

## Exploiting expertise in monumental building diagnosis

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**ABSTRACT:** An expert-system prototype for the assessment of the state of health of monumental buildings is the ultimate goal of a long-term research in progress. Several modules of this prototype are already operating and are reviewed in the paper. Attention is then focused on Bayesian schemes for updating knowledge networks on the basis of new pieces of information. The theory is illustrated with reference to a problem of seismic vulnerability assessment.

### 1. INTRODUCTION

The ultimate objective of a long-term research activity sponsored by the Earthquake Mitigation Group (GNDT) of the Italian Research Council (CNR) is the preservation of the historical heritage present in every monumental building. In order to keep intact its architectural value, the building vulnerability to seismic events must be decreased by a minimum amount of structural modifications. For this purpose, the exploitation of the entire expertise on the subject is recommended and this is made possible by the adoption of artificial intelligence (AI) techniques. The building of an appropriate knowledge-based expert-system (ES) prototype is therefore a preliminary step for further research.

A review of the expert-system modules already operative will first provide an idea of the framework within which the research is proceeding. The rest of the paper is devoted to the illustration of a probabilistic network scheme and its implementation in an AI environment. It is the core of the whole procedure: this Bayesian updating approach was recognized in fact to be the more appropriate way for introducing a form of uncertainty treatment (Casciati-Faravelli, 1990 and 1991). The illustrative example makes reference to the assessment of the seismic vulnerability of existing masonry buildings.

### 2. MODULES FOR AN ES PROTOTYPE

The expert system prototype is conceived to be able to retrieve information from past records, linking all the

relevant facts, and to use such information in specific inspection and simulation tasks, involving mathematical models, probabilistic analysis, graphical interfaces, user interaction, and so on (Faravelli-Gherardini, 1992 a). From the software engineering site an open development environment was selected. Among several commercial competitors, the product NEXPERT (Nexpert Object, 1987) is actually standing up, the main reasons being explained in (Casciati-Faravelli, 1990 and 1992) and (Faravelli-Gherardini, 1991 and 1992 a). The elements to be incorporated in the expert-system prototype are:

1. The data on the building under investigation and the site where it is located - The ES module drives the inspector in measuring the appropriate parameters and in investigating the aspects which dominate the seismic vulnerability of the building. Several updated versions of the ES module were successively made available (Casciati-Faravelli, 1989 a and b, 1992). A version allowing for uncertainty treatment in a form consistent with the subsequent vulnerability assessment (see point 5.) is presently in progress.
2. The data bank of the seismic events recorded by accelerometric networks. - A structural dynamic analysis requires the availability of some ground acceleration records in order to characterize the whole structural response. They must be selected as the ones which satisfy some requisites determined on the basis of seismological considerations. The methodology for building a digital strong-motion catalogue and for selecting a particular ac-

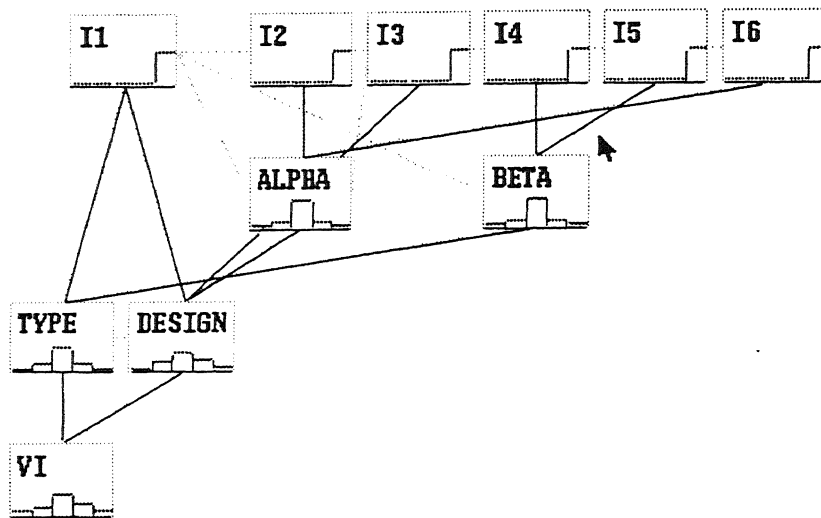


Figure 1: Causal network for vulnerability index assessment. Each node has five possible states. Direction of causality is from top to bottom. For each node, the bar diagram provides the marginal probability distribution in the initial equilibrium state.

celerogram is illustrated in (Casciati-Colombi-De Canio, 1991).

3. The data bank of the seismic events collected in the seismic catalogue. - The relevant ES module queries this data bank for the site under investigation. This consultation is of basic importance when the restoration of historical buildings considered since it provides: i) the sequence of the seismic excitations which engaged the building; ii) the location of the seismic sources around the site; iii) an idea of the actual seismic intensity at the site independently of the relevant macro zonation analysis. Details on the resulting ES module are given in (Faravelli-Gherardini, 1991). In summary, the geographical maps are bitmapped through a simple scanner; zoom in a classical sense is not available, but appropriate mouse clicking permits one to select a map of higher (or lower) scale which includes the site marked by the pointer; each map is preprocessed and this leads to the conversion of pixel coordinates to geographical coordinates. A click over the site of interest gives eventually rise to the list of epicentres inside a circle with assigned radius from the clicked coordinates.
4. The historical files of damage-repair which are often available, for ancient buildings, at city and/or region archives. - The relevant ES module queries

this second data bank for the building under investigation to retrieve the recorded damages, if any, which occurred during the life of the building. Knowledge of the global behaviour of the building during the seismic events collected in 3. is the basis for identifying the structural model to be considered in the dynamic analyses. The best configuration to be given to the damage data is discussed in (Faravelli-Gherardini, 1992 b), where the items of a paper-form studied at a European level for architectural purposes are digitalized and rearranged to meet structural goals. Graphical facilities similar to the ones illustrated in 3. were also implemented: they allow the zoom from the city map to the building and from the building to the topical area.

5. A diagnosis on the 'state of health' of the building on the basis of the pieces of information collected in 1. and retrieved by the consultations in 2., 3. and 4. - The ES module performs this task by computing the relevant vulnerability index (Casciati-Faravelli, 1991). Ongoing studies are aiming at introducing in this module the possibility of a validation of the result by conducting appropriate structural analyses. A basic point is to reach a vulnerability assessment allowing for the uncertainty implicit in the expertise and the one associated with the collected pieces of evidence. The next section of this paper provides the

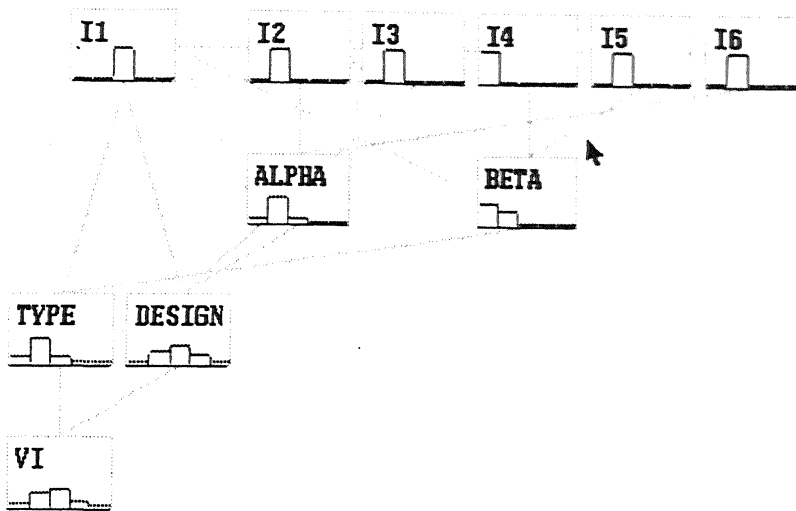


Figure 2: Marginal probabilities in the nodes of the causal network after evidence from the building inspection.

governing relations of the theory of causal probabilistic networks (Lauritzen-Spiegelhalter, 1988; Gherardini, 1990; Gherardini-Mayer, 1991) which will be successively used for modelling the expertise uncertainty. A diagnosis on probabilistic terms will be the final result and represent the original contribution of the paper.

6. The design of the appropriate countermeasures to be introduced when module 5. produces an alert. The development of this ES module is still at an early stage.

All the previous modules were developed within the working environment NEXPERT (Nexpert Object, 1987). Its object oriented view of data makes use of objects and sub-objects; classes and subclasses; definition of properties at class level; inheritance of properties and/or values; dynamic creation/deletion of objects and links. The reasoning representation is pursued by classical rules in the form *IF premises (LHS) THAN hypothesis and DO action (RHS)*. All the hypotheses are Boolean variables referable to in both the LHS and RHS of any rule. Backward and forward reasoning is supported and inference strategies are definable in a dynamic way.

### 3. GOVERNING PROBABILISTIC RELATIONS

As underlined in 5., the pieces of evidence (from monitoring measurements and/or data bank consultation) must be filtered through a propagation network of a probabilistic nature to enter the decision criterion.

One assumes here that:

- the network is identified. A causal network is a directed graph  $G = (V, E)$ : it consists of a set of nodes  $V$  and a set of directed links  $E$ , where the nodes represent conceptual entities and the links express a causal (in a very broad sense) relationship between these entities. In an object oriented framework nodes and objects coincide.
- the probabilistic structure of the relevant conditional probabilities is known. In a probabilistic framework the entities nodes will be in some way associated to random variables. In (Faravelli-Gherardini, 1992 a) they were assumed to have a binary nature (events which can or cannot occur with a given probability  $p$  or  $1 - p$ , respectively). In this paper, five possible discrete events are associated with each entity.

Due to the causality links, the probability distribution sitting on each node is a conditional probability distribution specifying, for each possible configuration of states of the parent variables, the probability of the child variable being in each of its five states.

The qualitative structure, which specifies the causality dependencies, and the quantitative behaviour (probabilistic assessment) of these dependencies form a CPN (Causal Probabilistic Network). It is a static model for a certain knowledge domain. Situations evolving dynamically arise if, at a certain stage, one collects data which specify the value of a subset

Table 1: Numerical values and weights presently adopted in Italy in the assessment of the vulnerability index.

Class	A	B	C	D	Weight
Item	Associated numerical values				
1	0	5	20	45	1.0
2	0	5	25	45	0.25
3	0	5	25	45	1.5
4	0	5	25	45	0.75
5	0	5	15	45	(1)
6	0	5	25	45	0.5
7	0	5	25	45	(2)
8	0	5	25	45	0.25
9	0	15	25	45	(3)
10	0	0	25	45	0.25
11	0	5	25	45	1.0

(1) Minimum between 1 and the ratio of 50 over the percentage of stiff and well connected horizontal structural elements.

(2) 1 in the absence of portico; 0.5 otherwise.

(3) 1.5 for high values of the dead load and low percentage of actually supported roof; 0.75 when either one or the other of the two situations occur; 0.5 otherwise.

of variables in the network. The problem is how this knowledge would modify our conditional probability assessments for the variables which have not been observed? Straight computational procedures to give these problems an answer can become very unefficient in large networks. Alternatively, a correct and efficient probability updating exploits local relations between nodes in the graph: an 'a priori' cumbersome computation is therefore reduced to a sequence of very simple steps, involving only small subsets of variables at a time. At the end, when no further updating is necessary, a new equilibrium is found that reflects the new state of the world. The Nexpert implementation of both the CPN structure and the probability propagation algorithm consists of two steps (Gherardini-Mayer, 1991; Faravelli-Gherardini, 1992 a):

- the original network is transformed into a tree whose nodes are 'cliques', i.e. well defined complete subsets  $C_i$ , of the original node set  $V$ ; these cliques are ordered in such a way that, if any two of them, say  $C_i$  and  $C_j$ , have common elements, than these elements can be found in the intersection set of any two adjacent cliques on the path that connects  $C_i$  and  $C_j$  in the tree. It can be proved that this property guarantees that any new information, on an individual node, is propagated in a consistent and non redundant way;

Table 2: Conditional probability tables for the numerical example. The rows whose elements are zero are not reported. The index  $i_j$ ,  $i_j = 1, \dots, 5$ , denotes one of the 5 possible state for the  $j$ -th node listed in the relation  $\text{Prob} [e_0 | e_1 \dots e_j \dots]$ .

$i_0$	$i_1$	Prob $[\alpha_{i_0}   I_{6i_1}, I_{2i_2}]$					Prob $[\beta_{i_0}   I_{5i_1}, I_{4i_2}]$					
1	1	.9	.8	.1	.0	.4	.9	.4	.1	.0	.0	
	2	.2	.1	.0	.0	.0	.6	.1	.0	.0	.0	
	5	.4	.2	.0	.0	.1						
	1	.1	.2	.8	.6	.6	.1	.6	.8	.0	.6	
	2	.8	.8	.6	.2	.8	.4	.8	.0	.0	.4	
2	3						.5	.0	.0	.0	.0	
	5	.4	.2	.0	.0	.1	.8	.6	.0	.0	.1	
	1	.0	.0	.1	.4	.0	.0	.0	.1	.0	.4	
	2	.0	.1	.4	.8	.2	.0	.1	.1	.0	.6	
	3	.5	.4	.1	.0	.2	.5	.8	.5	.0	.6	
3	4						.6	.4	.0	.0	.0	
	5	.6	.8	.8	.4	.8	.2	.4	.4	.0	.8	
	1						.0	.0	.0	.8	.0	
	2						.0	.0	.2	.8	.0	
	3	.5	.6	.8	.8	.8	.0	.2	.8	.4	.4	
4	4	.4	.4	.2	.1	.4	.4	.6	.6	.1	.8	
	5	.0	.0	.2	.6	.1	.0	.0	.6	.6	.1	
	1						.0	.0	.0	.2	.0	
	2						.0	.0	.0	.2	.0	
	3	.0	.0	.1	.2	.0	.0	.0	.1	.6	.0	
5	4	.6	.6	.8	.9	.6	.0	.0	.4	.9	.2	
	5						.0	.0	.0	.4	.0	
	$i_2 =$	1	2	3	4	5	1	2	3	4	5	
	$i_0$	$i_1$	Prob $[\tau_{i_0}   \beta_{i_1}, I_{1i_2}]$					Prob $[V I_{i_0}   \tau_{i_1}, \delta_{i_2}]$				
	1	1	.9	.8	.2	.0	.5	.9	.4	.0	.0	.0
2		.2	.1	.0	.0	.0	.6	.1	.0	.0	.0	
5							.2	.0	.0	.0	.0	
1		.1	.2	.8	.8	.5	.1	.6	.4	.0	.0	
2		.8	.8	.6	.4	.8	.4	.8	.4	.0	.0	
2	3	.4	.2	.0	.0	.1	.8	.4	.1	.0	.0	
	4						.6	.2	.0	.0	.0	
	5						.2	.0	.0	.0	.0	
	1	.0	.0	.0	.2	.0	.0	.0	.6	.8	.0	
	2	.0	.1	.4	.6	.2	.0	.1	.6	.6	.0	
3	3	.6	.8	.8	.5	.8	.0	.6	.8	.4	.0	
	4	.4	.4	.1	.0	.2	.4	.8	.6	.1	.0	
	5						.8	.6	.4	.0	.0	
	1						.0	.0	.0	.2	.6	
	2						.0	.0	.0	.4	.6	
4	3	.0	.0	.2	.5	.1	.0	.0	.1	.6	.4	
	4	.6	.6	.8	.5	.8	.0	.0	.4	.8	.4	
	5	.2	.2	.2	.1	.2	.0	.4	.6	.6	.1	
	1						.0	.0	.0	.0	.4	
	2						.0	.0	.0	.0	.4	
5	3						.0	.0	.0	.0	.6	
	4	.0	.0	.1	.5	.0	.0	.0	.0	.1	.6	
	5	.8	.8	.8	.9	.8	.0	.0	.0	.4	.9	
	$i_2 =$	1	2	3	4	5	1	2	3	4	5	

- from the original conditional distributions, one derives for each clique a belief table which is (up to a normalizing constant) the joint probability distribution on the nodes belonging to the clique. The propagation scheme then consists of: 1) collection of evidence, in which the new information entered from different branches is propagated to a single node acting as a root, and 2) distribution of evidence, which means that updating factors are propagated back from the root to all other cliques.

When the procedure reaches an equilibrium state, the marginal distributions on single nodes can eventually be derived.

#### 4. ASSESSING THE VULNERABILITY INDEX

##### 4.1 Present practice for masonry buildings

The expert system module performing the site inspection for any given masonry building classifies in four classes (from A to D as the quality decreases) 11 different items: 1) building structural organization; 2) resistant system quality; 3) total shear strength; 4) building site; 5) horizontal elements; 6) map configuration; 7) vertical configuration; 8) interwall distance; 9) roof type; 10) non-structural elements; 11) maintenance condition. A weighted sum of the numerical values in Table I expressing the seismic performance of the previously listed items defines the vulnerability index.

From Table 1 the vulnerability index comes out to range between 0 and 405.

##### 4.2 A new AI procedure

Assume that the classification of the items is made. A CPN which proceeds from to the assessment of a probabilistic vulnerability index is adopted in this section. Of course the CPN is not unique and a large calibration of the required probability matrices would be required before its practical exploitation. Nevertheless, the exemplification shows that the procedure is working and a probabilistic assessment of the seismic vulnerability index is possible.

Under the previous premises, the readability of the example suggests to manage matrices of reasonable size. Thence, without loss of generality, two of the items with lower weight (8 and 10) are neglected in the following reasoning together with items 5, 7 and 9 whose introduction requires more than a single node (due to the particular definition of the relevant weight). Under these assumptions the resulting

Table 2: continued

$i_0$	$i_1$	$P[\delta_{i_0} \alpha_{i_1}, I_{3,2}, I_{1_1}]$					$P[\delta_{i_0} \alpha_{i_1}, I_{3,2}, I_{1_2}]$				
1	1	.9	.6	.0	.0	.0	.8	.4	.0	.0	.0
	2	.6	.2	.0	.0	.0	.4	.1	.0	.0	.0
2	1	.1	.4	.4	.0	.8	.2	.6	.2	.0	.6
	2	.4	.8	.2	.0	.6	.6	.8	.2	.0	.6
3	3	.8	.6	.0	.0	.2	.8	.6	.0	.0	.2
	4	.6	.4	.0	.0	.0	.4	.2	.0	.0	.0
3	1	.0	.0	.6	.4	.2	.0	.0	.8	.4	.4
	2	.0	.0	.8	.4	.4	.0	.1	.8	.2	.4
4	3	.2	.4	.8	.0	.8	.2	.4	.6	.0	.8
	4	.4	.6	.4	.0	.8	.6	.8	.4	.0	.8
5	5	.8	.6	.0	.0	.2	.6	.4	.0	.0	.2
	1	.0	.0	.0	.6	.0	.0	.0	.0	.6	.0
4	2	.0	.0	.0	.6	.0	.0	.0	.0	.8	.0
	3	.0	.0	.2	.8	.0	.0	.0	.4	.8	.0
5	4	.0	.0	.6	.8	.2	.0	.0	.6	.6	.2
	5	.2	.4	.8	.6	.8	.4	.6	.6	.4	.8
5	3	.0	.0	.0	.2	.0	.0	.0	.0	.2	.0
	4	.0	.0	.0	.2	.0	.0	.0	.0	.4	.0
5	5	.0	.0	.2	.4	.0	.0	.0	.4	.6	.0
	$i_2=$	1	2	3	4	5	1	2	3	4	5
$i_0$	$i_1$	$P[\delta_{i_0} \alpha_{i_1}, I_{3,2}, I_{1_3}]$					$P[\delta_{i_0} \alpha_{i_1}, I_{3,2}, I_{1_4}]$				
1	1	.2	.0	.0	.0	.0	.4	.2	.0	.0	.0
	1	.8	.8	.0	.0	.4	.4	.2	.0	.0	.0
2	2	.8	.6	.0	.0	.2	.4	.2	.0	.0	.0
	3	.4	.2	.0	.0	.0	.0	.0	.0	.0	.0
3	4	.2	.0	.0	.0	.0	.0	.0	.0	.0	.0
	1	.0	.2	.8	.2	.6	.6	.8	.4	.0	.8
3	2	.2	.4	.6	.0	.8	.6	.8	.2	.0	.8
	3	.6	.8	.4	.0	.6	.8	.6	.0	.0	.4
4	4	.8	.8	.1	.0	.4	.6	.4	.0	.0	.0
	5	.4	.2	.0	.0	.0	.0	.0	.0	.0	.0
4	1	.0	.0	.2	.8	.0	.0	.0	.6	.8	.2
	2	.0	.0	.4	.8	.0	.0	.0	.8	.6	.2
5	3	.0	.0	.6	.8	.4	.2	.4	.8	.6	.6
	4	.0	.2	.8	.6	.6	.4	.6	.6	.4	.8
5	5	.6	.8	.6	.4	.8	.8	.8	.4	.1	.6
	1	.0	.0	.0	.2	.0	.0	.0	.0	.2	.0
5	2	.0	.0	.0	.2	.0	.0	.0	.0	.4	.0
	3	.0	.0	.0	.2	.0	.0	.0	.2	.4	.0
5	4	.0	.0	.1	.4	.0	.0	.0	.4	.6	.2
	5	.0	.0	.4	.6	.2	.2	.2	.6	.9	.4
$i_2=$		1	2	3	4	5	1	2	3	4	5

vulnerability index ranges between 0 and 225. The evaluation process is therefore schematically simplified into:

1. six input nodes ( $I_1, I_2, I_3, I_4, I_5$  and  $I_6$ ) corresponding to the items of evaluation 1, 2, 3, 4, 6 and 11. For each node five states are considered: the fifth state *unknown* is in fact added to the ones

corresponding to the classes A, B, C and D.

2. one output node providing the probability distribution of the vulnerability index (*VI*), for which five possible discrete states are assumed: (0,12.5), (12.5,48.125), (48.125,96.25), (96.25,173.125) and (173.125,225).

Two nodes ( $\delta$  and  $\tau$ ) belonging to an intermediate layer are then introduced in order to facilitate the reasoning. Also two further nodes ( $\alpha$  and  $\beta$ ) are added in order to cover logical links:

- Design ( $\delta$ ) = global evaluation of the structural design with state ranges (0,8.75), (8.75,32.125), (32.125,66.25), (66.25,120.625) and (120.625,157.5);
- Type ( $\tau$ ) = global evaluation of the architectural design with state ranges (0,3.75), (3.75,15), (15,30), (30,42.5) and (42.5,67.5);
- $\alpha$  = joint effect of the quality of the resistant system and of the maintenance condition assumed to be independent of *I1* and *I3*; the relevant state ranges are (0,3.125), (3.125,12.5), (12.5,25.), (25.,43.75) and (43.75,56.25) ;
- $\beta$  = joint effect of the building site and of the map configuration assumed to be independent of *I1*; the relevant state ranges are (0,3.125), (3.125,12.5), (12.5,25.), (25.,43.75) and (43.75,56.25).

The structure of this knowledge-base is represented by the directed graph in Figure 1. The quantitative knowledge comprises numerical assessments of the probability of each state conditional on all possible configurations of parents. Assessments are given in Table 2, representing the results of present expertise on the basis of a rough calibration with the approach of Section 4.1. The marginal probabilities in the initial equilibrium state, with unknown classification for all the input nodes, are provided in Figure 1.

Assume now that one has the results of the building inspection in the form: item 1 in class C; 2 in B; 3 in B; 4 in A; 6 in B and 11 in class B. The causal probabilistic network, after this evidence, provides the distribution of the vulnerability index *VI* given in Figure 2.

## 5. CONCLUSIONS

This paper illustrates how one can achieve a probabilistic assessment of the 'state of health' of the structure on the basis of pieces of evidence collected by inspection and by querying, in case, historical data

Table 2: continued

$i_0$	$i_1$	$P[\delta_{i_0}   \alpha_{i_1}, I3_{i_2}, I1_5]$				
1	1	.6	.2	.0	.0	.0
	2	.2	.0	.0	.0	.0
	1	.4	.8	.2	.0	.6
	2	.8	.8	.0	.0	.4
2	3	.6	.4	.0	.0	.1
	4	.4	.2	.0	.0	.0
	1	.0	.0	.8	.2	.4
	2	.0	.2	.8	.2	.6
3	3	.4	.6	.6	.0	.8
	4	.6	.8	.2	.0	.6
	5	.6	.4	.0	.0	.0
	1	.0	.0	.0	.8	.0
	2	.0	.0	.2	.8	.0
	3	.0	.0	.4	.8	.1
4	4	.0	.0	.8	.6	.4
	5	.4	.6	.6	.4	.8
	3	.0	.0	.0	.2	.0
	4	.0	.0	.0	.4	.0
5	5	.0	.0	.4	.6	.2
	$i_2=$	1	2	3	4	5

bases. The diagnosis is reached by propagating evidence in a network representing the available expertise. In this first implementation each piece of evidence to be collected by inspection is represented by a Boolean variable.

The next steps in the development of this causal network scheme will introduce 1) a greater number of discrete states for the output (the vulnerability index) and 2) a discrete variable idealization for each input component. This will make possible to incorporate the results of the effort presently in progress towards a probabilistic description of the pieces of evidence arising from 'in situ' investigation.

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