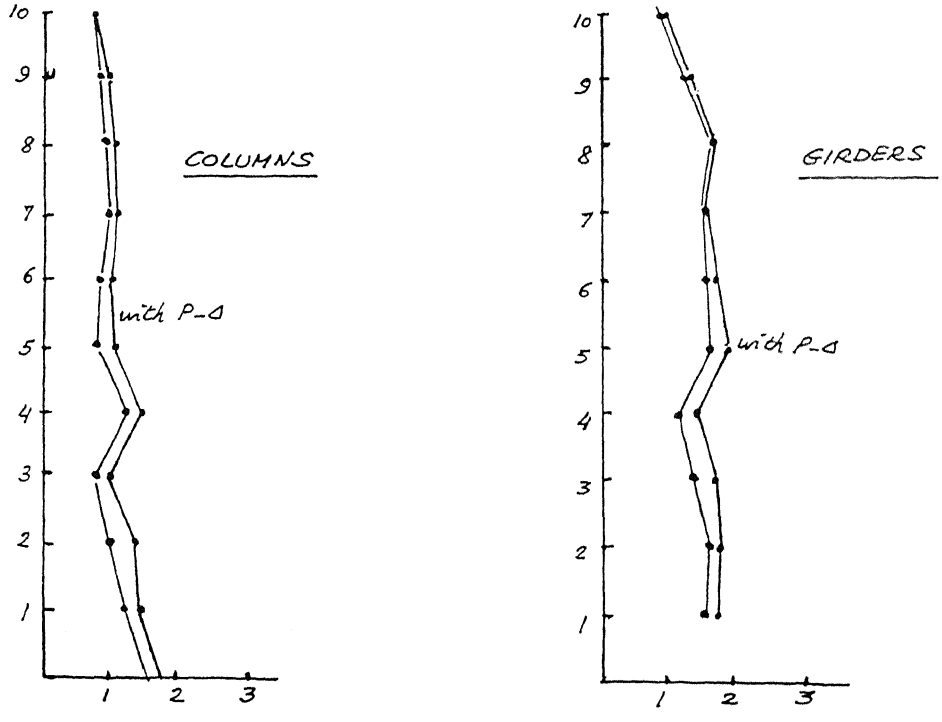


Fig. 14 P-Δ Effects on Ductilities

first is a dual component model where each member is assumed to consist of two elements in parallel, one fully elastic and the other with plastic hinges forming at the selected sections when the moment reaches a certain value which may be constant or a function of the axial load through an interaction diagram. The plastic hinge disappears when there is a reversal in the deformation or when the combination of the axial force and the bending moment fall inside the interaction diagram. The second is a single component model where the member is assumed to remain always elastic but nonlinear springs are placed at the connections between the members and the joints. The dual component model was proposed by Clough and Benuska in 1966, while the single component model is usually attributed to Giberson (1969). In some cases both models are combined using the end springs, in addition to the two elements to simulate slippage of the reinforcing bars at the joints.

Point hinge models cannot account for the spreading of yielding along the member or for the coupling between axial and bending effects that takes place once inelastic behavior starts, due to the shifting in the neutral axis. Incremental axial forces are then no longer a function only of the increment in axial deformations but depend also on the increments of curvature; incremental moments are equally a function of both axial and curvature increments. This coupling is directly related to the phenomenon of alternating plasticity, well known in the static analysis of steel members under cyclic loading. When a steel section is subjected to a constant axial load and the bending moment is cycled causing yielding on one side of the section and then on the other, the axial strain at the centroidal axis will increase continuously, while the curvature changes between fixed limits. This is shown in Figures 15 and 16 for a simplified wide flange section in which the stresses in the flanges are assumed to be constant and the stresses on the web are neglected. If, on the other hand, the axial strain were kept constant and the moment (or curvature) cycled, the axial force on the section would change. The spreading of yielding and the axial-bending coupling are accounted for automatically in fiber models in which the time history of stresses and strains is monitored at fibers across selected sections of each member and the nonlinear stiffness matrix (tangent or secant) of the member is assembled at each time or load step from the sectional properties. This model was used by LaTona (1970), Adams (1973), Mark (1974), and Papaleontiou (1992). A variation of this formulation is provided by one dimensional finite element models in which the stiffnesses are monitored at the sections corresponding to the Gaussian quadrature points. These sections can again be divided into fibers or, alternatively, the stiffnesses are computed from the M-P-Φ curves for the section. These models cannot account though for shear effects and coupled axial-shear-bending behavior.

A further degree of sophistication is provided by two dimensional finite element models in which each member is discretized using plane stress elements with nonlinear constitutive equations appropriate to simulate the behavior of the concrete. If the steel is correctly modelled and placed at the proper location, instead of smearing it over a concrete element, and if the plasticity model is a reasonable one, these models can account for the full coupling between axial force, shear and bending moment. They have only been used, however, for the detailed analyses of single members (beams, columns or shear walls) and not for complete frames.

4.3 Present procedures - Masses

Dynamic models of frames used for seismic analyses consider often a single degree of freedom per story (horizontal displacement in the plane of the frame), neglecting the axial deformation of the girders as well as vertical and rotational inertia forces. The stiffness matrix is modified, applying a kinematic condensation to achieve the first effect and a static condensation for the second. Masses are then lumped at the floor levels on the basis of tributary areas. The mass matrix is then a diagonal matrix. For nonlinear analyses the static condensation must be applied in each step once inelastic behavior occurs. Alternatively the mass matrix of the frame can be assembled from consistent mass matrices of the members similar in form to the stiffness matrix. This provides then coupling between transverse and rotational masses and makes unnecessary the use of the static condensation. If the solution is carried out in the frequency domain, one can, on the other hand, derive dynamic stiffness matrices for each member corresponding to a continuously distributed mass along its length. Studies conducted by LaTona et al (1969) showed that the first natural frequencies and corresponding mode shapes related to lateral displacements can be well reproduced by all models. Higher modes involving substantial joint rotations are poorly reproduced by lumped mass models, even when providing rotatory inertia terms. The solutions obtained for these modes using consistent mass matrices provide a much better approximation although they are not exact. Similar studies were conducted by Papaleontiou (1992). Figure 17 shows the time history of the horizontal displacement at the top of a 4x3 frame using distributed masses, consistent mass matrices and lumped masses. All solutions were obtained in the frequency domain and the excitation consisted of both a horizontal and a vertical earthquake component. It can be seen that the horizontal displacements, which are only affected by the horizontal base motion are nearly indentical for the three mass models. Figure 18 shows, on the other hand, the time history of the axial force in the outside column of the upper story. In this case, neither the consistent nor the lumped masses provide an exact solution but the former gives results which are much closer to those of the continuous distributed masses. While lumped masses can be used to determine effects controlled primarily by the first few modes of the structure (particularly the first one) and the horizontal base excitation, it is advisable to use consistent masses (or combination of consistent and lumped masses in some cases) when the results are controlled or influenced by higher modes (as may occur when computing local effects and forces in some members) or when the effect of the vertical earthquake component can be important.

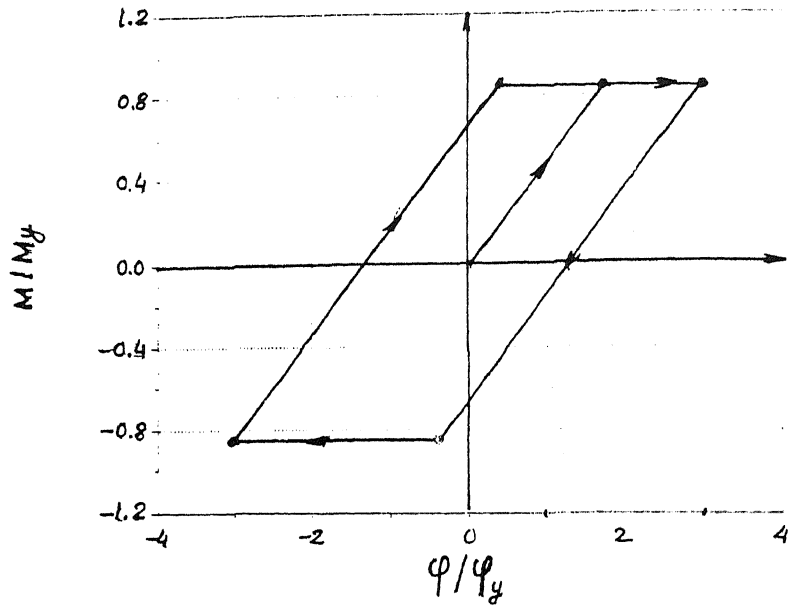


Fig. 15 Moment Curvature Loops

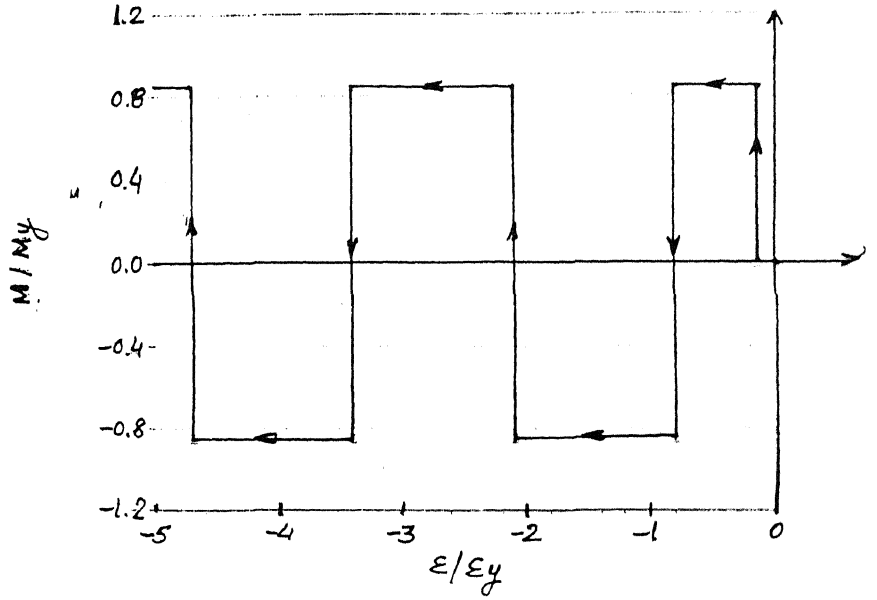


Fig. 16 Axial Strain Accumulation

4.4 Present procedures - Damping

Modal analyses, whether rigorous or spectral modal analyses, allow to specify values of damping directly in each mode. If the selected values of damping are supposed to reproduce the energy dissipation that would take place in the structure when conducting linear analyses but trying to simulate some nonlinear behavior and therefore the loss of energy associated with hysteresis. The values of the effective damping could reach 10% or more. For nonlinear analyses, the hysteretic losses of energy are automatically accounted for in the computations and the damping is intended to reproduce only the energy dissipation occurring through other mechanisms. In either case, the actual values of damping and their variation with frequency (with the number of the mode) are hard to know. As a result, it is often assumed that the damping is the same

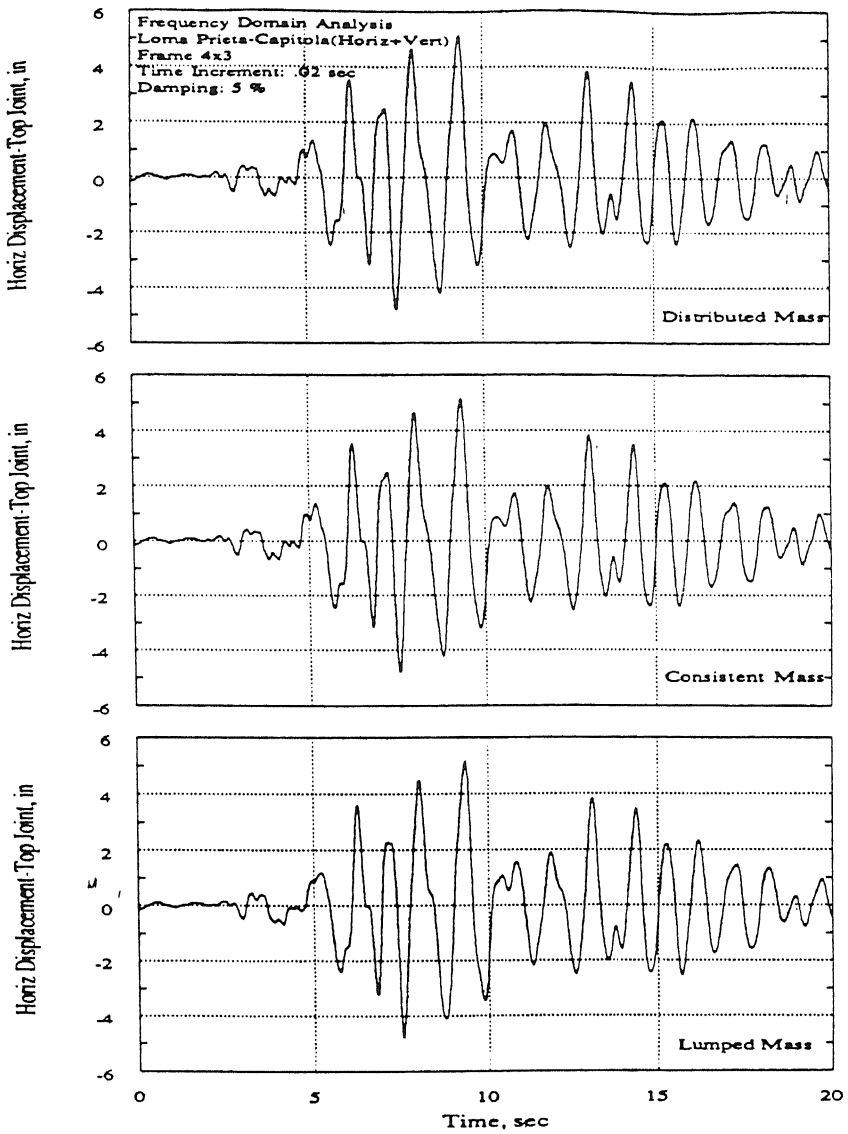


Fig. 17 Effect of Mass Model on Top Horizontal Displacement - Frame 4x3

in all the modes. Direct analyses in the time or frequency domains require the specification of a damping matrix. A number of solutions are available to assemble damping matrices that will produce desired variations of damping with frequency while maintaining the existence of classical modes of vibration. A damping matrix proportional to the mass matrix can provide a damping with decreases with frequency; a damping matrix proportional to the stiffness matrix results instead in damping which increases linearly with frequency; a third popular solution is the so-called Rayleigh damping. In this case, the damping matrix consists of two terms, one proportional to the mass matrix, the other proportional to the stiffness matrix. As a result, the damping is the sum of a term inversely proportional to frequency and another term directly proportional to frequency. This matrix involves two constants which can be selected to specify the desired value of damping at two frequencies; the damping will increase for frequencies smaller than the lower of these two frequencies and for frequencies higher than the other. One can in general obtain a damping matrix that will produce any desired value of damping in each mode, but the three solutions above, and particularly the third one, have become very popular because of their simplicity.

Direct assembly of a damping matrix is, on the other hand, straightforward, when there are well defined energy dissipation mechanisms such as a viscoelastic material, actual viscous dashpots or

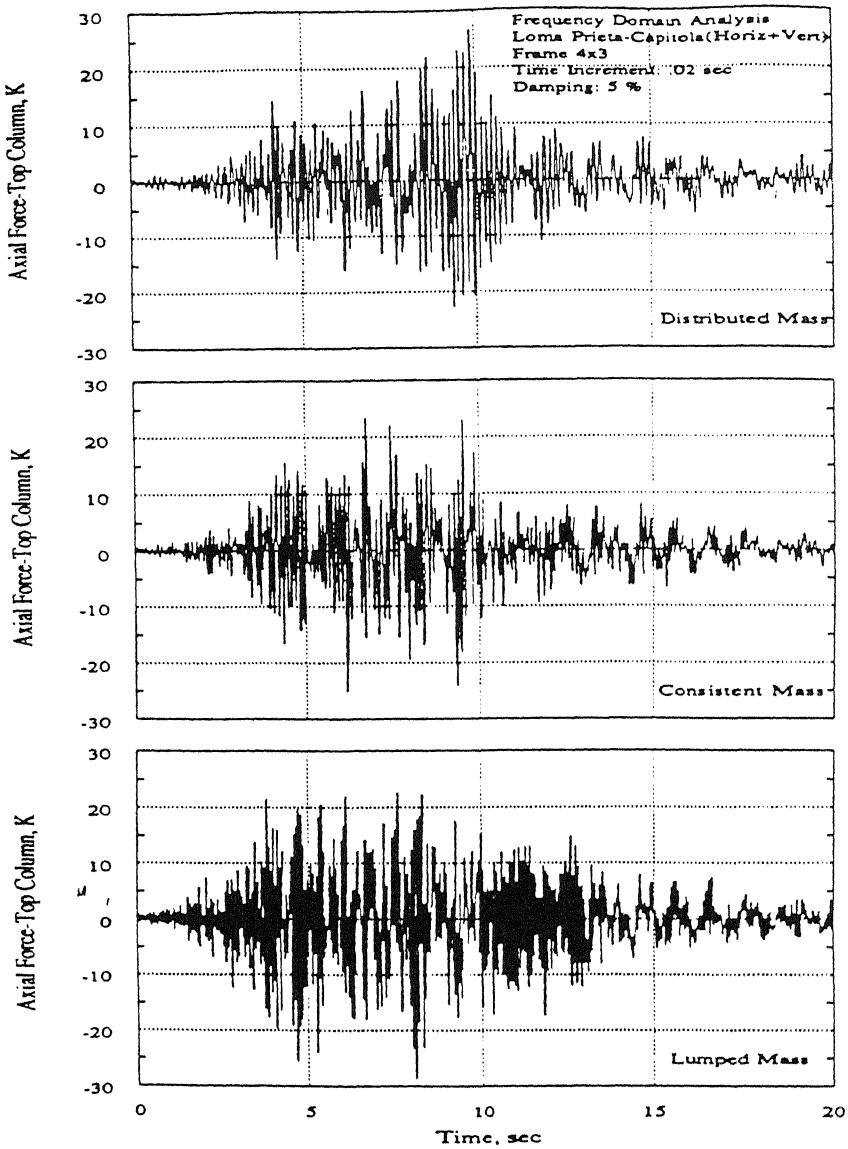


Fig. 18 Effect of Mass Model on Axial Force in Top Column - Frame 4x3

radiation damping (as associated with the dynamic response of a foundation). In this case, or when different components of the structure have different values or types of damping, the system may not have classical modes and it may be difficult, therefore, to assign values of modal damping or conduct any modal analysis. One could, of course, in all cases find complex modes and perform an eigenvector expansion of the solution but the physical interpretation of these modal solutions is lacking. This situation is encountered in soil structure interaction analyses and a number of approximate procedures to solve the problem using the undamped normal modes and equivalent values of modal damping have been suggested in the literature and are commonly used.

4.5 Limitatons, uncertainties and possible improvements

The main limitations of the present structural models are associated with the simulation of three dimensional behavior, the treatment of damping and the modelling of nonlinear behavior. As mentioned earlier most computer programs for seismic structural analysis are intended for plane frames or walls or three dimensional structures where the slabs act as rigid diaphragms in their planes so that there are only three degrees of freedom per floor, two horizontal displacements and a torsional rotation. Unemori (1978) and Vardanega (1981) found in their studies that the effect of inplane flexibility of the slabs is important in the distribution

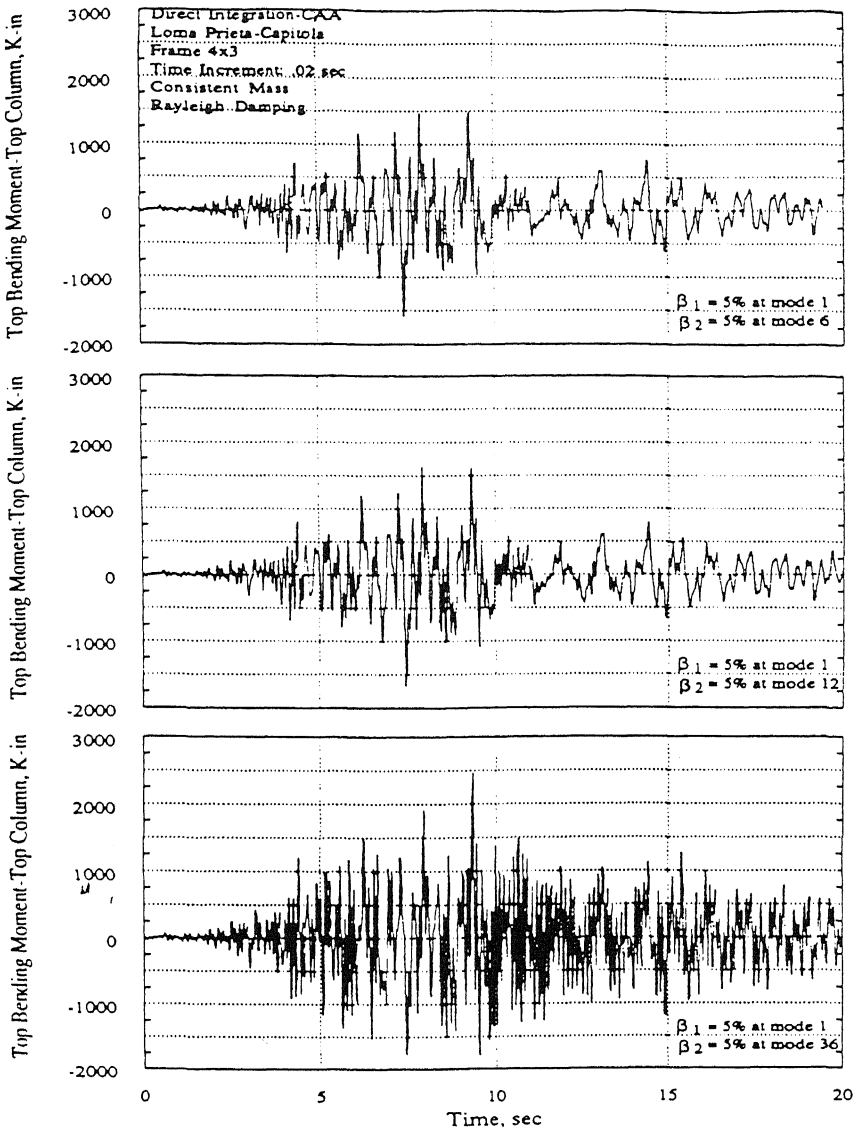


Fig. 19 Effect of Frequency Range on Rayleigh Damping - Frame 4x3

of lateral forces among the different elements when dealing with relatively short buildings (up to five stories) or in the bottom stories of taller buildings consisting of shear walls. Other three dimensional effects such as the coupling between bending moments in two orthogonal directions, the two shear forces, axial force resulting from three earthquake components and torsion are neglected. These effects may play an important role in the initiation of yielding and cracking, in the final failure mechanism and in the amount of damage that can occur once the elastic limit has been exceeded.

The amount and type of damping used for seismic analysis receives relatively little attention. Values of between 2 and 10% have been used in practice for the damping in the first mode with mass or stiffness proportional damping matrices and for the damping at one or two selected frequencies with the Rayleigh damping. The values of 5 and 7% have been particularly popular. Papaleontiou (1992) investigated the effect of different damping matrices on the response of several frames to the Loma Prieta Capitola records. Figure 19 compares the time histories of the bending moment at the top of the external column in the top floor using Rayleigh damping with 5% damping at the frequency of the first mode and at the frequencies of the 6th,12th and 36th modes. The results using for the higher frequency that of the 6th or the 12th mode are very similar but those using the 36th mode exhibit significant differences. Using modes 2

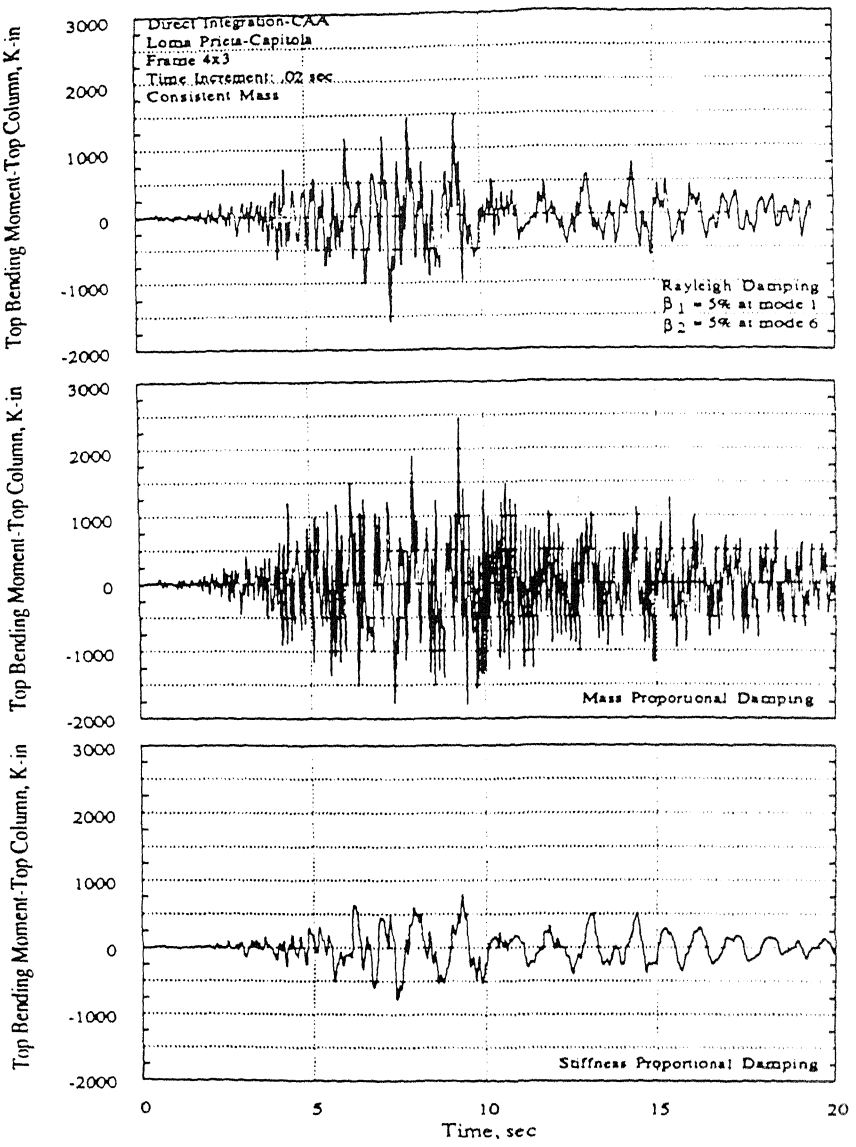


Fig. 20 Effect of Damping Model on Bending Moment in Top Column - Frame 4x3

to 5 for the high frequency produced results less accurate than using mode 6. In general, the use of the 6th natural frequency produced the best results (best agreement with the solution in the frequency domain using hysteretic type, frequency independent, damping which would correspond to equal damping in all the modes). Figure 20 shows the corresponding results for Rayleigh damping with 5% at the first and 6th natural frequencies, mass proportional damping with 5% at the fundamental frequency and stiffness proportional damping with 5% at the same frequency. It can be seen that in this case, when vertical vibrations make a non-negligible contribution to the response the solutions using mass or stiffness proportional damping are very different from those using constant modal damping (the results for this case with distributed mass were shown in Figure 4; the results with consistent masses, not shown here are very similar). Aziz (1976) studied the effect of different forms of damping on Anderson's 10 story frame subjected to the El Centro 1940 NS record. Figure 21 shows the maximum relative floor displacments using 2% damping in all modes, 5% constant modal damping, mass proportional damping with 5% in the first mode and stiffness proportional damping with also 5% in the first mode. Although the differences are not as pronounced as in the previous case since now the response is controlled mostly by the first mode, there are still larger than those reported before with or without P-Delta effects. Figure 22 shows the corresponding

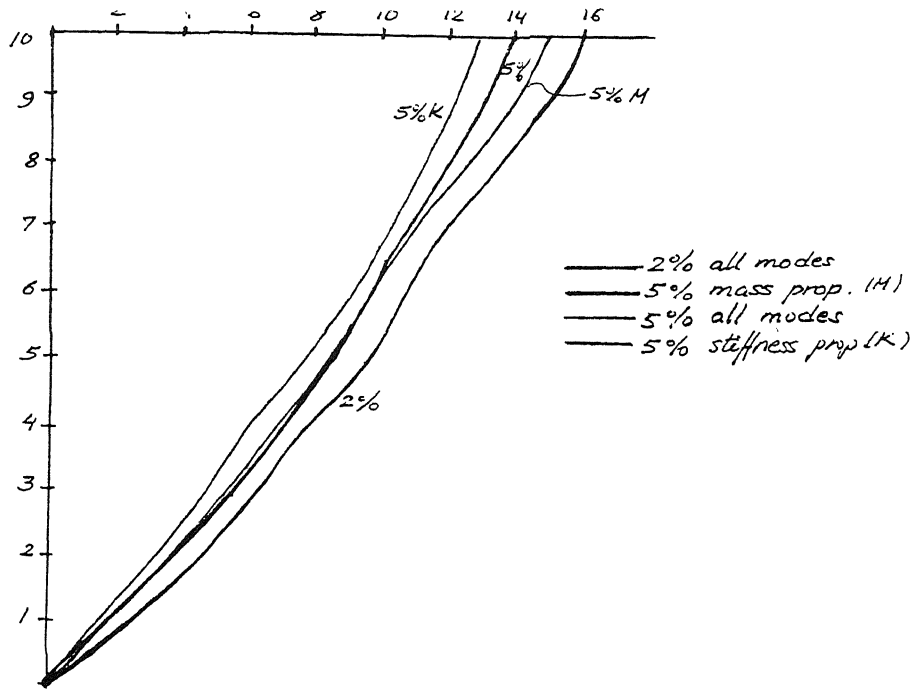


Fig. 21 Effect of Damping on Maximum Displacements - 10 Story Frame

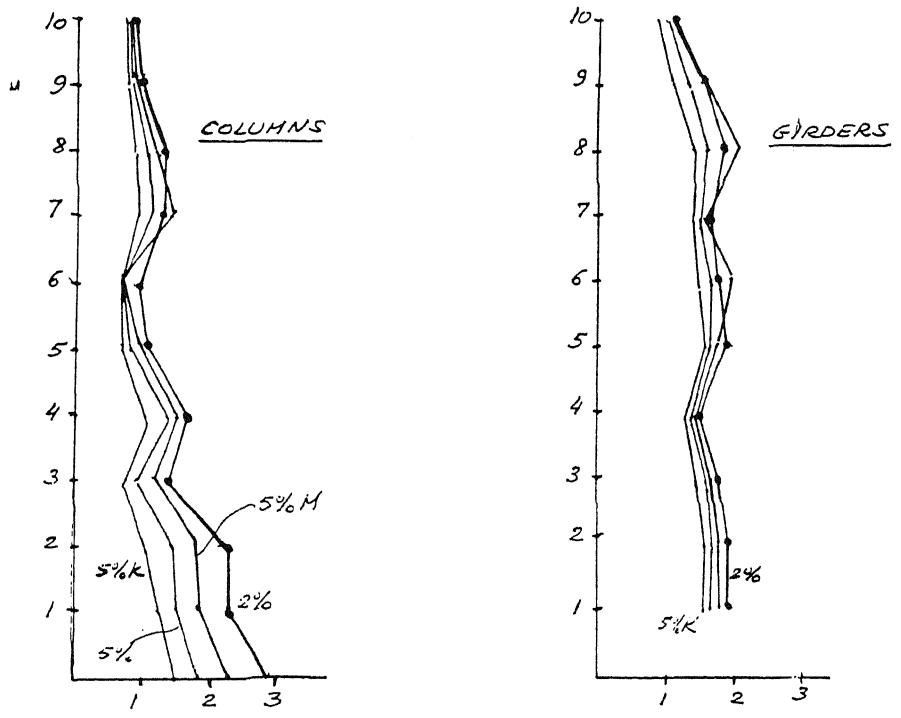


Fig. 22 Effect of Damping on Ductilities - 10 Story Frame

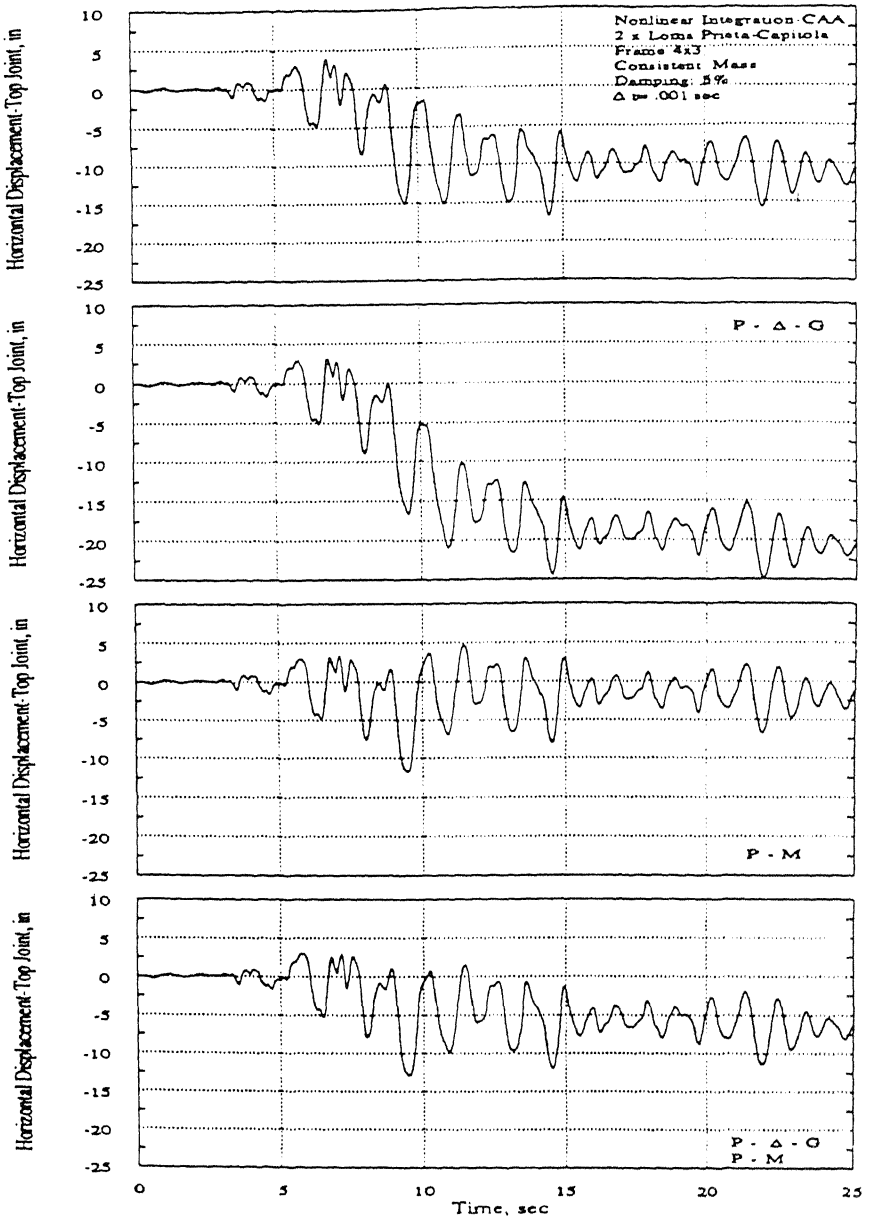


Fig. 23 Axial Effects on Inelastic Displacement - Frame 4x3

results for member ductilities. The difference in the ductility demands at the bottom of the first story columns using 2% constant modal damping and 5% stiffness proportional damping is a factor of almost 2.

The limitatons involved in the nonlinear models commonly used and particularly the point hinge models should be apparent from their description. There are a number of effects neglected or improperly modelled which are not very important with linear or nearly linear responses but which become significant as the nonlinearities increase. They must be accounted for if one want to estimate damage under a moderate to storng earthquake or to predict the failure load or failure mechanism. Figure 23 shows results obtained by Papaleontiou (1992) using the fiber model. They represent the time histories of the horizontal displacement at the top of the 4 story 3 bay frame using the Loma Prieta-Capitola records scaled by a factor of 2. Results were obtained neglecting all effects of axial loads and gravity; including gravity loads and P-Delta effects but neglecting the axial bending coupling; including gravity loads and axial-bending coupling but neglecting P-Delta effects, and including finally all effects. The first solution

produced a permanent plastic set of about 10 inches. Adding P-Delta effects increased this permanent set to about 20 inches. The solution accounting for axial-bending coupling without P-Delta effects had a very small permanent set of about 2.5 inches which increased to 6 or 7 inches when all effects are considered. When these results are compared, it is seen that the axial bending coupling effects are in fact more important than P-Delta effects. It is also interesting to notice that the time histories are almost identical in their details for all cases except for the plastic set. Results for a 10 story and a 16 story frame (both with 3 bays) exhibited similar trends. For the 20 story frame the axial effects were nearly neglibible in all cases even for twice the Loma Prieta earthquake.

To improve present procedures research work is necessary to develop models which can account with reasonable accuracy for three dimensional effects particularly in the nonlinear range. 3-D models are particularly needed when dealing with buildings made up primarily of shear walls, like those found in Chile for instance, where it is extremely hard to establish an equivalent frame that would provide satisfactory results. Structural models should include also nonstructural components which are now neglected but which can contribute to the initial stiffness of the building as well as to the damage to structural elements once they start cracking or breaking. Improvements on the way damping is handled require on the other hand better understanding of the nature of all possible mechanisms of energy dissipation. The uncertainties or simply lack of knowledge of what is an appropriate variation of the damping with frequency and what are reasonable values when considering only the damping not associated with nonlinear structural behavior make it very difficult to decide what is the most realistic model. The values of damping to be used for nonlinear analyses should be clearly small. The previous discussion on the use of hysteretic, frequency independent damping for soil amplification studies would seem to indicate that values of damping used to simulate nonlinear behavior in linear analyses should probably decrease with increasing frequency rather than remain constant.

Uncertainties will also exist on the as built condition of the structure: actual dimensions of the members, amount of reinforcement and its position, properties of the materials, etc.. These uncertainties can be reduced through improved quality control of materials and the construction process. It is also possible, and increasingly easier to determine the actual conditions and material properties for low levels of strain using nondestructive testing techniques. A number of smaller points where uncertainties exist and further studies may still be necessary are related to the selection of appropriate values of the modulus of elasticity and the moment of inertia of the cross section for reinforced concrete members and linear dynamic or static analyses, the methods to account for the bending stiffness of the slabs when isolating frames (for different types of slabs) or ways to account for the in plane flexibility of the slab in simplified models.

5. SOIL-STRUCTURE INTERACTION

5.1 Present Procedures

For many years seismic structural analyses were performed assuming that the design earthquake was the motion occurring at the base of the structure and that it was the same for all supports. The effect of the flexibility of the soil on the dynamic response of the structure and the possibility of having slightly different motions (particularly out of phase motions) at the various supports are now widely recognized. The effects can be of two kinds: a modification of the characteristics of the seismic motions due to the geometry of the foundation, known as kinematic interaction, and a modification of the effective natural period and damping of the structure (the soil-structure system), known as inertial interaction.

The motion that a foundation will experience before the structure is built or if the structure had no mass, will be different in most cases from that which would occur at the same level in the soil by itself (free field). The only case when these two motions will be equal is when dealing with a surface foundation and vertically propagating waves. Rigid surface foundations subjected to a train of plane waves propagating at arbitrary angles of incidence will experience a reduction in the amplitude of the motions in the high frequency range, as reported by Yamahara (1970), but also the appearance of torsional components of motion as described by Newmark (1969), who suggested an approximate procedure to compute them. More rigorous analyses for surface foundations of rectangular shape and various angles of incidence of the waves have been carried out by Wong and Luco (1978) and Dominguez (1978), among others. The effects are analogous to those caused by incoherence of the seismic motions. For a mat foundation that can be considered to act as a rigid diaphragm, the parameter controlling the importance of these effects is a dimensionless frequency equal to the product of the circular frequency by the slab's length in the direction of propagation of the waves, divided by the apparent velocity of propagation of the waves in the horizontal direction (phase velocity). If one assumed a train of waves travelling horizontally through the soil the apparent velocity of propagation would be the shear waves velocity of the soil; this is not, however, a realistic assumption for soft soil deposits where the apparent velocities of propagation should be of the order of the shear wave velocity of the underlying rock. Apparent velocities backfigured from data recorded by instrumental arrays show values of 2.5 to 4 km/sec.. Wave passage effects may be more important for foundations on spread footings since in this case the motions of the various footings will be out of phase but there will not be the compensating effect of the reduction in the translational motion by filtering of the high frequencies. Studies by Unemori (1978) showed that these effects tends to be confined to the lower stories, that they depend on the inplane stiffness of the floor slabs and that they can result in increases of 20 to 40% in the shear forces in the outside frames or walls with respect to the values that would be obtained if all the elements had the same motion.

When dealing with embedded foundations, kinematic effects will occur even for vertically

propagating waves. Studies by Elsabee and Morray (1977) for circular foundations embedded at various depths show that there are again two main effects: a filtering of the translational motions for high frequencies and the appearance of rotational components of motion (rocking around a horizontal axis). The most significant parameter in the definition of these effects is the natural frequency of the embedment layer and its relation to the natural frequency of the soil-structure system. For flexible structures with small natural frequencies (long periods) and little embedment in a firm soil, the embedment frequency will be much larger than the fundamental system frequency and the effects of embedment on the foundation motion will be small. For stiff structures deeply embedded in soft soils the effects can be significant. It should be pointed out, again, that these effects are quite different from the variation of the seismic motion with depth in the free field. For embedded foundations and seismic waves at arbitrary angles of incidence there will be a combination of reduced translational motions and both torsional and rotational components of motion. The torsion will be, however, smaller than for surface foundations. The rotational components could be important for slender structures offsetting the reduction in horizontal motions and resulting perhaps in increased accelerations at the higher levels.

Seismic building codes ignore entirely kinematic interaction effects although one could say that the accidental eccentricity required for buildings could take care of possible torsional effects due to wave passage or motion incoherence. Filtering or rocking effects are, however, neglected. Seismic regulations for nuclear power plants have allowed in some cases to account for kinematic interaciton while imposing some limits on the maximum reduction allowed, but have opposed it on other occasions specifying the design motions directly at the foundation level accounting for the excavation, but without structure, or, what is worse, at the foundation level in the free field. This type of specification leads to serious inconsistencies and illogical assumptions when considering several nearby buildings (which may be connected by pipes or galleries), founded at different levels.

The inertial interaction, or modification of the base motion due to the structural vibration, is the one most commonly associated with the name soil-structure interaction. Although in the seismic case the kinematic interaction effects can be as important as those of inertial interaction, or even more for deeply embedded foundations, the inertial interaction is of significance also in cases where the dynamic excitation is applied directly to the structure instead of coming through the soil (vibrations due to rotating machinery, response of offshore structures to hydrodynamic waves and current loads, etc.). Two different approaches could be used in principle to calculate or explain inertial interaction effects for the seismic case. The first one, similar to the one used for kinematic interaction is to determine the changes in the base motion due to the dynamic response of the structure, in order to arrive at a design earthquake, including all soil effects, which can be used for a direct analysis of the structure on a rigid base (the most usual type of dynamic analysis in practice). The second, more general, and providing an easier and more clear interpretation of the effect, is to modify the structural model to include the flexibility of the osil. From the study of a highly simplified model of the structure consisting of a one degree of freedom system, with an effective mass lumped at an equivalent height, and a spring that would provide the desired natural period (corresponding to the fundamental period of the building), it can be concluded that inertial interaction has two main effects: an increase in the effective natural period of the system, and a change, which is in most cases, but not always, an increase in the effective damping. The effect of the change in the period will depend on the shape of the response spectrum and the location of the natural frequency of the structure on a rigid base. For a smooth design spectrum as specified in seismic codes and regulations there will be no effect if the frequency of the structure by itself is in the range corresponding to the maximum spectral acceleration and it remains in this range when accounting for the flexibility of the soil; for more flexible structures the change would result in a decrease in the response acceleration. It should be remembered, however, that when dealing with an actual earthquake record the effect could be either beneficial or detrimental. The change in effective damping will be generally beneficial but there are a few cases (highly nonlinear structures on soils with very little internal damping) where the effect could be detrimental.

The ATC-3 recommendations allow to include inertial interaction effects in the design by computing the effective period and damping using the approximate formulae corresponding to a very simplified single degree of freedom system. The procedure requires the estimation of the stiffnesses of the foundation. These should be dynamic stiffnesses functions of frequency, but in practice, one would use the static values or the values corresponding to the natural frequency of the soil structure system (this may require some iterations). Seismic regulations for nuclear power plants allow on the other hand full inertial interaction analyses. Two general procedures have been used in practice to include soil structure interaction effects in the seismic analysis of nuclear plants: one in which the complete soil-structure system is analyzed directly as a whole, accounting simultaneously for kinematic and inertial interaction; the other a three-step or substructure approach in which one computes first the motions that would occur at the foundation level accounting for kinematic interaction when this is permitted, then the dynamic stiffness matrix of the foundation and finally a dynamic analysis of the structure, with its dynamic matrix expanded by that of the foundation, is performed for the motion computed in the first step (or the specified motion if the kinematic effect is not allowed). In the direct solution, the soil is often modelled using finite elements (or in some cases finite differences). The structure is modelled using linear members or finite elements but the structural model is rarely the full model that would be used for the seismic analysis of the structure by itself. In the 3 step approach one can use discrete or continuous solutions for each of the first two steps. Both finite elements and boundary elements have been extensively utilized for research and design purposes.

5.2 Limitations

Neglecting kinematic interaction effects as done in building codes and in seismic regulations where the design motion is specified directly at the foundation level, may be appropriatre for many typical buildings. It would be, however, unduly conservative for very stiff and massive structures such as nuclear power plants, shear wall buildings, or prefabricated panel buildings, with deep foundations on relatively soft soils. It may also be inadequate for very long buildings or for other elongated structures.

The procedure adopted in the ATC-3 recommendations to account for inertial interaction, while extremely simplified, may again be appropriate for many buildings for which the effect may not be large, when applied within the context of present codes and using smooth spectra. Its extrapolation to other cases must be done carefully, The key in the application of this proceudre is the estimation of appropriate values for the foundation stiffnesses, assuming rigid foundations (a maximum of 6 degrees of freedom corresponding to three translational motions and three rotations). A number of formulations and computer programs are now available to compute dynamic stiffnesses of foundations, surface or embedded, with any desired degree of accuracy. The accuracy is generally controlled by the size of the mesh used for the discretization. The programs assume in general a horizontally stratified soil deposit, linear soil behavior and perfect contact between the foundation and the surrounding soil. There are also a number of approximate formulae which can be suggested, from the results of parametric studies, and which can be used for preliminary estimates. These formulae have implicit all the above simplifications and assume in addition a homogeneous soil layer or half space. Their use for actual analyses of important structures is not recommended. In spite of all the available information, both in terms of accurate analysis procedures and simplified formulae, there have been recently a number of studies intended to assess the importance of soil structure interaction effects on different types of structures, which have used arbitrary values of foundation stiffnesses, and often entirely unacceptable models. In many cases the effect has been studied by simulating the flexibility of the soil through the use of an additional floor, as a means to use existing computer programs without modifying them (in spite of the fact that the modifications needed to account properly for soil structure interaction effects is relatively simple). In these instances and in many others, in which springs have been used to reproduce the foundation, the effect of the effective damping has been entirely neglected, accounting only for the change in the period. The values of the springs have been often selected based on formulae for rigid mats, even in cases where the building was supported on spread footings and pile foundations have been modelled as an equivalen single pile without any justification. The validity of these studies is highly questionable.

Of the two procedures commonly used for detailed soil structure interaction analyses of nuclear power plants, the direct approach, in which the combined soil structure system is included in the model, would be the most appropriate one (or in fact the only valid one) if nonlinear effects were to be properly reproduced. The procedure requires the determination of compatble motions along the boundaries of the finite domain selected for the study (both bottom and lateral boundaries). These motions are normally computed using one dimensional deconvolution analyses with the limitations discussed previously, particularly when attempting to simulate nonlinear soil behavior using the iterative linear approach, instead of true nonlinear analyses. The main additional limitations are the use in many cases of two dimensional or pseudo three dimensional models (as implemented in well known computer programs such as LUSH or FLUSH), the selection of highly oversimplified structural models and the use of the iterative linear scheme to account for nonlinear behavior even under two or three dimensional states of stresses (without a proper constitutive model for the soil). The importance of the errors introduced by using a two dimensional model depends on each particular case and is hard to generalize It was suggested at one time that the soil structure interaction analyses could (or even should) be conducted with this type of model and a very simple structural representation (a simple stick model or an equivalent two dimensional block of plane finite elements) in order to compute the base motions, and then a more rigorous dynamic analysis of the structure on a rigid base with these computed motions as input could be performed. Inconsistencies in the models of any of the components (in this case the structure) used in separate analyses is one of the major potential sources of errors and should always be avoided. It is equally wrong to use a detailed model of the structure with a simple spring to reproduce the soil and to use a sophisticated extensive model of the soil with a simple spring mass system to reproduce the structure. It should be remembered finally that when the direct approach is used, both interaction effects are included unless one determines first the boundary motions that would produce the desired input at the foundation level without structure (this is never done). The three step approach provides more flexibility, as long as one keeps consistent models, and allows to perform more economically studies to investigate the effect of variations (or uncertainties) in the different parameters. It is based, however, on the concept of superposition of the various effects, and it requires therefore a linear system. Nonlinearities must be approximated through the use of equivalent material properties, consistent with the expected levels of strain.

Both approaches assume generally a horizontally layered soil deposit, vertically propagating shear or compressional waves, perfect contact between the foundation and the soil and the existence of an isolated structure. The consideration of sloping or irregular layer interfaces complicates the solution procedures but is possible if one had the necessary information on the characteristics of the soil deposit. The study of arbitrary trains of plane waves does not represent any serious complication but requires also the definition of the earthquake input in terms of the types of waves, their amplitude and frequency content. Vertically propagating waves are used primarily because of lack of information on a better alternative. Travelling waves could complicate, however, the simulation of nonlinear soil effects in the free field

depending on the approach followed. A number of studies have been conducted to assess the importance of separation effects between a mat foundation and the soil for surface and embedded foundations. Scaletti (1978) found that the effects are generally beneficial but they may produce increased vertical accelerations along the axis of the structure and may require more detailed analysis and reinforcement of the slab to prevent its cracking. Very comprehensive work by Wolf and his colleagues reached similar conclusions and showed that separation will start to take place at lower acceleration levels when dealing with rock than with soft soils. It is possible therefore to account for separation effects in the analysis but it is not considered normally necessary. The effect of adjoining structures has also received a limited amount of attention. Studies by Richardson (1969), Gonzalez (1977) and Savidis (1977) have shown that the effects may be more important when considering a small structure next to a massive one, but that they are not generally large. The combined system of two structures and the soil has a fundamental frequency which is different from that of one isolated structure and the soil and therefore the peaks in the transfer functions corresponding to structural response may be shifted by the presence of the other structure or two peaks may appear where there would have been only one. Horizontal, vertical and rotational motions are also coupled by the existence of another structure and a foundation subjected for instance to purely vertical vibrations may experience some horizontal and rocking motions if there is a nearby structure. This would be important if the other modes had not been considered at all in the design but such is not normally the case. Even so, it would appear that additional research is needed in this area particularly when considering many closely spaced buildings with very different characteritics and possibly different foundation types as may be encountered in a major metropoltan area. The motions reaching one particular building are going to be clearly influenced by the vibrations of the others.

5.3 Uncertainties and possible improvements

The major uncertainties in soil structure interaction analyses are related again to soil properties, the as built condition of the foundation, the types of waves constituting the earthquake (for the kinematic interaction) and the characteristics of adjoining buildings. As in the study of the effect of local soil conditions on the earthquake motions, the soil properties required are both the values of the elastic modui for low levels of strain, in situ, and their variation in the vertical and horizontal directions, and the appropriate constitutive models to account for their variation with levels of strain. For embedded foundations the conditions of the soil in the backfill, its degree of compaction and the existing state of stresses are often unknown, yet, key factors in controlling increases in stiffness, and particularly damping, due to the embedment or the possible separation between the foundation and the soil. Studies by Scaletti (1978) showed that depending on the assumed backfilll conditions separation effects could vary widely. These uncertainties can again be decreased by proper measurement of soil properties in situ, using geophysical techniques, as well as in the laboratory and by careful quality control and supervision of the construction process. An area which requires additional work in this respect is the in situ, nondestructive evaluation and monitoring of the condition of a foundation. In Mexico City, after the 1985 earthquake, some buildings that had been damaged were repaired and strengthened, but their number of floors were reduced because of lack of knowledge on the status of the foundations and their ability to support increased loads.

There is still, in general, a lack of adequate communication between the structural designer and the geotechnical engineer, which must be solved. The design of foundations for static loads is based largely on semi-empirical procedures which account properly, even if not explicitly, for nonlinear soil behavior. This is particularly true in the case of piles where Winkler type models are used with nonlinear springs given by P-y (horizontal) or T-z (vertical) curves. The analysis of foundations for dynamic loads (whether caused by earthquakes or by vibratory forces exerted directly on the structure) is based on the other hand on more rigorous but linear elastic analysis procedures. The derivation of models which can account properly for both nonlinear and dynamic effects is still in need of further research and represents one of the main areas of potential improvement.

Most studies on embedded foundations have assumed uniform embedment over the complete perimeter of the building. In many cases, however, the soil elevation changes along the perimeter and the embedment is therefore non uniform. The simplified rules that have been applied in practice to account for this situation (multiplying for instance the stiffnesses by the ratio of the embedded perimeter to the total one) lack appropriate justification. Studies by Huei Tsyr Chen (1980) have shown that they provide a better approximation for the damping (imaginary part of the stiffness) than for the stiffness itself (real part). The actual degree of embedment along the perimeter may be uncertain at the time the design is performed. One must thus make some assumptions and then estimate the possible changes introduced by the as built conditions. More research on the effect of non uniform embedment is also needed.

6. METHODS OF ANALYSIS

6.1 Present Procedures

Seismic building codes rely still primarily on static analyses using equivalent lateral loads for the design of simple, regular structures, although dynamic analyses are being increasingly required for tall, important or irregular buildings. Whether static or dynamic these are always linear analyses. The nonlinear behavior that will take place under the selected design earthquakes is accounted for in a roundabout way through the use of a seismic design coefficient which simulates an inelastic response spectrum rather than an elastic one. Approximate rules to construct smooth inelastic spectra for single degree of freedom elasto-plastic systems were proposed by Newmark and Hall in 1973, on the basis of studies of systems with 5%

initial damping (in the elastic range) subjected to the 1940 El Centro earthquake. Further study by Sehayek (1976) showed that these rules applied very well to the average of nine earthquake records for elasto-plastic systems with 5% damping, althought they did not provide a uniform degree of conservatism over the complete frequency range. Studies for other inelastic systems and different values of initial, elastic, damping were conducted by Riddell and Newmark (1979) and Al Sulaimani (1982) who concluded that the details of the force deformation relationship had little effect on the value of the maximum inelastic displacement. Tarquis (1988) has shown, on the other hand, that this is not true when dealing with earthquake records on soft soil deposits, such as the 1985 Mexico City motions in the lake zone, which exhibit a very pronounced predominant frequency. Defining the ductility of a single degree elasto-plastic system as the ratio of the maximum inelastic displacement to the yield displacement, the inelastic spectra can provide the resistance (yield force) of the system required to limit the maximum ductility to a specified value, or the ductility required to limit the force in the system to a given value. Newmark and Hall's rules suggest that systems with an initial, elastic, natural frequency in the range of maximum spectral velocities, or smaller, will have the same maximum displacement as an elastic system with the same frequency. The ductility of such a system will thus be equal to the ratio of the maximum displacement of the elastic system to the yield displacement of the inelastic one. The yield force needed to limit the ductility to a desired value will be the maximum force induced in the elastic system divided by the specified ductility. Systems with initial natural frequencies in the range of maximum spectral acceleration will have on the other hand, approximately the same maximum strain energy as the corresponding elastic systems with the same frequency. Their resistance must then be equal to the maximum force developed in the elastic system divided by the square root of twice the desired ductility minus one. Finally, very stiff systems will have essentially the same response acceleration as the ground motion. These systems must be designed to remain elastic. To construct an inelastic acceleration (or pseudo-acceleration) response spectrum following the rules of Newmark and Hall, one would have to divide the smooth elastic design spectrum by a reduction factor function of the allowable ductility. The factor is one for low period structures, the square root of two times the ductility minus one over the range of constant, maximum, response accelerations, and equal to the ductility for longer periods. While these rules form the basis for most recent codes, the reduction factor is kept constant over the complete range of periods, and equal to some measure of ductility multiplied by additional, vaguely defined, factors to account for the effect of higher modes (the fact that the total mass is used in conjunction with the first mode response). Dynamic analyses performed according to the codes use a spectral modal analysis.

Seismic design of nuclear power plants is always based on dynamic analyses. Modal spectral analyses were conducted when dealing directly with the specified design spectra. This led, however, to difficulties when trying to incorporate soil structure interaction effects, particularly because modal damping values were arbitrarily assigned upper bounds, which could not reproduce properly radiation effects, whereas no limitations existed when performing direct time domain or frequency domain analyses. In these cases it was necessary, on the other hand, to generate artificial earthquakes that would match the target response spectra. Dynamic analyses in the frequency domain, which had been relatively rare in structural dynamics, became very popular in seismic soil-structure interaction studies because of the frequency dependence of the foundation stiffnesses, when using the three step, or substructure, approach, and the existence of very accurate transmitting (or absorbing) boundaries when modelling the soil with finite elements in the one step solution of the combined soil-structure system. All these solutions require linear behavior of the structure (assumed in the design of nuclear power plants) as well as linear behavior of the soil and the soil-structure interface. Nonlinear soil effects are normally approximated using equivalent properties consistent with the expected level of strains, while separation effects are neglected. True nonlinear analyses would require a solution in the time domain. Time domain solutions have been used primarily for research purposes extending the finite element mesh to a sufficient distance of the foundation to ensure that, with the available damping, the waves reflected at the boundary reach again the foundation with a very small amplitude. A number of alternative procedures to solve soil structure interaction problems in the time domain have been recently suggested by Wolf (1988). Time domain solutions are the most usual ones, on the other hand, when studying the nonlinear response of structures (in most cases on a rigid base). Analysis in time or frequency are normally conducted with a direct solution of the equations of motion considering all the dynamic degrees of freedom. It is possible, however, to use a modal analysis in both cases (even with nonlinear models).

The concept of ductility as a measure of inelastic deformation is clear when dealing with a single degree of freedom elasto-plastic system. In research studies with multidegree of freedom models other measures of ductility are often used. In close-coupled shear beam type models consisting of lumped masses connected by inelastic springs, the ductility of each spring, considered as a single degree of freedom system, represents the interstory ductility. In the most usual point hinge models one can define for the ends of each member either a rotation or a moment ductility. The rotation ductility is simply the ratio of the maximum response rotation to a specified yield rotation; the definition of this yield rotation requires the assumption of a mode of deformation for the member. The assumption of equal rotations at both ends of the member is the one most commonly made. The definition of the moment ductility is generally based on the assumption of a bilinear direct relationship between the moment and the rotation at each member end. The bilinear relationship is the most common one for dual component models but the moment at a member end is then a function not only of the rotation at the same point but also of the rotation at the other end. The direct moment-rotation relationship is automatic for the single component model although its physical interpretation is not always clear. Fiber

models can compute curvature or strain ductilities as well as moment ductilities involving only section properties and section behavior. Two dimensional finite element models with appropriate nonlinear constitutive equations can provide a variety of alternate measures of inelastic deformation as well as equivalent one dimensional measures of strain ductility.

6.2 Limitations

The main limitation of present procedures for the seismic design of buildings are associated with the use of the maximum displacement of a single degree of freedom system as a measure of the damage to a complex multidegree of freedom structure. The use of maximum displacements (or distortions) as measures of the dynamic response is imposed by the definition of the earthquake in terms of a response spectrum and the subsequent use of modal spectral analysis. Yet it is clear that even for a single degree of freedom system, damage is not a function only of the maximum deformation but also of the number of times this value is reached, the maximum excursion in the opposite direction, the energy dissipated, etc.. While the maximum displacement is nearly independent of the exact force deformation characteristics of the system (at least when dealing with smooth inelastic spectra), and of the duration of the excitation, damage for a degrading system will increase with the number of cycles of vibration of the same amplitude.

The definition of ductility becomes less straightforward when dealing with trilinear, multilinear or general curvilinear systems. It is not clear in this case whether the denominator should be the deformation corresponding to first yield (when such a deformation is clearly apparent, with a sharp change in the slope of the force deformation relationship), or some intermediate effective value that would provide an equivalent bilinear system with the same maximum strain energy. The difference between these definitions may be small when dealing with very large ductility ratios but could be significant when dealing with small or moderate values (of the order of 2). The basic question is which definition corresponds better to observed, acceptable, damage in structural members.

The extrapolation of the ductility of some equivalent single degree of freedom elasto-plastic system (with the same elastic fundamental frequency, and a yield force equal to the value of the base shear at the time of first yielding) to predict the damage of a frame, or a complete building, is another ambiguous, unexplained, step. The value of the ductility used to construct the inelastic design spectrum (seismic design coefficient) is often referred to as system ductility. Its relation to the local ductilities of the frame's members is a function of the type of structure and the particular design. The use of other ductilities represents an improvement but does not eliminate all the problems and limitations. Interstory ductilities are attractive and have become popular because they have a more rational meaning when dealing with multistory buildings. They assume, however, that the frame or building behaves as a close coupled system, which would be true if the girders were infinitely stiff but will be erroneous in general. It should also be noticed that the force deformation relationship for the equivalent springs representing each story would be multilinear as shown by Piqué (1976). The maximum ductility of the springs in a close coupled system will generally be much larger than the system ductility (Mobasseri-1986). Member end ductilities represent a further improvement although they maintain the basic limitation of trying to define damage from maximum distortions. The additional, and important limitation is the assumption that the end moments are uniquely related to the rotations of the same points which is not physically true nor consistent with the mathematical models used for the analyses (the exception is when placing rotational springs to account for the additional rotation caused by loss of bond and slippage of the reinforcing bars and modelling this rotation). As pointed out earlier, both moment and rotation ductilities are used in practice, and the tacit assumption seems to be that they are equivalent. Figures 24 and 25 show the rotation and moment ductilities in the members of a 4 story and a 10 story frame respectively, as reported by Aziz (1976). Although the general trends are the same for both definitions, it can be seen that individual values, and the maxima in particular, can be very different. For these two frames, the moment ductilities are larger than the rotation ductilities in the columns, while the opposite is true for the girders. The moment ductility in the bottom columns of the 10 story frame (designed by Anderson, 1969) is more than twice the rotation ductility. The question is which one of these two definitions is more realistic or better correlated with damage. The assumption of equal end rotations normally made to define the yield rotation is rarely valid for the columns and does not apply to the girders if one considers the static gravity loads already acting on the system. Another problem associated with the definition of ductility in the columns is that the yield moment, and yield rotation, depend also on the value of the axial load which is changing with time.

Curvature ductilities are more meaningful from the physical point of view than any of the previous definitions since the moment is uniquely related to curvature for a section in flexure. The problem is that the nonlinear analyses programs most commonly used will not compute curvatures. Experiments can provide on the other hand either curvatures (measuring strains at the extreme fibers) or rotations.

6.3 Sources of errors

While there are uncertainties associated with the meaning of some of the variables used, or the validity of some procedures, most of the uncertainties associated with the methods of analyses can be better classified as errors and can be reduced by a more careful and detailed execution. Modal spectral analyses have the error associated with the combination of the modal maxima, which cannot be eliminated entirely. It can be reduced, however, using one of the available techniques to account for possible correlation between modes with very similar periods. True modal analyses can have, on the other hand, the same degree of accuracy as direct solutions if all the modes are considered. When using a reduced number of modes, the question is what is the number needed to

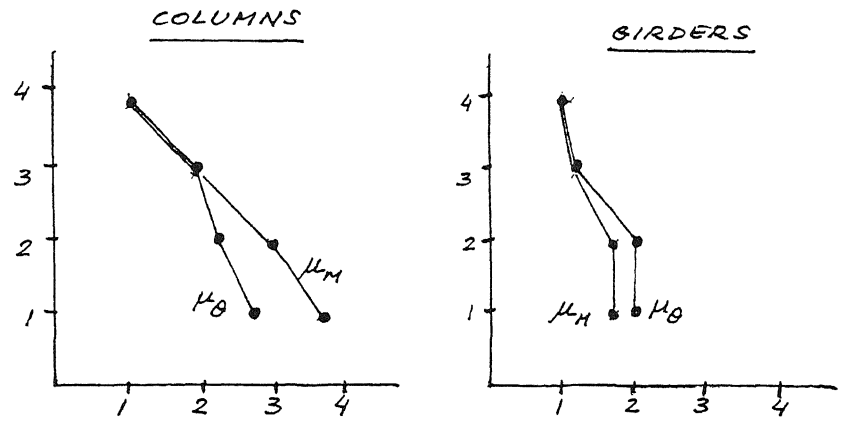


Fig. 24 Moment and Rotation Ductilities
4 Story Frame

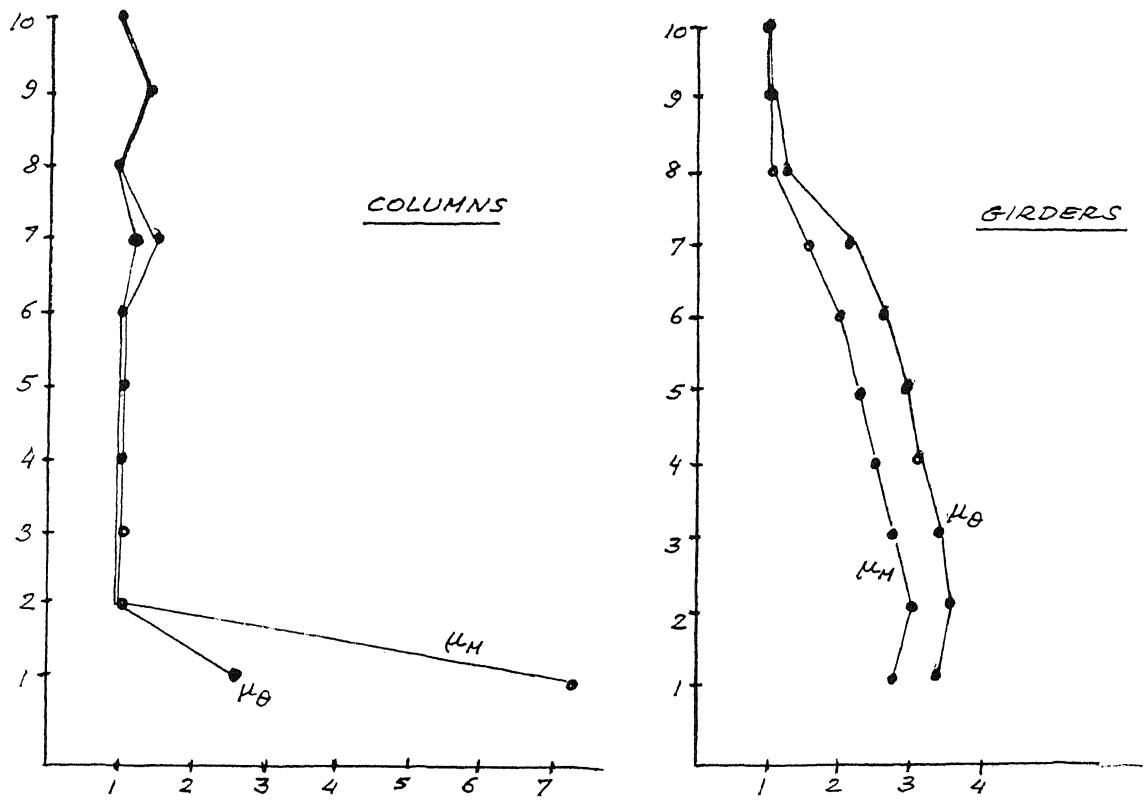


Fig. 25 Moment and Rotation Ductilities
10 Story Frame

obtain a reasonable accuracy. When accounting for instance with both horizontal and vertical vibrations it will be necessary to include at least the first vertical mode. Modal analysis has been an extremely popular technique in structural dynamics not only because it uncouples the equations of motion (for linear analyses) and may require only a reduced number of modes, but also because the response in each mode provides a substantial insight into the dynamic behavior and pattern of deformation of the system. Some of these advantages, and particularly the latter, are lost when dealing with systems which do not have real modes in the classical sense (soil-structure systems tend to belong to this class) or with systems which have a

large number of degrees of freedom and many very close and only slightly different modes (earth dams are typical examples). In these cases not only the modes may lose their physical meaning but it may be very hard to decide how many modes are necessary to obtain a reasonable accuracy, particularly when dealing with local effects (forces in some specific members). Thus, while there are many occasions in which modal analysis constitutes an excellent method of solution, there are others in which direct solution of the coupled equations of motion, in frequency or time, is a far more reliable and simpler procedure. Trying to force modal solutions in all cases by introducing approximations of unknown consequences, or selecting the number of modes on the basis of the accuracy of global response parameters, is not advisable.

Solutions in the frequency domain are affected by the frequency increment and the number of points (or maximum frequency) used. The adequacy of the selected parameters can be easily estimated from a visual observation of the Fourier spectrum of the excitation and the amplitude of the transfer function of the desired effects. Direct solutions in the time domain are affected by the selected numerical integration procedure and the time step of integration. Explicit integration schemes like the central difference formula, which are conditionally stable require normally time steps of integration, based on stability considerations, which guarantee accurate results. They may be extremely small for very stiff systems and models with a very large number of degrees of freedom. Implicit methods which are unconditionally stable have become popular in a number of applications. When these methods introduce fictitious damping into the system, as in the case of Wilson's theta method or Houboldt's method, the results can be on the unconservative side (smaller than the true solution) if the time step is not sufficiently small. Even for methods which do not introduce fictitious damping, like the constant average acceleration procedure, the stability of the solution does not guarantee its accuracy. For some time, earthquake records were digitized at time steps of 0.01 seconds and this was the maximum time increment used for seismic analyses. In recent years the records are digitized at increments of 0.02 seconds and this time step is often used for the solution with unconditionally stable integration schemes. Figures 26, 27 and 28 show the time histories of the bending moment at the top of the upper story columns of the 4 x 3 frame studied by Papaleontiou (1992) obtained using the constant average acceleration, Houboldt's and Wilson's methods, and time steps of 0.02, 0.01 and 0.005 seconds respectively. It can be seen that the results of all three methods are nearly identical for the smallest time step (Figure 27). For larger time steps the results obtained with the constant average acceleration method provide slightly higher peaks, while Houbolt's method gives results smaller than the real values. The solution obtained with this method is particularly bad for the time step of 0.02 seconds. The results with Wilson's method tend to be intermediate between those of the other two procedures. Figure 29 shows the time history of the displacement at the top of the same 4 x 3 building when performing a nonlinear analysis for the Capitola record of the Loma Prieta earthquake multiplied by the factor of 2 (the previous results were for the same earthquake without scaling). Although the general shape of the response is almost identical in all cases as far as the details of the vibration are concerned, the permanent plastic set is sensitive to the method of integration and the time step. It should be noticed that in this case a time step of 0.001 seconds is needed to get the same results with all three methods. Nonlinear analyses will generally require smaller time steps (larger number of modes when performing modal analyses) to obtain reliable results.

The main improvements in this area must come in the definition of more realistic measures of damage than the system ductility or even the local ductilities used at present. Damage indices should be based on the values of parameters which can be accurately measured in experiments and computed in the analyses. They will have to be validated through extensive studies comparing analytical predictions to laboratory test data and observations of damaged buildings during or after actual earthquakes. These measures of damage should reflect the effect of duration of the excitation, possible degradation in stiffness and strength, energy dissipated as a fraction of the total amount of energy that the structure can dissipate, etc.. Some new definitions of damage have been proposed in recent years which satisfy various of these requirements. Additional work is needed, however, to increase their reliability.

7. ADDITIONAL CONSIDERATIONS

The previous discussion has dealt primarily with technical problems and issues which must be resolved to improve our ability to model and analyze structures subjected to earthquakes and to predict with some accuracy the degree of damage they may experience. Research in earthquake engineering, as in any other technical or scientific field, or for that purpose in any form of human endeavor, must fight in addition, a number of nontechnical problems if real progress is to be achieved. Among them are the dangers of oversimplification, overgeneralization, credulity, defeatism and complacency, and excessive commercialism.

Simplification is in itself desirable. Simplified models are necessary to identify key parameters, to understand their effect on basic behavior, and to obtain preliminary estimates for an initial design or to check the results of more sophisticated models. They are particularly useful to explain the importance of a phenomenon or different variables to students or to a nontechnical audience. Thus courses on Structural Dynamics will always start with a thorough coverage of the fundamental behavior of single degree of freedom systems, and a look at the transfer function (or dynamic amplification factor) for one of these systems illustrates immediately when dynamic effects may or may not be important and when dynamic analyses may be required. The fact that we now have available extraordinary computational capabilities at relatively low cost does not imply that one must always use the most complicated models available. It does not imply either that we must always use the simplest models. As Albert Einstein states, "Everything should be made as simple as possible but not simpler."

The two main forms of oversimplification are

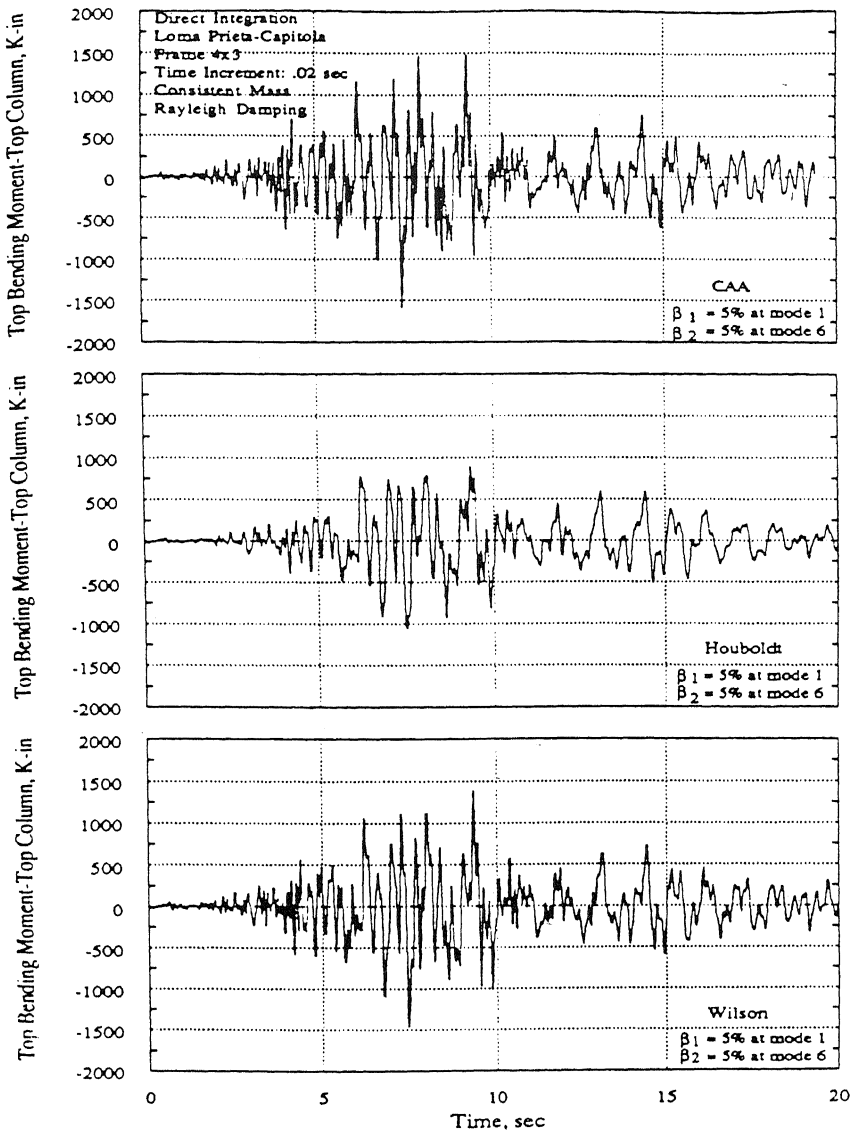


Fig. 26 Comparison of Integration Schemes - Direct Integration at $\Delta t = 0.02$ sec.

associated with the urge to make everything one dimensional and linear. It appears that the human mind has trouble dealing with more than one variable at a time: every physical phenomenon must be characterized by a single parameter just as one person must be blamed for all social or political ills. In earthquake engineering, the design earthquake must be characterized by a single variable related to the peak or effective ground acceleration; a soil deposit must be reproduced as a simple degree of freedom system and a single degree of freedom is also sufficient to represent the structure or the combined soil structure system; finally damage can be assessed from the computation of a single quantity which is the system ductility. It must be mentioned again that each one of these simple models may be of great value for some specific uses, but to make progress in the accuracy of the damage prediction, one must realize that more realistic and complex models are going to be necessary. Similarly, there is a strong desire to have only linear relationships. Nonlinear problems are linearized through a variety of techniques with different degrees of approximation. It must be realized that no linear system can reproduce all the features of the response of a nonlinear system even when this one has a unique and well-behaved solution. The discrepancies will naturally increase with increasing degree of nonlinearity.

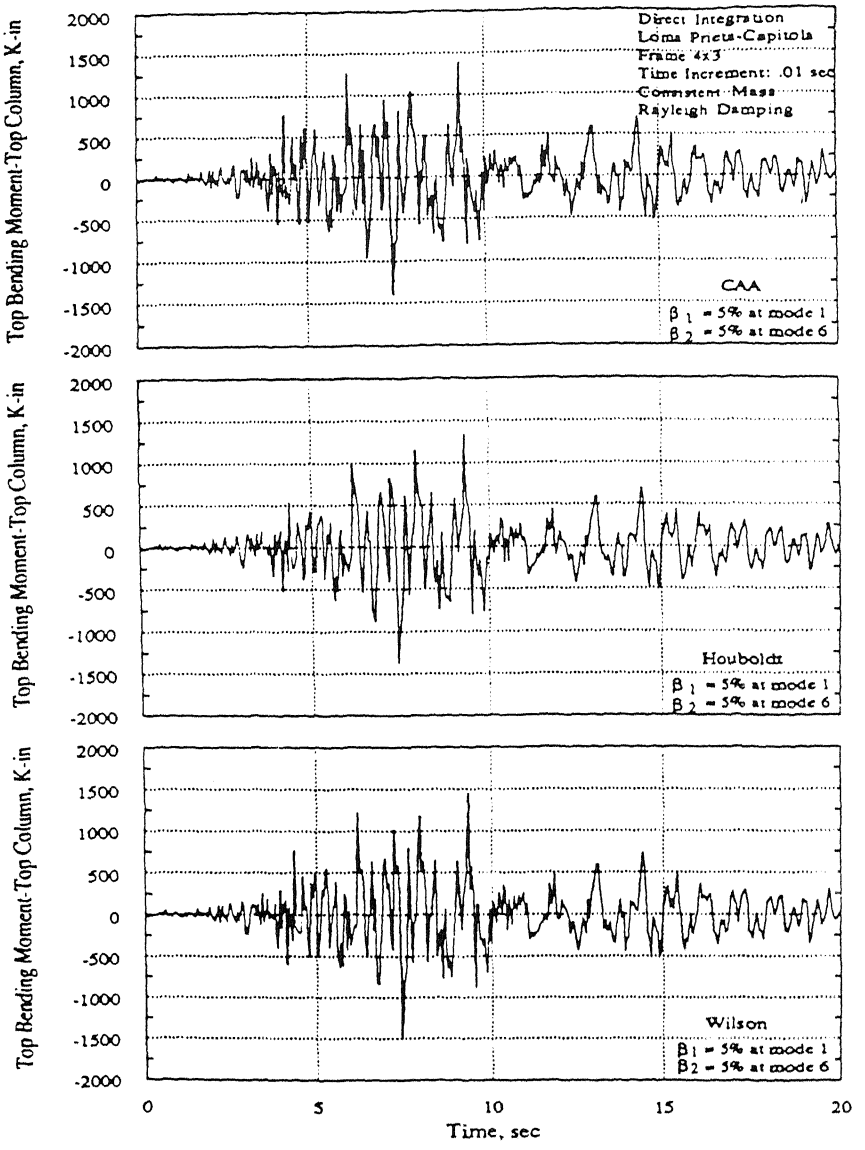


Fig. 27 Comparison of Integration Schemes at $\Delta t = 0.01$ sec.

Generalization is also necessary if results of a finite number of laboratory experiments are going to be used to predict the behavior of a class of buildings or if the conclusions from parametric studies are going to be valid for other sets of values than those considered. It is important, however, to make clear the limits of applicability of any study, the assumptions and simplifications involved and the errors associated with the instrumentation or the numerical procedures used. The results of experiments conducted on plane frames loaded in their plane cannot lead to the conclusion that three dimensional effects are not important in structural behavior under earthquakes, just like one cannot deny the existence of high frequency terms in a particular problem if the recording instruments used in the experiments filter them out. Contrary to popular belief, particularly among empiricists, the fact that results of an experiment do not agree entirely with theoretical predictions, is not necessarily an indictment of the theory but perhaps an indication of a poorly planned or inadequate testing procedure. On the other hand, theories which are mathematically correct, and even elegant, may be of little practical value if they do not have a solid physical basis and they do not reproduce the results of physical observations. They may provide in short a correct solution to the wrong problem. Generalization is particularly difficult in some cases

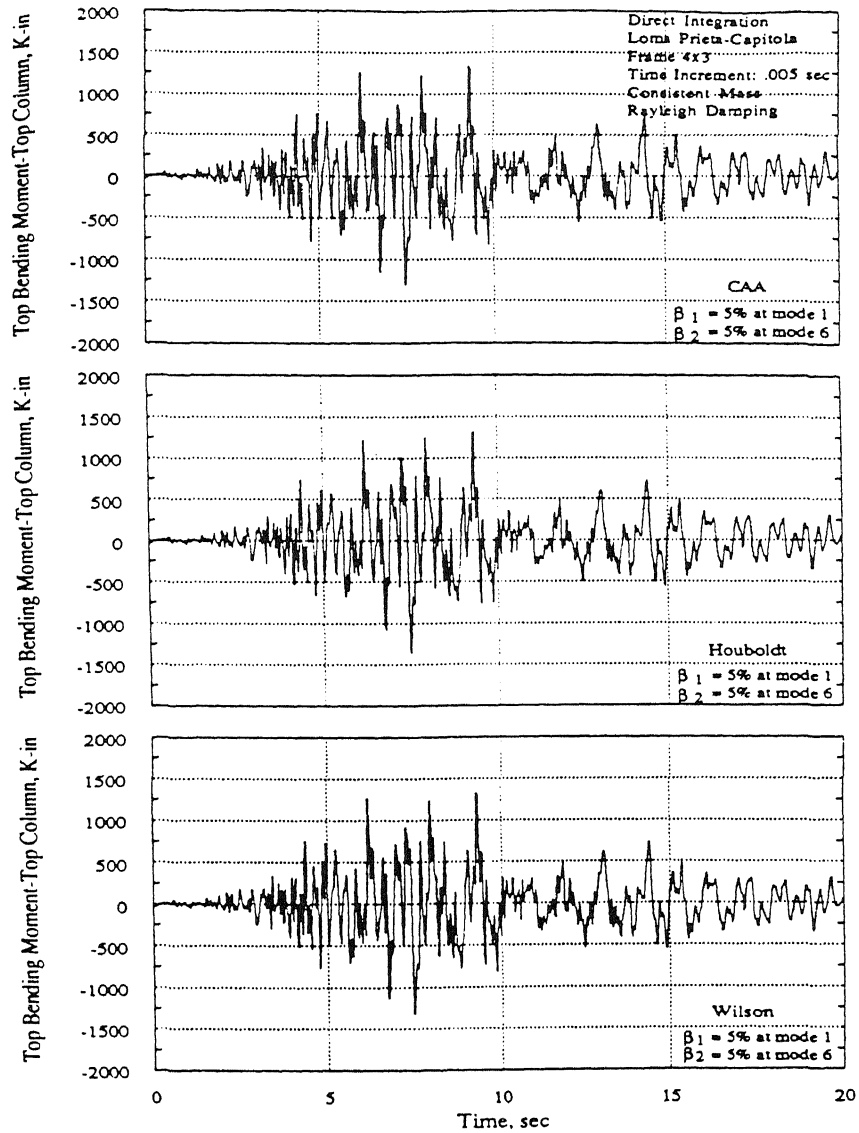


Fig. 28 Comparison of Integration Schemes at $\Delta t = 0.005$ sec.

where there are a large number of parameters, or effects, influencing the results. This is the case for instance of soil structure interaction problems or nonlinear analysis.

Every researcher or educator feels flattered if his statements are automatically accepted without need to prove them or at least justify them adequately. Yet teaching students to accept statements rather than question them is not a sound educational policy. Students should learn not only what to think but more importantly how to think, not only how to do anything but also why, and not just how to use computer programs, but rather the formulation behind these programs and its limitations. The desire to apply immediately in practice the latest research results or methodologies, often of limited scope, particularly when dealing with very important structures such as nuclear power plants, the lack of an adequate period of time over which these results or methodologies can be evaluated and a consensus on their validity reached, and the existence of controversial issues and contradictory opinions, has led many engineers and regulators to rely on the opinions of a few selected researchers as the truth. While this is understandable and it may be better than relying on committees to reach the scientific truth through democratic compromise (as done normally for codes), it can also be overdone and there are dangers associated with

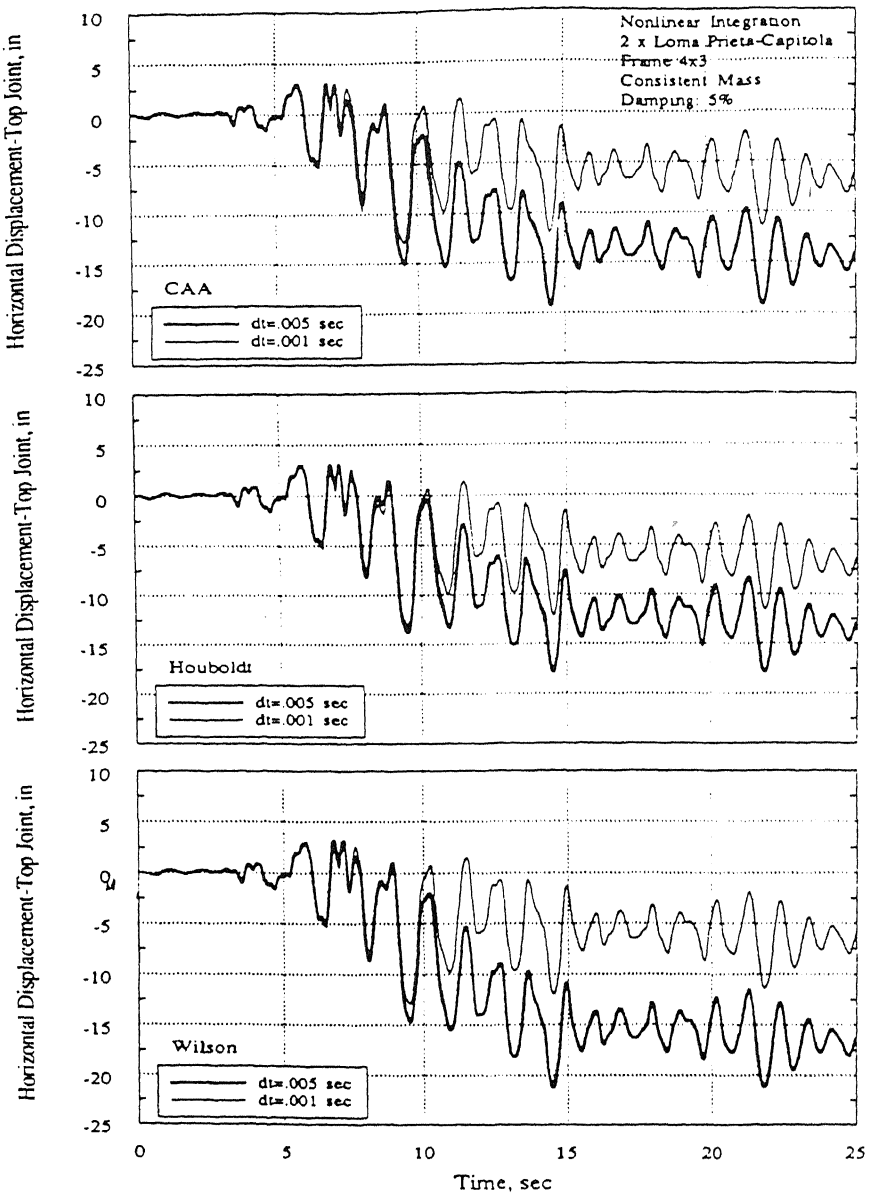


Fig. 29 Comparison of Integration Methods - Horizontal Displacement - Frame 4x3

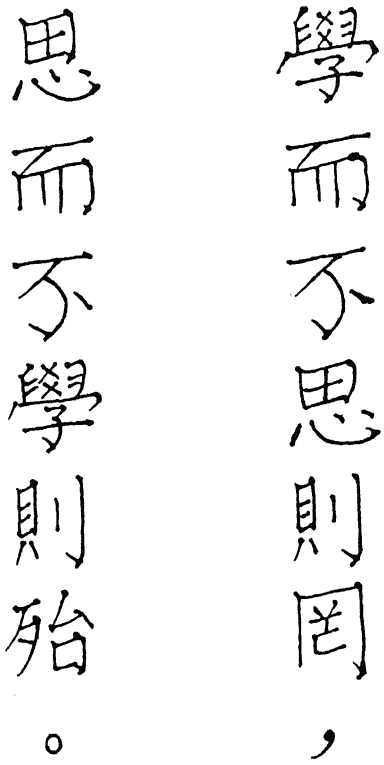
granting infallibility to ordinary human beings. As a result of this credulity, we have seen some methods imposed as the only acceptable ones and later on condemned as erroneous; when results of two different methods are different one of them has been assumed to be correct without proof; methods have been shown to be wrong by using them incorrectly, and sophisms have been accepted as legitimate proofs. So, for instance showing that one alternative, logical or illogical, was incorrect has been taken as proof that a second one was true, forgetting conveniently that there were more than two alternatives.

Negating research needs because the problem is too complex or involves too many variables is a form of defeatism which must be resisted. It can take different forms: it is not unusual, for instance, to hear that further research or refinement on a particular topic is not warranted considering all the uncertainties involved in the problem. While it is logical to give low priority in research to developments that may reduce errors from 20 to 10 or 5% in favor of areas where results cannot be obtained within 100% of the solution, one should also realize that the accumulation of many different errors, each one of 20%, may lead to

highly unreliable results, particularly if they are all in the same direction. It is also stated at times that the use of a probabilistic formulation makes further research unnecessary since the errors will be included and accounted for with the other uncertainties. This argument cannot apply to systematic errors and negates the fact that uncertainties can only be reduced or eliminated through a better understanding of the physical phenomena. Finally, resisting the application in engineering practice of new research results or methodologies under the pretext that it would require extensive retraining of design engineers is an underestimation of the education and intellectual capability of these engineers which represents an insult to the profession. Equally dangerous is the overcomplacency with the state of the art at any particular time. The feeling that code procedures have reached all the complexity that is ever needed or warranted and that further changes or improvements are unnecessary, has been a popular and persistent one since the publication of the first seismic code. In spite of which, codes have been steadily changing and improving.

In the present environment, seismic research, like any other kind of research, must be marketed and sold. The submission of properly justified research proposals and their evaluation by knowledgeable peers is a necessary and desirable procedure to allocate reasonably limited resources. Yet this process tends to favor sometimes projects that follow well established and predictable paths at the expense of more innovative, and riskier, research efforts. The main problem arises when the packaging of research programs for marketing purposes takes precedence over their substance. This leads automatically to overgeneralization of the potential of the expected results, exaggeration of the capabilities and range of applicability of the proposed procedures and omission of any implied limitations. On occasions it can lead also to the use of sophisms to justify a given approach or to the unfair discrediting of other more powerful methods.

All these are important issues that require our concern. Yet, it would not be appropriate to conclude this review on a negative note. While some amount of duplication is unavoidable and the research effort has been at times repetitive, and while there have been periods over which important available knowledge has been consistently disregarded, there is no doubt that the understanding of earthquakes, their causes and mechanisms, and their effects on structures, has increased considerably, thanks to all the research work which has been conducted. And much more remains to be learned through continued research . In conducting this research and applying its results to engineering practice one must keep in mind that, as Confucius said, to learn without thinking is useless; to think without having learnt before is dangerous.



BIBLIOGRAPHY

Adams, J. 1973. Nonlinear behavior of steel frames. Ph.D. Thesis. MIT Civil Engineering Department

Al Sulaimani, G. J. and Roesset, J.M. 1985. Effect of damping on inelastic response spectra. Proceedings 5th Canadian Conference on Earthquake Engineering.

Ananostopoulos S.A. 1972. Nonlinear dynamic response and ductility requirements of building structures subjected to earthquakes. MIT Civil Engineering Department Report R72-59.

Anderson, J. C. 1969. Seismic behavior of multistory frames designed by different philosophies. Ph.D. Thesis. U. of California-Berkeley, Civil Engineering Department.

Anderson, J.C. and Bertero, V.V. 1970. Seismic behavior of multistory steel frames. Symposium on Earthquake Engineering. Indian Society of Earthquake Technology.

Applied Technology Council. 1978. Recommended design provisions for building ATC-3.

Aziz, T. 1976. Inelastic dynamic analysis of building frames. MIT Civil Engineering Department Report R76-37.

Blume, Newmark and Corning. 1960. Design of multistory reinforced concrete buildings for earthquake motion. Portland Cement Association.

Boubenider, R. 1988. Foundation effects on the inelastic response of frames to seismic excitation. M.S. Thesis. The University of Texas at Austin.

Chen, Huei-Tsyr. 1981. Reliability of nonlinear seismic analysis of frames. M.S. Thesis. The University of Texas at Austin.

Chen, Huei-Tsyr. 1984. Dynamic stiffness of uniformly embedded foundations. The University of Texas at Austin Department of Civil Engineering Report GR 84-9.

Chen, Huei-Tsyr. 1984. Dynamic stiffness of non uniformly embedded foundations. The University of Texas at Austin Civil Engineering Department Report GR 84-10.

Cheng, Fu-Ping. 1986. Dynamic response of circular foundations in an elasto plastic soil medium. The University of Texas at Austin Civil Engineering Department Report GR 86-3.

Chopra, A.K. and Gutierrez, J.A. 1974. Earthquake analysis of multistory buildings including foundation interaction. Earthquake Engineering and Structural Dynamics. Vol. 3.

Clough, R. W. and Benuska, K.L. 1966. FHA Study of seismic design criteria for high rise buildings. HUD, TS-3.

Constantopoulos, I.V. 1973. Amplification studies for a nonlinear hysteretic soil model. MIT Civil Engineering Department Report R 73-46.

Dominguez, J. 1978. Dynamic stiffness of rectangular foundations. MIT Civil Engineering Department Report R 78-20.

Dominguez, J. 1978. Response of embedded foundations to traveling waves. MIT Civil Engineering Department Report R 78-24.

Elsabee, F. and Morray, J.P. 1977. Dynamic behavior of embedded foundations. MIT Civil Engineering Department Report R 77-33.

Ettouney, M.M. 1976. Nonlinear soil behavior in soil structure interaction analysis. MIT Civil Engineering Department Report R 76-9.

Garcia, F and Roesset, J.M. 1970. Influence of damping on response spectra. MIT Civil Engineering Department Report R 70-4.

Gazetas, G. 1975. Dynamic stiffness functions of strip and rectangular footings on layered soils. S. M. Thesis. MIT Civil Engineering Department.

Geberson, M.F. 1969. Two nonlinear beams with definitions of ductility. Journal of the Structural Division ASCE. ST2.

Gonzalez, J. J. 1977. Dynamic interaction between adjacent structures. MIT Civil Engineering Department Report R 77-30.

Hansen, R. J. 1970. Seismic design for nuclear power plants. The MIT Press.

Housner, G. W. 1955. Properties of strong ground motion earthquakes. Bulletin of the Seismological Society of America. Vol. 45 No. 3.

Housner, G.W. 1959. Behavior of Structures during earthquakes. Journal of the Engineering Mechanics Division. ASCE. No. EM4.

Housner, G.W. and Jennings, P.C. 1969. Generation of artificial earthquakes. Journal of the Engineering Mechanics Division. ASCE. Vol. 90 No. EM1.

Iguchi, M. 1973. Seismic response with consideration of both phase differences of ground motion and soil structure interaction. Proceedings. Japan Earthquake Engineering Symposium.

Jakub, M. 1977. Nonlinear stiffness of foundations. MIT Civil Engineering Department Report R 77-35.

Jones, T. J. and Roesset, J.M. 1970. Soil amplification of SV and P waves. MIT Civil Engineering Department Report R 70-3.

Kausel, E. 1974. Forced vibrations of circular foundations on layered media. MIT Civil Engineering Department Report R 74-11.

Kausel, E. and Roesse, J.M. 1974. Soil structure interaction for nuclear containment structures. Proceedings ASCE Power Division Specialty Conference, Boulder, Colorado.

Kausel E. and Ushijima R. 1979. Vertical and torsional stiffness of cylindrical footings. MIT Civil Engineering Department Report R 79-6.

Kim, Y.S. 1987. Dynamic response of structures on pile foundations. Ph.D. Dissertation. The University of Texas at Austin.

La Tona, R.W. and Roesset, J. M. 1969. Analysis of building frames for natural frequencies and natural mode shapes. MIT Civil Engineering Department Report R 69-49.

La Tona, R.W. 1970. Nonlinear analysis of building frames for earthquake loading. MIT Civil Engineering Department Report R 70- 65.

Luco, J.E. 1974. Impedance functions for a rigid foundation or a layered medium Nuclear engineering and design.

Mark, K. 1976. Nonlinear dynamic response of reinforced concrete frames. MIT Civil Engineering Department Report R 76-38.

Mohraz, B. 1976. A study of earthquake response spectra for different geological conditions. Bulletin of the Seismological Society of America. Volume 66.

Mobasseri, M. 1986. The relation between local and overall ductility demands in multidegree of systems. Ph.D. Dissertation. The University of Texas at Austin.

NEHRP 1988. Recommended provisions for the development of seismic regulations for new buildings. FEMA 95.

Newmark, N.M. 1967. Design criteria for nuclear reactors subjected to earthquake hazards. Proceedings IAEA Panel on Aseismic Design and Testing of Nuclear Facilities. Japan Earthquake Engineering Promotion Society. Tokyo.

Newmark, N.M. 1969. Torsion in symmetrical buildings. Proceedings 4th W.C.E.E. Santiago, Chile.

Newmark, N.M., Hall, W.J. and Mohraz, B. 1973. A study of vertical and horizontal earthquake spectra. Report WASH-1255. USA-AEC. Washington, D.C.

Newmark, N.M and Hall, W.J. 1973. Procedures and criteria for earthquake resistant design. Building Science Series 46. National Bureau of Standards.

Novak, M. 1973. Vibrations of embedded footings and structures. Preprint 2029. National Structural Engineers Meeting. ASCE. San Francisco.

Pais, A. and Kausel, E. 1985. Stochastic response of foundations. MIT Civil Engineering Department Report R 85-6.

Papaleontiou, C. 1992. Dynamics analysis of building structures under combined horizontal and vertical vibrations. Ph.D. Dissertation. The University of Texas at Austin.

Parmelee, R.A. 1967. Building foundation interaction effects. Journal of the Engineering Mechanics Division ASCE. Volume 93 No. EM2.

Pique, J.R. 1976. On the use of simple models in nonlinear dynamic analysis. MIT Civil Engineering Department Report R 76-43.

Richardson, J. D. 1969. Forced vibrations of rigid bodies on a semi-infinite elastic medium. Ph.D. Thesis. University of Nottingham.

Riddell, R. and Newmark, N.M. 1979. Statistical analysis of the response of nonlinear systems subjected to earthquakes. University of Illinois Civil Engineering Department Report SRS 468.

Roesset, J. M. and Kausel, E. 1976. Dynamic Soil Structure Interaction. 2nd International Conference on Numerical Methods in Geomechanics. ASCE. VPI. Blacksburg.

Roesset, J.M. and Kim, Y.S. 1987. Specification of control motions for embedded foundations. Proceedings 5th Canadian Conference on Earthquake Engineering.

Roesset, J. M. and Scaletti, H. 1979. Nonlinear effects in dynamic soil-structure interaction. Proceedings 3rd International Conference on Numerical Methods in Geomechanics. Aachen.

Roesset, J.M. and Whitman, R.V. 1969. Theoretical background for amplification studies. MIT Civil Engineering Department Report 69-15.

Sanchez Salinero, I. 1982. Static and dynamic stiffnesses of single piles. The University of Texas at Austin Civil Engineering Department Report GR 82-31.

Sanchez Salinero, I. 1983. Dynamic stiffness of pile groups: approximate solutions. The University of Texas at Austin Civil Engineering Department Report GR 83-5.

Sarrazin, M.A. 1970. Soil structure interaction in earthquake resistant design. MIT Civil Engineering Department Report 70-59.

Savidis. S. and Richter. T. 1977. Dynamic interaction of rigid foundation. Vol. II. Proceedings IX International Conference on Soil Mechanics and Foundation Engineering. Tokyo.

Scaletti, H. 1975. Frequency analysis of structures with foundation interaction. M.S. Thesis. MIT Civil Engineering Department.

Seed, H. B. and Idriss, I. M. 1969. The influence of soil conditions on ground motions during earthquakes. Journal of the Soil Mechanics and Foundations Division. ASCE. Volume 95. No SM1.

Sehayek, S. 1976. Effect of ductility on response spectra for elasto-plastic systems. MIT Civil Engineering Department Report R 76-42.

Tarquis, F. 1988. Structural response and design spectra for the 1985 Mexico City earthquake. The University of Texas at Austin Civil Engineering Department Report GD 89-1.

Unemori, A.L. 1978. Generalized dynamic behavior of crosswall building systems. Sc. D. Thesis. MIT Civil Engineering Department.

Vardanega, C. 1978. Soil structure interaction effects on the dynamic response of shear wall buildings. M.S. Thesis. MIT Civil Engineering Department.

Veletsos, A.S. and Verbic, B. 1974. Basic response functions for elastic foundations. Journal of the Engineering Mechanics Division. ASCE Volume 100-No. EM2.

Veletsos, A.S. and Wei, Y.T. 1971. Lateral and rocking vibration of footings. Journal of the Soil Mechanics and Foundations Division. ASCE. Volume 97.

Waas, G. 1972. Earth vibration effects and abatement for military facilities. Report S71-19 USAE-WEJ.

Wolf, J.P. 1976. Soil structure interaction with separation of base mat from soil. Nuclear Engineering and Design.

Wolf, J. P. 1985. Dynamic soil structure interaction. Prentice-Hall.

Wolf, J.P. 1988. Soil structure interaction analysis in time domain. Prentice Hall.

Wong, H.L., and Luco, J.E. 1978. Dynamic response of rectangular foundations to obliquely incident seismic waves. International Journal of Earthquake Engineering and Structural Dynamics.

Yamahara, H. 1970. Ground motion during earthquakes and the input loss of earthquake power to an excitation of buildings. Proceedings U.S. - Japan Seminar on Earthquake Engineering with Emphasis on Safety of School Buildings. NSF.