

## Seismic force reduction and displacement amplification factors

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**ABSTRACT:** Seismic force reduction factor (*FRF*) and displacement amplification factor (*DAF*) are derived. These factors are functions of the structural ductility factor and structural overstrength factor. The force reduction factors used in three seismic codes are compared. A single-*FRF* approach is used in the UBC, while a compound-*FRF* approach is used in the NBC of Canada and Eurocode. A comparison of *DAF*'s showed that the ratios between *DAF* and *FRF* are 0.375, 0.6, and 1.0 in the UBC, NBCC, and Eurocode, respectively. Dynamic analyses of two instrumented buildings in California (one 13-story steel moment frame and one 3-story eccentrically braced frame) were conducted to verify the appropriate relationship between *DAF* and *FRF*. It was concluded that the Eurocode's approach of using a *DAF* which is equal to *FRF* is reasonable for estimating the maximum story and roof drifts.

### 1. INTRODUCTION

Modern seismic codes recognize that a well designed and properly detailed structure, when deformed into the inelastic range under severe earthquakes, can dissipate a significant amount of energy. To compute the design seismic force level for strength design, building codes allow the designer to reduce the elastic seismic force demand by a force reduction factor (*FRF*). Since a reduced level of seismic force is used in design, the computed design displacements from an elastic analysis have to be amplified in order to estimate the actual deformations that may develop in severe earthquakes. Building codes usually use a displacement amplification factor (*DAF*) for this purpose.

Although both the *FRF* and *DAF* play an important role in seismic codes, a review of the 1991 Uniform Building Code (*UBC* 1991) of the U.S.A., the 1990 National Building Code (*NBCC* 1990) of Canada, and the 1988 Eurocode (*Structures* 1988) indicates that the format of the *FRF* is very different. Furthermore, the ratios between the *DAF* and the *FRF* also vary considerably from one code to the other. In this paper, the first objective is to present a rational formulation of the *FRF* and *DAF*. Based on this derivation, the second objective is to compare the approach of establishing the value of the *FRF* in each code. The third objective is to compare the *DAF* of each code by examining the ratio between the *DAF* and *FRF*. Since significant differences in this ratio exist among the three codes, dynamic analyses of two instrumented buildings in California were performed to establish an appropriate relationship between the *DAF* and *FRF*. Based on this study, recommendations for selecting a

reasonable *DAF* are presented.

### 2. DERIVATION OF *FRF* and *DAF*

Consider the typical lateral force versus deformation relationship of a structural system shown in Fig. 1. The elastic force demand, expressed in terms of a base shear ratio, for a severe design earthquake is expressed as  $C_{su}$ . Idealizing the actual structural response curve by the linearly elastic-perfectly plastic curve in Fig. 1, the structural ductility factor can be defined as

$$\mu_s = \frac{\Delta_{max}}{\Delta_y} \quad (1)$$

where the deformation is expressed in terms of story drift  $\Delta$ . As a result of ductility, the structure has a capacity to dissipate hysteretic energy. Therefore the elastic design force can be reduced to a yield strength level ( $C_y$ ) by the factor  $R_\mu$ :

$$R_\mu = \frac{C_{su}}{C_y} \quad (2)$$

Note that the yield strength level refers to the structural collapse level, not the level of first significant yielding. Usually a certain amount of equivalent viscous damping ratio, say 5 percent, is considered in the computation of this reduction factor. The reserve strength that exists between the actual structural yield level ( $C_y$ ) and the code's prescribed first significant yield level ( $C_s$ ) is defined in terms of the overstrength factor  $\Omega$ :

$$\Omega = \frac{C_y}{C_s} \quad (3)$$

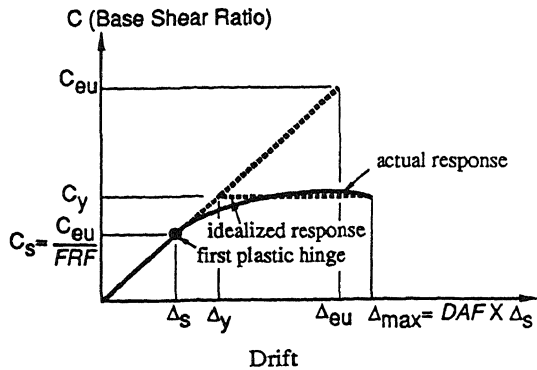


Fig. 1 General Structural Response

Structural overstrength results from redundancy, multiple load combinations, code minimum requirements regarding proportioning and detailing, etc. (Blume 1977).

Based on these definitions, the force reduction factor (*FRF*) for strength design can be derived from Fig. 1 as follows (Uang 1991):

$$FRF = \frac{C_{eu}}{C_s} = \frac{C_{eu}}{C_y} \frac{C_y}{C_s} = R_\mu \Omega \quad (4)$$

The displacement amplification factor (*DAF*) can also be derived from Fig. 1 as follows:

$$DAF = \frac{\Delta_{max}}{\Delta_s} = \frac{\Delta_{max}}{\Delta_y} \frac{\Delta_y}{\Delta_s} \quad (5)$$

where  $\Delta_{max}/\Delta_y$  is the structural ductility factor (see Eq. 1) and  $\Delta_y/\Delta_s$  from Fig. 1 is equal to

$$\frac{\Delta_y}{\Delta_s} = \frac{C_y}{C_s} = \Omega \quad (6)$$

Therefore Eq. 5 can be expressed as

$$DAF = \mu_s \Omega \quad (7)$$

From the above derivations, it is observed that both the *FRF* and *DAF* are functions of the structural overstrength factor, structural ductility factor, and damping ratio — the effect of damping is generally included in the ductility reduction factor  $R_\mu$ .

### 3. COMPARISON OF *FRF*

Table 1 lists the *FRF* and *DAF* used in three codes, where  $R_w$ ,  $R$ , and  $q$  factors are used in the UBC, NBCC, and Eurocode, respectively. Since the UBC is the only code which specifies a seismic design force level for working stress design, to compare it with the other two codes for strength design the *FRF* ( $= R_w$ ) and the *DAF* ( $= 3R_w/8$ ) have been divided by a load factor 1.5 (Minimum 1990). The *FRF* used in each

Table 1 *FRF* and *DAF* in Building Codes

Building Code	<i>FRF</i>	<i>DAF</i>	$\frac{DAF}{FRF}$
UBC (1991)	$\frac{R_w}{1.5}$	$\frac{3R_w}{8(1.5)}$	0.375
NBCC (1990)	$\frac{R}{0.6}$	$R$	0.6
Eurocode (1988)	$q$	$q$	1.0

code is discussed as follows.

**UBC:** The UBC combines the contributions of structural ductility and overstrength into a single *FRF*. Although the contribution of overstrength is not specified in the code, the high values assigned to the *FRF* appear to suggest that this beneficial contribution has been considered (see Table 2 for *FRF* values assigned to steel moment-resisting frames).

Table 2 *FRF* for Steel Moment-Resisting Frames

Building Code	<i>FRF</i>
UBC (1991)	$\frac{R_w}{1.5} = \frac{12}{1.5} = 8.0$
NBCC (1990)	$\frac{R}{U} = \frac{4}{0.6} = 6.7$
Eurocode (1988)	$5\Omega \leq 8.0$

Although simplicity may be the primary reason to use a single *FRF*, the disadvantage is that designers do not know (and are not required by code to verify) whether the structure has "sufficient" overstrength to limit the ductility demand.

**NBCC:** The NBCC expresses the *FRF* as follows:

$$FRF = \frac{R}{U} \quad (8)$$

where  $U$  ( $= 0.6$ ) is defined as a calibration factor. A comparison of Eqs. 8 and 4 indicates that the NBCC uses the following factors:

$$R_\mu = R \quad (9a)$$

$$\Omega = \frac{1}{U} = \frac{1}{0.6} = 1.67 \quad (9b)$$

That is, the NBCC uses a "compound" *FRF* which includes explicitly a structural ductility reduction factor ( $= R$ ) and a constant structural overstrength factor ( $= 1/U$ ) of 1.67. Although numerically the NBCC approach does not have any advantage over the UBC single-*FRF* approach, using a compound *FRF* provides an opportunity to "fine-tune" individual contributions in the future. For example, using a constant

*FRF* value for the entire period range has long been criticized (Bertero 1986). If desired, the period-dependent nature of the ductility reduction factor  $R_\mu$  (Newmark and Hall 1982, Riddell et al. 1989) can be incorporated easily into the existing *FRF* formula. Furthermore, the compound-*FRF* approach can incorporate easily the need, if desired, to use a higher structural overstrength factor if designers compute it. Then designers should be allowed to use a much larger *FRF*. For example, a multistory steel frame whose design is controlled by the drift limit is likely to have an  $\Omega$  much larger than 1.67.

*Eurocode*: For steel moment frame design, the following *FRF* is used (*European* 1988):

$$FRF = 5 \left( \frac{\alpha_\mu}{\alpha_1} \right) \leq 8 \quad (10)$$

Since  $\alpha_1$  and  $\alpha_\mu$  are the base shear ratios which correspond to the formation of the first plastic hinge and collapse mechanism, respectively, it follows from Eq. 4 that the Eurocode uses the following factors:

$$R_\mu = 5 \quad (11a)$$

$$\Omega = \frac{\alpha_\mu}{\alpha_1} \quad (11b)$$

The Eurocode, like the NBCC, uses a compound *FRF*. A constant ductility reduction factor is specified for each lateral-load-resisting system. Nevertheless, the Eurocode requires designers to compute a structural overstrength factor; otherwise this factor has to be taken as 1.2, which is more conservative than the 1.67 of NBCC. It should be noted that an *FRF* like the one in Eq. 10 implies that an iterative procedure has to be used in order to determine design seismic forces; this is not the case for the UBC and NBCC.

#### 4. EVALUATION OF *DAF*

Table 1 also lists the *DAF* used in three codes. Since each code uses a different *FRF*, the most logical way to evaluate the *DAF* would be to normalize it by the *FRF*. The last column of Table 1 indicates that, on one extreme, the *DAF* is equal to the *FRF* in the Eurocode. At the other extreme, the *DAF* is only three-eighths the *FRF* value in the UBC. The reliability of the *DAF* used in these codes is assessed as follows.

From Eqs. 4 and 7, the ratio between *DAF* and *FRF* is

$$\frac{DAF}{FRF} = \frac{\mu_s \Omega}{R_\mu \Omega} = \frac{\mu_s}{R_\mu} \quad (12)$$

Note that the structure overstrength factor does not play a role in this ratio. For a single-degree-of-freedom system, the ratio between  $\mu_s$  and  $R_\mu$  can be

expressed as follows (Newmark and Hall 1982):

- (1) In the velocity and displacement amplification regions:

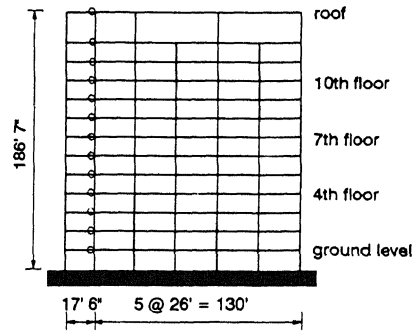
$$\frac{\mu_s}{R_\mu} = \frac{\mu_s}{\mu_s} = 1 \quad (13a)$$

- (2) In the acceleration amplification region:

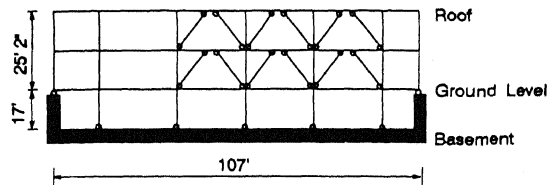
$$\frac{\mu_s}{R_\mu} = \frac{\mu_s}{\sqrt{2\mu_s - 1}} \geq 1 \quad (13b)$$

If a multistory building tends to deform as a single-degree-of-freedom system (which appears to be a valid assumption if the structure tends to deform into a global mechanism in severe earthquakes), it follows from Eqs. 12 and 13 that the *DAF* should not be less than *FRF*. Note that the *DAF* used in the UBC and NBCC is smaller than the *FRF*.

To further verify whether the *DAF* should be no less than *FRF* for multistory buildings, a parametric study has been conducted on the dynamic responses of two multistory buildings located in California. These two buildings have been instrumented by CDMG (Shakal et al. 1989). The first one is a 13-story office building with a steel moment-resisting space frame, which is designated as CSMIP No. 57357 by CDMG. Fig. 2a shows the elevation of an interior frame.



(a) CSMIP No. 57357



(b) CSMIP No. 58496

Fig. 2 Elevations of Two Steel Structures

The fundamental period identified from the responses recorded during the 1989 Loma Prieta Earthquake is

2.2 seconds. The second building, designated as CSMIP No. 58496, is a 3-story steel hospital building with an eccentrically dual braced system (see Fig. 2b); the measured fundamental period is 0.3 seconds. The dynamic response of each frame was analyzed by the computer program DRAIN-2D (1973). As input motions, the recorded building base motions together with the eight historical earthquake records (see Table 3) were used.

Table 3 Earthquake Records

Earthquake	Station	Comp.
Imperial Valley (1940)	El centro	S00E
Washington (1949)	Olympia	S86W
Kern County (1952)	Taft	S69E
Parkfield (1966)	Cholane	N85E
San Fernando (1971)	Pacoima Dam	S16E
Imperial Valley (1979)	I.V.C.	S40E
Loma Prieta (1989)	Carrolita	S00E
Loma Prieta (1989)	Santa Cruz	S90E

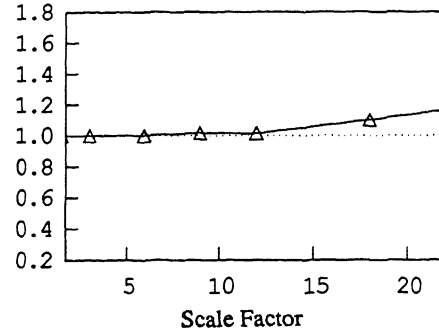
By scaling the intensity of each earthquake record to different levels, the roof drift and story drifts from the inelastic dynamic analyses were compared to those that would develop if the structure were to respond elastically. Referring to Fig. 1, the ratio between the inelastic drift and elastic drift is the same as the ratio between  $DAF$  and  $FRF$  because:

$$\frac{\Delta_{max}}{\Delta_{eu}} = \frac{DAF \times \Delta_e}{FRF \times \Delta_e} = \frac{DAF}{FRF} \quad (14)$$

Both the ratios obtained from roof drift and story drift were evaluated. The information obtained from the ratio of story drift can be used to determine a suitable  $DAF$  for estimating the story drift, while the ratio of roof drift can be used to determine an appropriate  $DAF$  for estimating the required building separations in order to avoid pounding, which during the Loma Prieta Earthquake has been shown to be a problem for many buildings (Kasai and Maison 1990). For each scaled earthquake, the scale factor is defined as the ratio between the average pseudo-acceleration around the fundamental period of the structure and the UBC prescribed design base shear ratio. For each scale factor, results for nine earthquakes were then averaged.

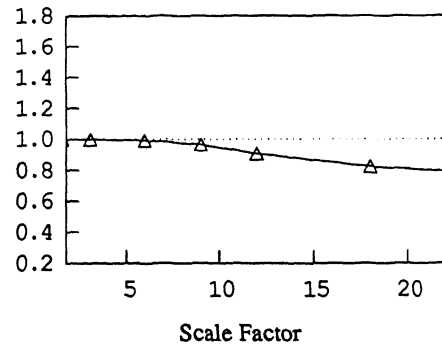
Consider first the 13-story building. For story drift estimation, Fig. 3a shows that the  $DAF$  versus  $FRF$  ratio tends to increase to a value slightly above one as the intensity of the earthquake is increased. For roof drift estimation, the ratio in Fig. 3b tends to drop below one as the intensity is increased. Nevertheless, within the practical range of interest, the scale factor which corresponds to the UBC severe design earthquakes is equal to 12 ( $= R_w$ ) for this type of framing

$$\frac{\Delta_{max}}{\Delta_{eu}} = \left( \frac{DAF}{FRF} \right)$$



(a) Ratio of Story Drifts

$$\frac{\Delta_{max}}{\Delta_{eu}} = \left( \frac{DAF}{FRF} \right)$$



(b) Ratio of Roof Drifts

Fig. 3  $\Delta_{max}/\Delta_{eu}$  Ratios for Building CSMIP No. 57357

system (UBC 1991). In this range a  $DAF/FRF$  ratio equal to one is appropriate for design purposes. In other words, the values of the  $DAF$  and  $FRF$  are identical.

Next consider the 3-story eccentrically braced dual system. Figure 4 shows the ratios for both the roof drift and story drift. For such a stiff structure, Fig. 4a shows a trend similar to that observed in Fig. 3a for estimating maximum story drift. For roof drift, Fig. 4b indicates that the  $DAF/FRF$  ratio may be larger than one when the scale factor is large. But by considering the practical range of interest, which again corresponds to a scale factor equal to 12 ( $= R_w$ ) for this type of braced dual system, a  $DAF/FRF$  ratio equal to one as recommended for the long-period moment frame is also valid.

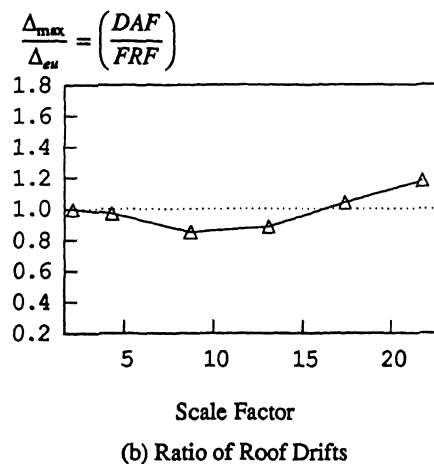
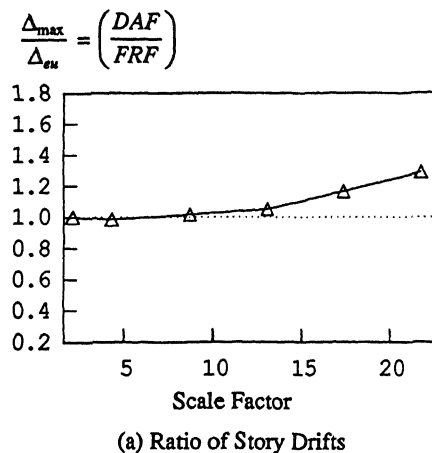


Fig. 4  $\Delta_{max}/\Delta_{eu}$  Ratios for Building CSMIP No. 58496

## 5. CONCLUSIONS

The main conclusions of this study are summarized below.

(1) Simple expressions for the seismic force reduction factor (*FRF*) and displacement amplification factor (*DAF*) have been presented. These two factors are functions of the structural overstrength factor, structural ductility factor, and equivalent viscous damping ratio — the effect of damping is generally included in the ductility reduction factor  $R_{\mu}$ .

(2) The UBC uses a single-*FRF* approach; the contributions from structural ductility and structural overstrength are not specified in the code. NBC of Canada uses a compound-*FRF* approach; a structural ductility reduction factor (= *R*) is specified for each lateral-load-resisting system, and a constant structural overstrength factor equal to 1.67 is used for all systems. The

Eurocode also uses a compound-*FRF* approach. A structural ductility reduction factor is specified for each lateral-load-resisting system. But designers have to compute the structural overstrength factor; otherwise a conservative value of 1.2 should be used.

(3) The *DAF* versus *FRF* ratios are 0.375, 0.6, and 1.0 for the UBC, NBCC, and Eurocode, respectively. Based on a simple derivation, the *DAF/FRF* ratio should not be less than one if the structure tends to deform into a global mechanism in severe earthquakes. Dynamic analyses of two instrumented buildings with distinct fundamental periods (2.2 versus 0.3 seconds) were also conducted to evaluate the UBC and NBCC approaches of using a *DAF* which is smaller than *FRF*. The results show that the *DAF* for estimating story drift is slightly larger than *FRF*; to estimate roof drift in order to avoid pounding, this study shows that the *DAF* can be slightly lower than the *FRF*. For simplicity, it is recommended that a *DAF* equal to *FRF*, which is the approach used by the Eurocode, be used for estimating both the story drift and roof drift.

## 6. ACKNOWLEDGEMENTS

This research was funded by the National Science Foundation (Grant No. CES-8810563) and the California Department of Conservation, Division of Mines and Geology, Strong Motion Instrumentation Program (Grant No. 1090-526). Dynamic analyses performed by Mr. Ahmed Maarouf are greatly appreciated. Any opinions, discussions, findings, conclusions, and recommendations are those of the author and do not necessarily reflect the views of the sponsors.

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