Seismic hazard uncertainties in rational building code decisions

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ABSTRACT: Fundamental uncertainties (ambiguity) in seismic hazard analyses vary widely, both between lower and higher peak ground accelerations at a given site and, in many countries, from site to site. This paper examines how these uncertainties can be included in seismic provisions of building codes. The reasonings are based on a recently-developed normative theory of ambiguity in rational decision making. This model includes the possibility of applying a penalty to safety goals to account for fundamental uncertainties in the failure probabilities. It is not clear, however, that the costs of such a penalty are justified in public policy decisions. Although ambiguity-aversion is generally in line with people's preferences and can be considered "prudent," it can also result (like the classic risk aversion) in a suboptimal allocation of risk management resources.

1. FUNDAMENTAL UNCERTAINTIES VS RANDOMNESS IN COST-BENEFIT ANALYSIS

1.1 Cost-benefit analysis under uncertainty

When balancing costs and benefits in building code decisions, one faces two fundamental issues: (1) what cost of human safety is acceptable and (2) how should uncertainties about the risk itself be accounted for? For safety regulations in the U.S., "value of life" figures in the order of 2 million dollars are generally accepted (Paté, 1979) implying, for instance, a willingness-to-pay of $20 per person per year to eliminate an annual individual risk of 10^{-5}. The focus of this paper is on the second question: what should be the treatment of fundamental uncertainties in the economic assessment of risks and benefits. Should the benefits of seismic code provisions be evaluated as the expected value of the avoided losses? Should there be a provision for society's "risk aversion" (i.e., a willingness to pay a "risk premium" to avoid large amounts of human and monetary losses)? Should there be an additional provision for uncertainties about the probabilities of different levels of losses?

A distinction is made here between the uncertainty faced when the probability of prospective outcomes are "firm" and unambiguous (aleatory uncertainties or randomness) and the risk uncertainties about the probabilities themselves (ambiguity or epistemic uncertainties) due to lack of fundamental knowledge. Epistemic uncertainties sometimes emerge as disagreements among experts and are reflected by the dispersion of the probabilities encoded as expert opinions. In seismic hazard analyses, epistemic uncertainty comes from lack of knowledge about the seismic sources, about mechanisms of earthquake occurrence and maximum magnitudes for the different sources, and about attenuation functions. Furthermore, for each code option, there are fundamental uncertainties about the seismic capacities of different systems. The focus here is on the seismic loads.

1.2 Ambiguity in seismic risk mitigation

Rationality is defined by the axioms of expected utility decision theory (von Neuman and Morgenstern, 1944), which allows for risk aversion and the systematic payment of a premium
to avoid the possibility of large losses. EUDT implies that, when the annual probability of exceedence of given levels of peak ground accelerations (pga's) are themselves uncertain, the relevant characteristic for decision making is the mean of the probability distributions of the probability of different loss levels for each possible seismic code option. Consequently, in that framework, it is the mean failure probability that must meet the safety goals regardless of the dispersion of its distribution (Rosenblueth and Lomnitz, 1976).

In reality, descriptive decision research shows that people are often averse to ambiguity (Ellsberg, 1961; Einhorn and Hogarth, 1986) and, for example, are willing to trade an ambiguous probability of failure with a given mean for a firm failure probability higher than this mean (Der Klurghlan, 1988). Several normative theories, that diverge from EUDT, have been proposed to introduce aversion to (or preference for) ambiguity in rational decision making (e.g., Fishburn, 1990). Such a model was recently developed and its implications for building code decisions under epistemic uncertainties examined (Pate-Cornell and Pischbeck, 1990).

In this model, epistemic and aleatory uncertainties are separated, combining all fundamental uncertainties on one hand and all aleatory uncertainties (probabilities of failure for each set of models and parameter values) on the other. The classical utility function for "firm" probabilities of outcomes ($u_1$) is applied to the aleatory level leading to a distribution of expected utilities due to epistemic uncertainties. A second utility function ($u_2$) is then applied to this distribution of expected values of $u_1$'s. This model is supported by a set of axioms of rationality, similar in nature to the EUDT axioms within each level, but requiring at the interface between the two levels a second utility function to represent non-indifference to ambiguity. The model leads to the maximization of nested utilities ($\max (E_{u_2} (E_{u_1} (X)))$, $X$ being the outcome random variable). This is the framework which is used here to examine the implications of aversion towards ambiguity for seismic provisions of building codes.

1.3 Variable standards as a function of fundamental uncertainties

The uncertainties in seismic hazard analyses vary widely, both between lower and higher peak ground accelerations at a given site, and, in many countries, from site to site (McGuire and Shedlock, 1981; Bernreuter et al. 1988; EPRI, 1988; Whitman, 1989). For example, in the United States, many believe that the uncertainties vary largely from East to West, these uncertainties being greater in the East in the central part of the distribution (annual probabilities of exceedence of given levels of pga's), and greater in the West at the higher end of the distribution. Probability-based codes use a relatively high probability of ground motion intensity (10% in 50 years) coupled with a safety factor that is implicit in the other code provisions. Consequently, as is discussed further, the current safety decision process appears to provide a higher level of safety in the Western States. This apparently higher safety level, however, can be justified by an aversion to ambiguity (or risk uncertainties). Whether or not, however, such ambiguity aversion is prudent and desirable in regulatory decisions is as debatable as the desirability of risk aversion.

1.4 Invariance of the risk assessment method

It is essential to note that introducing a distinction between epistemic and aleatory uncertainties in the preference structure does not affect the risk assessment method and does not imply any change in the probability computation. It affects, however, the preferences of the rational decision maker among different options that may show the same probability distributions for the outcomes (loss levels) but with different levels of epistemic uncertainty.

A helpful analogy is that of a driver who prefers to drive on straight roads than on winding roads when the conventional wisdom is that he "should" prefer the shortest road. To choose among different routes between two locations, he may want to add the lengths of the straight segments and apply to the result his straight-road preference function. He may then add the lengths of the
winding segments and apply to the result his preference function for the winding roads. The total length of the road remains unchanged (length is additive by axiomatic definition) but the lengths of the alternative roads alone are insufficient for the driver to decide which one he prefers. In the same way, the probability distribution for potential losses associated with each possible policy option does not provide sufficient information for a decision maker who is ambiguity averse. He needs, in addition, a breakdown of the uncertainties (aleatory and epistemic) in order to make a rational decision according to his preferences. The outcome distribution, however, is computed by collapsing all uncertainties according to the axioms of probability in the same way as all lengths are added to obtain the total length of any road.

2. RATIONALITY AND RISK MANAGEMENT IN THE PUBLIC SECTOR

2.1 Minimization of the expected values of the total loss

In the simplest form of (probabilistic) cost-benefit analysis, the objective is to minimize the expected value of the total costs (ECosts(Code i)). Each code option is characterized by design parameters that imply an annual probability of failure \( p_i \). This failure probability is a decreasing function of the initial costs \( C_i \): equivalent uniform annual costs for code \( i \); \( C_F \): cost of failure. (For simplicity, the reasoning is reduced here to one typical building). The choice of a code level reflects a tradeoff between the costs of increasingly stringent codes and the decreasing probability of building failure. The rational, expected-value decision maker chooses code \( i_1^* \) such that:

\[
i_1^* = \min_i \text{ECosts(Code } i) = \min_i \left\{ (1-p_i) \times C_i + p_i \times (C_i + C_F) \right\}
\]  

(1)

2.2. Maximization of expected utility

Equation 1 implies that the decision maker is willing to face high losses provided that the probabilities are low enough. In the long run, this strategy is in principle the one that "dominates" the others, i.e., that leaves the decision maker with the maximum amount of resources provided that he/she does not go bankrupt. For centuries, such a strategy was the only one considered "rational". The theory was enriched since the eighteenth century to recognize the rationality of other attitudes towards risk: EUDT allows, for example, systematic payment of a risk premium which implies that the decision maker will, in the long run, accept higher payments (losses) or lower gains in expected value than the risk neutral in return for a decrease in the probability of high losses. In this framework, purchasing an insurance policy is a rational decision even though the decision maker pays a premium that is systematically higher than the expected value of his/her potential losses.

The normative value of EUDT relies on its axioms. Several equivalent sets of such axioms have been proposed (e.g. Savage, 1954; Howard, 1970). All imply the separation of probabilities and utilities of the outcomes. Howard's, for example, include: orderability of outcomes, monotonicity and continuity of utility functions, existence for each distribution of prospects (e.g., losses in earthquakes) of a certain equivalent, and decomposability (or compounding) of a problem involving a sequence of random variables.

Instead of a utility function, the disutility of the costs (minus the utility) is used here for convenience. Consider, for example, an increasing marginal disutility \( u(.) \) characteristic of a risk-averse decision maker. He chooses code \( i_2^* \) such that:

\[
i_2^* = \min_i \text{EU(Code } i) = \min_i \left\{ (1-p_i) u(C_i) + p_i u(C_i + C_F) \right\}
\]  

(2)

2.3. Ambiguity aversion in rational code decisions

In reality, the \( p_i \)'s often come from the combination of a set of possible models and parameter values and can be decomposed into two components: the probabilities of different "models of the world" and the probability of failure conditional on each of these models for each code option. "Models of the world" include hypotheses about seismic sources and values of key parameters such as the
maximum magnitude from each source. An example of seismic hazard analysis with explicit representation of a spectrum of hypotheses is presented in Figure 1 for the site of the Limerick power station in Pennsylvania (NUS Corp., 1983).

![Figure 1: Limerick, Pennsylvania: annual frequency of exceedence versus peak ground acceleration for different hypotheses about seismic sources (Source: NUS Corp., 1983)](image1)

Figure 2: Separation of aleatory and epistemic uncertainty in a building code decision tree (Source: Paté-Cornell and Fischbeck, 1991a)

The separation of uncertainties can be displayed in a decision tree as shown in Figure 2: a set of n exhaustive, mutually exclusive models are indexed in j each with a probability q_j. Conditional on model j, for code i, the probability of building failure is r_{ij}. Once hypotheses and parameter values have been specified, this probability of failure represents only aleatory uncertainties (randomness). The probabilities p_i of equation 2 can then be replaced by \sum_j q_j r_{ij}.

If the decision maker is averse to ambiguity, his preferences (according to these modified axioms) can be represented by two separate disutility functions: the first one (u_1) for the cost outcomes associated with randomness (first level), the second one (u_2) for the expected disutilities of these first-level prospects distributed as the considered set of models (see Figure 3).

![Figure 3: Ambiguity aversion in a rational code decision model (u_1: disutility function representing attitude towards randomness; u_2: disutility function representing aversion towards fundamental uncertainties) (Source: Paté-Cornell and Fischbeck, 1991a)](image2)

It has been shown elsewhere (Paté-Cornell and Fischbeck, 1991a) that aversion towards ambiguity implies that the decision maker is indifferent between facing an uncertain (or "soft") probability of failure with mean \bar{p}_i and a "firm" probability of failure \bar{p}' higher than \bar{p}_i. \bar{p}' is the certain probability equivalent (CPE). The difference between \bar{p}_i and \bar{p}' represents the penalty that the decision maker is willing to pay to reduce risk uncertainties. This penalty has been explicitly computed by equating the utility of one "firm" prospect (failure with probability
p_1^\prime\prime) and the two-level prospect involving ambiguity (ibid.).

3. RELEVANCE OF RISK UNCERTAINTIES FOR SEISMIC CODE DECISIONS

The acceptance of such a penalty to compensate for local uncertainties in seismic hazard has important implications for building code decisions: if the code is based on a maximum threshold of failure probability (or "safety goal"), this threshold will be adjusted for these uncertainties. In the United States, for example, these uncertainties are larger in the West than in the East in the relevant range of peak ground accelerations. Given the seismic hazard curves, the process by which the seismic design criterion is set (see Figure 4) relies on a reference criterion that is a peak ground acceleration corresponding to a probability of exceedance (p*) of 1/475 per year (i.e., a probability 0.10 of exceedance in 50 years). p* yields the local pga levels corresponding to this probability of exceedance. In both cases, the same multiplicative safety factor \( \gamma \) is then applied to obtain the final design criteria to be used in each region (pga_{E}^{**} in the East and pga_{W}^{**} in the West). These criteria, in turn, correspond to annual probabilities of exceedance \( P_{W}^{**} \) and \( P_{E}^{**} \).

Because the slope of the hazard curve is steeper in the West than in the East in the part of the hazard curve corresponding to the probability of exceedance \( p^{*} \), the use of the safety factor \( \gamma \) implies a greater drop in the probability of exceedance (\( P_{W}^{**} \) is much lower than \( P_{E}^{**} \)). For the same building (characterized, for instance, by its fragility curve), it thus appears that this process provides greater safety in the West than in the East.

However, the uncertainties about the probabilities of exceedance are larger in the West than in the East for high peak ground accelerations. For example, Figure 5 shows the results of the seismic hazard analysis for Diablo Canyon, California (PG&E, 1988): in the high range of PGA's, (3g's), the probability distribution for the annual probability of exceedance ranges over two orders of magnitude.

Figure 4: Schematic representation of seismic hazard curves; means and uncertainty band (e.g., one \( \sigma \))

![Figure 4](image_url)

Figure 5: Diablo Canyon, California: annual frequency of exceedance versus peak ground acceleration; curves representing approximate fractiles of total hazard (Source: Pacific Gas and Electricity, 1988)

The higher safety level provided in the West by the process described above therefore occurs in zones of the curve where the uncertainties are larger than in the East. It may be justified by the public's aversion to fundamental uncertainties and willingness to pay a premium for a greater safety margin.
4. CONCLUSION

Is it rational, wise, prudent, or economically sound to include in public policies a penalty for uncertainties about seismic hazard analyses for sites where less is known about the seismic mechanisms? Rationality depends on an axiomatic definition. Aversion to epistemic uncertainty is as rational as risk aversion (a classically admitted feature of rational decision making) under a different set of axioms. Economic efficiency is defined with respect to a chosen objective: risk aversion is "inefficient" (yet rational) in the sense that paying a systematic insurance premium implies a higher cost in the long run than absorbing the expected value of the losses and accepting the prospect of bankruptcy. In the same way, a penalty to account for fundamental uncertainty is "inefficient" but justified by preference (and willingness to pay) for a firmer estimate of the failure probabilities. If this is a desired objective, the process needs to be made internally consistent: the same attitudes towards risk and ambiguity must be applied everywhere.

Is it wise and prudent? It may be neither if it diverts resources from the mitigation of known risks for the mitigation of unknown ones. Note that the same is true of risk aversion which diverts resources from the mitigation of isolated accidents with individually small (but cumulatively large) losses for the mitigation of more visible ones with smaller total benefits. Yet, altogether, a process that includes some provisions for aversion towards fundamental uncertainties can be justified if it is done consistently and knowingly, and corresponds to the true preferences of the public and the decision makers.

5. REFERENCES


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