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Nonlinear response spectra for earthquake resistant design

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ABSTRACT: This paper summarizes the results of a comprehensive statistical study of nonlinear response spectra in which 124 ground motions recorded during various earthquakes was considered. Special emphasis is given to the influence of soil conditions on the inelastic strength and deformation demands of single-degree-of-freedom systems. The study included both a deterministic and a probabilistic approach. For each soil group, mean and mean plus one standard deviation spectral ordinates were computed for: inelastic strength demands, strength reduction factors, inelastic displacement demands, and inelastic displacement ratios. The paper also presents an evaluation of the dispersion and distribution of spectral ordinates, as well as probabilistic nonlinear spectra computed from the observed probability distribution. Results and conclusions from this study can be directly used in design of new structures, the evaluation of existing structures and for the evaluation of present seismic design provisions.

1. INTRODUCTION

There is a general consensus that the greatest source of uncertainty in the determination of the response of structures to earthquake ground motions is that associated with the prediction of the intensity and characteristics of the seismic input. Since the concept of response spectrum was developed in the late 30's response spectra have been widely used to estimate strength demands on structures imposed by earthquake ground motions.

A number of statistical studies have been conducted over the years with the purpose of improving the knowledge on design spectra. These studies have been improved in time as more earthquake ground motions have been recorded. Linear elastic response spectra provide a reliable tool to estimate the level of forces and deformations developed in structures responding elastically during the earthquake. There has been a good number of statistical studies that, by considering a large number of recorded ground motions, have investigated the characteristics of linear elastic response spectra (Newmark et al. 1973, Seed et al. 1974, Mohraz et al. 1975, Katayama et al. 1978, Kiremidjian et al. 1980). During strong earthquakes, however, present seismic design philosophy accepts structural and non-structural damage. Thus, buildings designed according to this philosophy are likely to experience significant inelastic excursions which produce deformations and reductions in seismic forces which cannot be predicted with the use of linear elastic models. Typically, statistical studies that have included nonlinear behavior have only considered a small number of recorded ground motions (Veletsos 1969, Ridell et al. 1979). Recently, Ridell et al. (1989) studied strength reductions due to nonlinear behavior by using 53 ground motions, mostly recorded in South America. Similarly, Krawinkler et al. (1990) studied strength reductions by using 33 ground motions recorded during the 1987 Whittier Narrows, California earthquake. However, the effect of soil conditions was not taken into account in any of these two studies. A complete bibliography review on statistical studies of linear and nonlinear spectra has recently been compiled by Miranda (1991).

The objective of this paper is to present a summary of the results of a comprehensive statistical study of inelastic strength and deformations demands of single-degree-of-freedom (SDOF) systems in which a large number of recorded ground motions was considered.

2. SYSTEMS AND MOTIONS CONSIDERD

In order to improve the present knowledge on the characteristics of ground motions and of current methods for estimation of seismic demands on inelastic systems, a comprehensive statistical study of nonlinear was conducted.

2.1 Method of analysis and systems considered

Constant ductility nonlinear spectra were com-

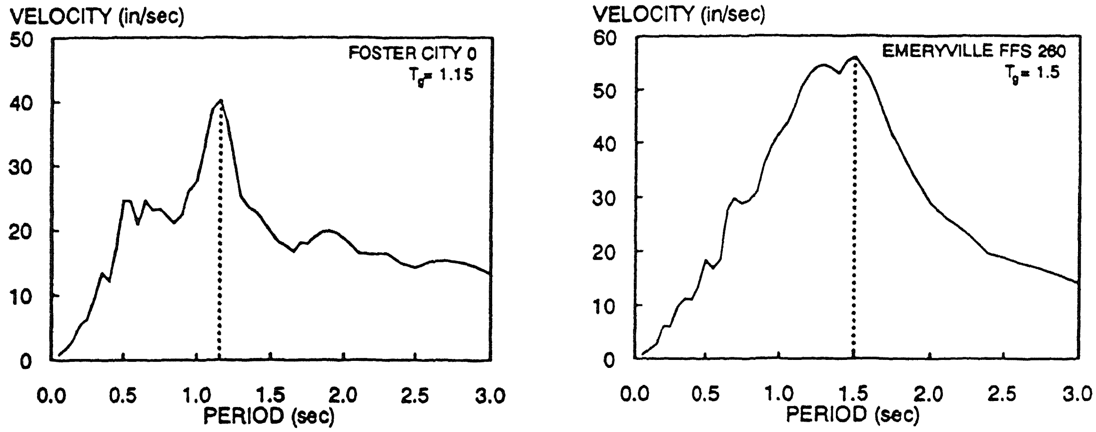

Figure 1. Predominant ground period computed for two soft soil sites in California.

puted for 124 ground motions by iterating on the yielding strength of SDOF systems until the target ductility ratio was reached. The spectra were computed for a set of 50 periods. The following values of displacement ductility, μ, were selected for this study: 1 (elastic), 2, 3, 4, 5 and 6. Iteration was done using the secant method and the iteration was successful when the computed ductility was within 1% of the specified (target) ductility. For cases where multiple roots exist, that is, systems with different yielding strengths exhibit the same ductility demand under a given ground motion, the computer program used in this study yields the root corresponding to the largest strength. This largest strength is the minimum strength required (i.e., strength that needs to be supplied) in order to limit the ductility demand to the target ductility.

Due to the large number of records and the computational effort involved in calculating constant ductility nonlinear spectra, the study was limited to bilinear systems with a post-elastic stiffness of 3% of the elastic stiffness and with a damping ratio of 5% of the critical. On each iteration, response time histories were computed by numerical step-by-step integration using the linear acceleration method with a variable time step to minimize energy equilibrium violations when changes in stiffness occur.

2.2 Ground motions considered

For this study 124 ground motions records were selected, with emphasis on those recorded in California and on those recorded during the last six years. The ground motions were classified into three groups according to the geologic conditions at the recording station. These groups were rock, alluvium and very soft soil. Complete listing of all records and their classification can be found in Miranda (1991).

3. STATISTICAL STUDY

As mentioned above the constant ductility nonlinear spectra were computed for 50 periods. For ground motions recorded on rock or alluvium sites a, fixed set set of periods between 0.05 and 3 seconds was used, while for motions recorded on very soft soil, the spectra were computed for a fixed set of T/T_g ratios, where T_g is the predominant period of the site. For this study T_g is defined as the period corresponding to the maximum spectral velocity of systems responding elastically. Figure 1 shows two examples on the computation of the predominant period for two sites in California.

After computing the constant displacement ductility nonlinear response spectra for all ground motions, statistical studies were conducted on the following parameters:

3.1 Normalized inelastic strength demands

Two sets of normalized inelastic strength demand spectra were computed for each record. The first set used peak ground acceleration (PGA) as normalizing parameter, then the normalized strength demand is given by η which is defined as

$$\eta = \frac{C_y}{\ddot{u}_{g_{max}} / g} \tag{1}$$

where $\ddot{u}_{g_{max}}$ is the peak ground acceleration, g is the acceleration due to gravity and C_y is the structure's yielding seismic coefficient defined as the yielding strength divided by the weight of the system.

The second set used effective peak ground acceleration, EPA, (as defined in the ATC-3-06 and NEHRP seismic provisions) as normalizing parame-

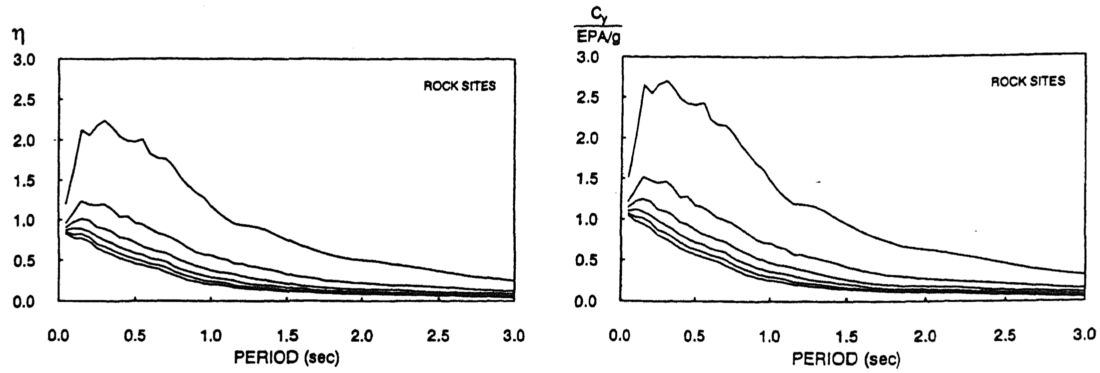

Figure 2. Mean strength demands of ground motions recorded on rock when normalized using PGA and EPA (μ=1,2,3,4,5,6 from top line to bottom line).

ter. Mean normalized spectra were computed for each soil group. Mean inelastic strength demand spectra for ground motions recorded at rock sites are shown in Fig. 2. In order to provide strength demand spectra with a smaller probability of exceedance, mean plus one standard deviation spectra were also computed. Study of the dispersion of the inelastic strength demands was conducted by means of computing coefficients of variation, COV, (defined as the ratio of standard deviation to the mean) for each level of ductility. An example of COV of strength demands (normalized by PGA) for ground motions recorded on rock is presented in Fig. 3.

3.2 Strength reduction factors

For each soil group and for each level of inelastic deformation (i.e. displacement ductility ratio), the strength reduction due to nonlinear hysteretic behavior in the system was computed. This reduction factor, R_μ, is defined as the ratio of the elastic strength demand (strength required to maintain the system elastic) to the strength demand on an inelastic system undergoing a certain ductility demand μ_i,

$$R_\mu = \frac{C_y(\mu=1)}{C_y(\mu=\mu_i)} \qquad (2)$$

For design purposes this reduction factor corresponds to the maximum reduction in strength that can be taken to limit the displacement ductility demand to a certain value μ_i.

This ratio was computed for a total of 31,000 different SDOF systems (the product of 124 ground motions, 50 periods and 5 levels of displacement ductility). Mean spectra were computed for each soil group and each level of ductility. Figure 4 shows mean values for ground motions recorded on rock and very soft soil for six levels of ductility.

In order to study the reliability of the computed strength reduction factors for each soil group and each level of ductility the statistical study included the computation of COV's as well as reduction factors spectra corresponding to mean minus one standard deviation. Examples of COV's for ground motions recorded on alluvium and for ductility ratio of 2 and 5 are shown in Fig. 5.

3.3 Normalized inelastic displacement demands

Using again PGA and EPA as normalizing parameters, mean displacement demands and their dispersion was computed for each soil group and each ductility ratio.

3.4 Inelastic displacement ratio

The inelastic displacement ratio is defined as the ratio of the maximum inelastic deformation of a system undergoing a certain ductility ratio to the max-

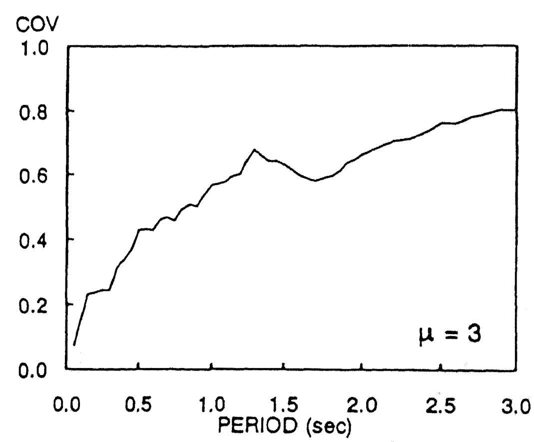

Figure 3. Coefficients of variation of strength demands (normalized by PGA) for ground motions recorded on rock and for μ=3.

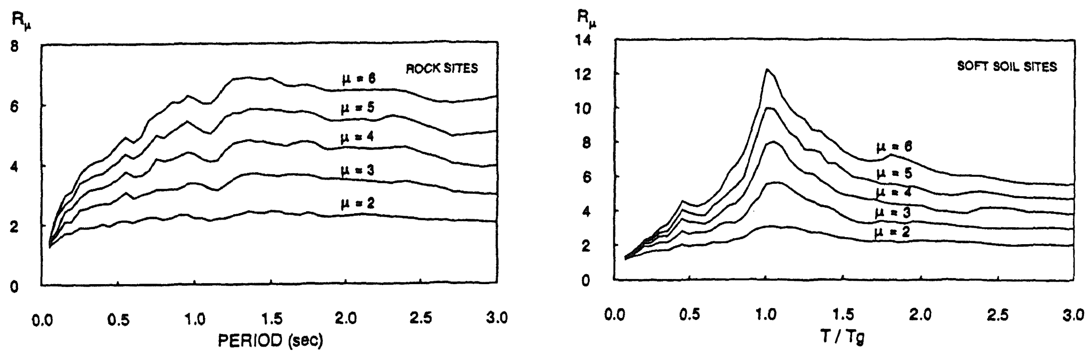

Figure 4. Mean of strength reductions due to nonlinear behavior for rock sites and for soft soil sites.

imum deformation of a system undergoing a ductility ratio of one (i.e., which remains elastic). An example of inelastic displacement ratio spectra corresponding to mean values and ground motions recorded on alluvium and soft soil sites are shown in Fig. 6.

3.5 Probability distribution of spectral ordinates

A more rational consideration of the uncertainty in seismic demands is to look at the frequency distribution of spectral ordinates. For each soil group and each normalizing parameter 300 frequency histograms were computed. Figure 7 shows an example for T=0.3 seconds and ductility of 3 for motions recorded on alluvium.

Attempts to fit the computed statistical data to various theoretical probability density functions were made by plotting the sorted data against the quantiles of five different theoretical probability distribution functions, PDF (Normal, Lognormal, Rayleigh, Gamma and Gumbel type I). An example using the Gamma distribution is shown in Fig. 8.

3.6 Probabilistic nonlinear response spectra (PNRS)

These spectra provide a rational way to design since they allow the designer the option of selecting the confidence level for which a certain response parameter is likely to be exceeded. Murakami and Penzien (1975) and more recently Conte et al.(1990) computed PNRS using artificially-generated accelerograms combined with the use of theoretical PDFs (Gumbel Type I in the former study and Lognormal in the latter study).

A PNRS is computed by obtaining the nonlinear response spectra for different probabilities of non-exceedance of a certain random variable (i.e., response parameter). If the selected random variable is the displacement ductility demand, then the probability of having a ductility demand less than or equal to a certain ductility μ_i is given by

$$P(\mu_i) = P(\mu \le \mu_i) = \int_{-\infty}^{\mu_i} p(\mu)\, d\mu \qquad (3)$$

where $p(\mu)$ is the PDF of μ. The corresponding pro-

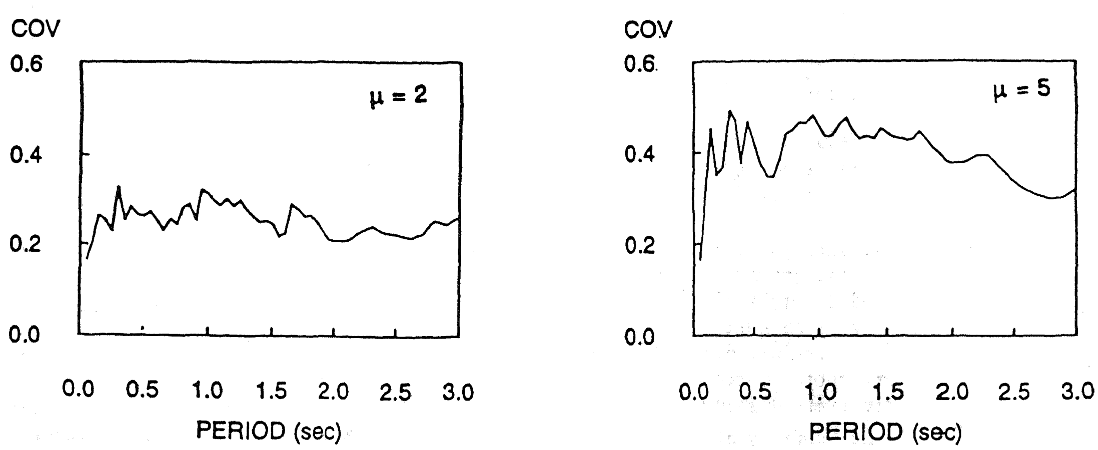

Figure 5. Coefficients of variation of strength reduction factors for alluvium sites and ductilities of 2 and 5.

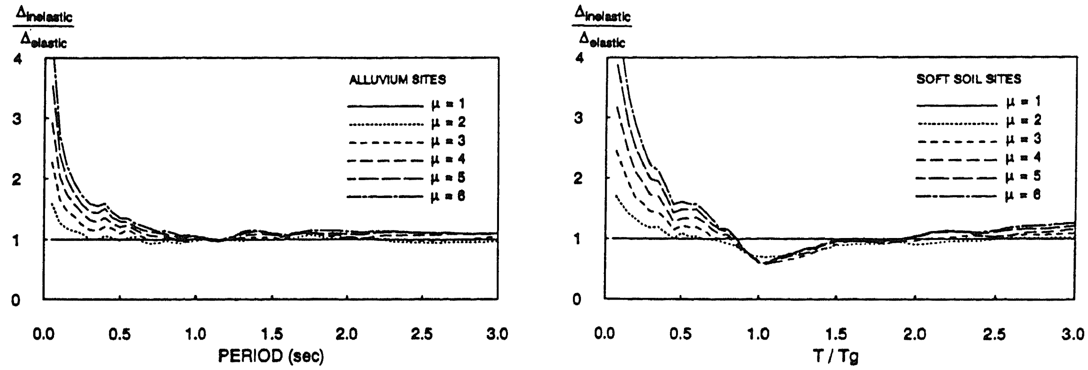

Figure 6. Mean of ratio of inelastic to elastic displacement demands for alluvium sites and for soft soil sites.

bability of exceedance is given by

$$P(\mu>\mu_i) = \int_{\mu_i}^{\infty} p(\mu)\, d\mu = 1-P(\mu_i) \qquad (4)$$

In this study, PNRS were computed using the previously computed constant ductility nonlinear spectra of 124 ground motions and using the actual (observed) probability distribution. The advantage of using the observed probability distribution instead of the a theoretical probability distribution is that the resulting PNRS are based on no assumption on the probability distribution of spectral ordinates. The PNRS were computed for six confidence levels (probability of non-exceedance of a certain ductility demand) namely p = 50, 60, 70, 80, 90, and 95%, which correspond to probabilities of exceedance (probability of ductility demands larger than the maximum specified) of 50, 40, 30, 20, 10, and 5%, respectively. For each period, ductility level, and confidence level the normalized strength demands (quantile) were computed through integration (summation) of the area under the observed PDF (i.e., frequency histogram).

Two sets of PNRS were computed, one for strength demands normalized by PGA, and the other for strength demands normalized by EPA. An example of the former for p=0.8 and ground motions recorded on rock is shown in Fig 9.

4. CONCLUSIONS

Based on the results obtained from the statistical study, a number of general conclusions are made

(i) The shape of nonlinear response spectra differs significantly from the shape of elastic response spectra. Thus, direct scaling by the use of a period-independent factor, of the elastic spectra to obtain

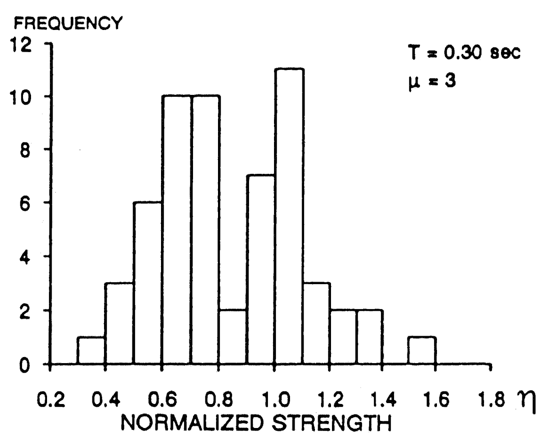

Figure 7. Frequency histogram of normalized strength demands for alluvium sites (T=0.4 sec and μ=3).

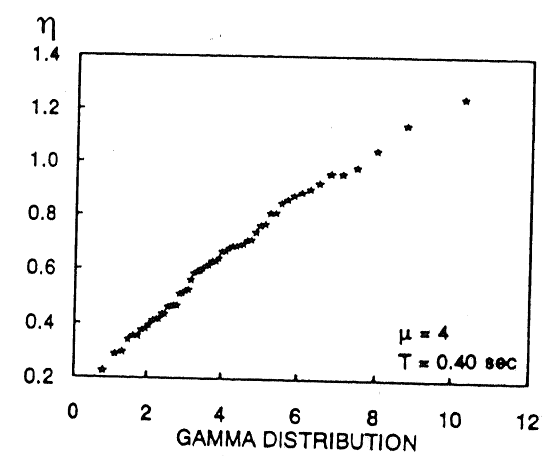

Figure 8. Gamma probability fit of normalized strength demands for alluvium sites (T=0.4 sec and μ=4).

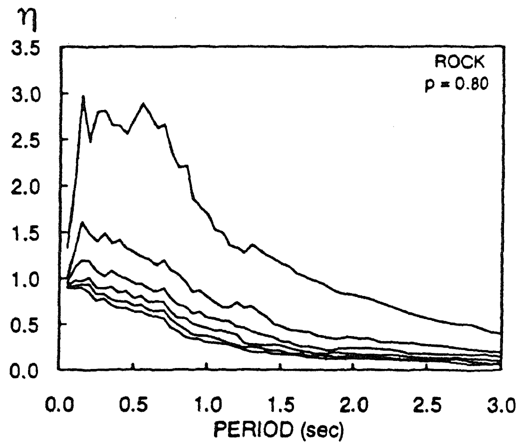

Figure 9. Probabilistic nonlinear spectra for strength demands (normalized by PGA) for rock sites and a 20% probability of exceedance (μ=1,2,3,4,5,6 from top to bottom).

inelastic strength demands is not rational nor conservative.

(ii) The use of either PGA or EPA to normalize inelastic strength demands results in an increase in dispersion (coefficient of variation) with increasing period. In both cases the dispersion was found to be independent of the level of inelastic deformation (i.e., ductility ratio).

(iii) Strength reduction factors were found to be strongly influenced by the level of inelastic deformation, local site conditions, and the period of vibration. Unlike the dispersion of inelastic strength demands, the dispersion of strength reduction factors was observed to be independent of the period of vibration. This dispersion was found to increase as the level of inelastic deformation increased.

(iv) Periods which limit the use of elastic analyses to estimate inelastic displacement demands were observed to depend on the level of inelastic deformation.

(vi) The reliable estimation of inelastic strength and deformation demands of structures on soft soils requires the estimation of the predominant site period.

(vii) It was found that with the exception of the extreme values, inelastic spectral ordinates may be assumed to have a Gaussian probability distribution. An improved fit of extreme values was obtained using a gamma probability distribution.

5. ACKNOWLEDGEMENTS

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