

# Nonlinear seismic behaviour of code-designed eccentric systems

A. Rutenberg & A. Benbenishti  
 Technion-Israel Institute of Technology, Haifa, Israel  
 O.A. Pekau  
 Concordia University, Montreal, Que., Canada

**ABSTRACT:** A comparative parametric study of the earthquake response of single storey asymmetric structures designed by the static provisions of several seismic codes is presented. Three different configurations of the resisting systems were considered and analyzed for four earthquake time-histories, assuming bilinear force-displacement relation. Strength distribution effects were separated from total strength effects by normalizing total strength to that of the associated symmetric system. Results show that the SEAOC/UBC and NBCC designs generally lead to lower ductility demand (DD) than the ATC/NEHRP and CEB designs, and more so for non-normalized systems. The presence of elements normal to the direction of excitation usually moderates peak DD, displacement and rotation, but the effect is not large. However, even the best design still results in larger DD than in symmetric systems.

## 1 INTRODUCTION

The nonlinear seismic behaviour of code-designed asymmetric structures has attracted a growing interest in recent years. Studies focus mainly on the differences among static seismic code provisions as manifested in the numerical value of the design eccentricity  $e_d$  which is defined as:

$$e_d = ae \pm \beta b \quad (1)$$

in which  $e$ =(static) eccentricity of the mass center CM from the rigidity center CR,  $b$ =width of building perpendicular to the direction of excitation (Fig. 1),  $\alpha$  and  $\beta$  are numerical coefficients. Table 1 gives  $e_d$  for a number of seismic codes. It can be seen that the values of  $\alpha$  and  $\beta$  depend on whether the element under consideration is located on the flexible ( $e^+_d$ ) or rigid side ( $e^-_d$ ) of the deck, as shown in Fig. 1.

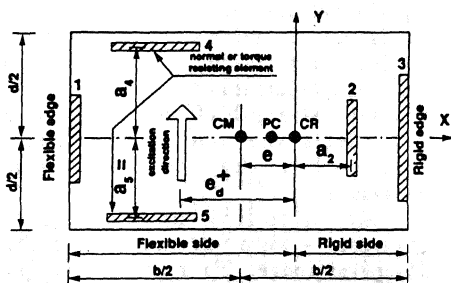


Figure 1. General asymmetric model.

Table 1. Design eccentricities per several codes

code	$e^+_d$	$e^-_d$
ATC (1978)	$e + 0.05b$	$e - 0.05b$
NBCC (1990)	$1.5e + 0.10b$	$0.5e - 0.10b$
CEB (1987)	$1.5 + 0.05b$	$e - 0.05b$
SEAOC (1975)	$e + 0.05b$	$e - 0.5b$ ( $e < 0.05b$ )
UBC (1979)		0 ( $e > 0.05b$ )

Once  $e_d$  is evaluated the forces  $F_{iy}$  acting at yield on elements 1, 2, 3 are obtained from the modified static formula

$$F_{iy} = F_{oy} \frac{K_{iy}}{\Sigma K_y} \left[ 1 \pm \frac{e^*_d a_i}{\Omega_o^2} \right] \quad (2a)$$

in which  $F_{oy}$ =total yield strength (or base shear) of the associated symmetric system in the Y direction,  $K_{iy}$ =lateral stiffness of element  $i$ ,  $\Sigma K_y$ =total y-direction stiffness,  $e^*_d = e_d/\rho$ ,  $a_i^* = a_i/\rho$ =normalized distance of element from CR,  $\Omega_o^2 = \Sigma K_i a_i^2 / (\rho^2 \Sigma K_y)$  and  $\rho$ =mass radius of gyration w.r.t. CM. Evidently  $\Sigma F_{iy} = \Sigma F_y > F_{oy}$ . Note that when normal elements (Nos. 4 and 5) are present, their contribution to  $\Omega_o^2$  should be considered, and the forces acting on them will be obtained from:

$$F_{ix} = \pm F_{oy} \frac{K_{ix}}{\Sigma K_y} \frac{e^*_d a_i}{\Omega_o^2} \quad (2b)$$

The difference between the post-yield response of symmetric and asymmetric systems lies mainly in the fact that due to rotation the peak ductility demand (PDD), i.e. the maximum ductility of the resisting members, and the maximum displacements  $y_{max}$  of asymmetric systems are usually larger, and so is their damage potential, as evidenced by the damage statistics of destructive earthquakes (e.g. Rosenblueth and Meli, 1986).

In the NBCC and the CEB code  $e$  in Eqn. 1 was calibrated using response spectrum and time history results in the linear range. On the other hand the traditional U.S. approach has followed the static formula (i.e.  $e_d = e$ ) and considered only the additional accidental eccentricity, with the notable exception that no reduction relative to the symmetric case is to be permitted. Thus, the relative success of the various code formulae in Table 1 to guard against excessive DD has been the subject of a number of recent investigations (e.g. Chandler & Duan 1991, Chopra & Goel 1991, Diaz-Molina 1988, Rutenberg et al. 1989, Tso & Ying 1990).

The differences among the codes lead to (1) different lateral strength distributions along the resisting elements and (2) different total lateral strengths. Therefore, it is expected that the non-linear dynamic response of systems designed by different codes is different even when their linear behaviour is similar.

An extensive parametric study was carried out on the nonlinear seismic response of code-designed asymmetric systems for a wide range of parameters. Three typical monosymmetric models either mass or stiffness eccentric having several values of  $e^*$  ( $=e/\rho$ ), torsional-to-lateral frequency ratio  $\Omega_o$ , and uncoupled lateral vibration periods  $T_y$ , with or without resisting elements normal to the direction of excitation were considered. Damping ratio of 2% and a bilinear force displacement relation with a secondary slope ratio of 2% were assumed. Strength was assigned to the elements of each model using the  $e_d$  formulae of the following codes: SEAOC (1975), ATC (1978), CEB (1987) and NBCC (1990). Note that the UBC (1979) adopted the SEAOC (1975)  $e_d$  formula. The 1988 editions of SEAOC, UBC and NEHRP use a modified  $e_d$  formula, but the resulting strength distribution does not appear to have been drastically changed.

The three models in Fig. 2 present a wide range of strength and stiffness distributions. The CM model allocates more strength to element 3 than the other two models, the shifting or mass eccentric (SME) model allocates the least, and locates element 3 at a larger distance from CM. The respective elements of the CR and CM models differ in strength but have similar yield displacements. Note that, in code-designed

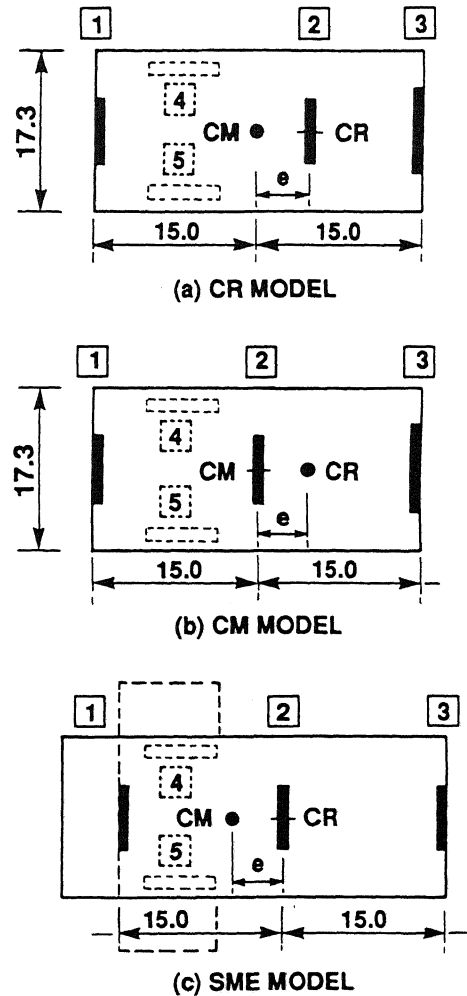


Figure 2. Three 3-element (& 5 element) models.

structures the plastic centroid (PC) is close to CM and its eccentricity varies within very narrow bounds, and is not studied herein. These systems were excited by the time histories of four representative earthquakes: El Centro 1940, Taft 1952, Bucharest 1977 and Mexico 1985.

The response parameters were PDD and  $y_{max}$ . In order to isolate the effects of asymmetry the response values were divided by the corresponding values in the associated symmetric systems. To separate total strength effects from strength distribution effects, the total lateral strength was normalized to that of the relevant symmetric system.

The models including normal (torque resisting) elements were variants of the three models described above. To make the comparison meaningful it was necessary to keep the elastic properties and the total Y-direction

strength intact. Since part of the rotational stiffness was transferred to elements 4 and 5 the stiffness of elements 1, 2, 3 had to be adjusted accordingly. This stiffness adjustments led to the allocation of different strengths to these elements (Eqn. 2). The following parameters were used: stiffness ratio  $\lambda = \Sigma K_x / \Sigma K_y = 0.5, 1.0, 1.5$ ; strength ratio  $\gamma = \Sigma F_x / \Sigma F_y = 0.5, 1.0, 1.5$ . Note that the *strength* of elements 4 and 5 is an independent parameter (it mainly depends on the X-direction seismic requirements).

## 2 RESULTS

### 2.1 Ductility demand (DD)

The element ductility demand (EDD) is the maximum inelastic displacement sustained by the element during an earthquake relative to its yield displacement. The need to define PDD arises from the fact that there are several resisting elements and PDD is the largest EDD of them all. Element 1 usually has the largest displacement in the linear and nonlinear ranges, but in code-designed structures its DD is usually lower than that of elements 3 or 2.

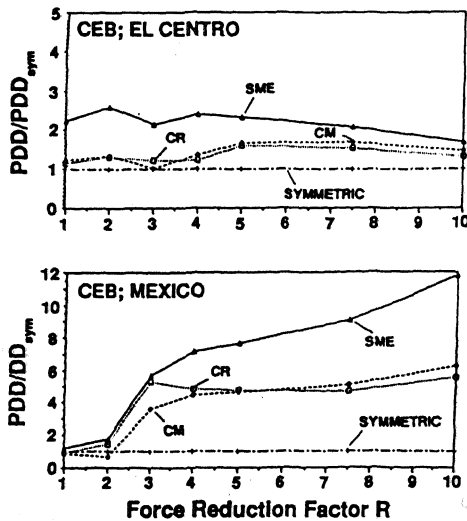


Figure 3. PDD/DD<sub>sym</sub> vs R: Comparison of models, CEB code, El Centro & Mexico,  $T_y = 0.5$  sec,  $e^* = 0.5$ ,  $\Omega_0 = 1.0$ .

1. Systems without normal elements ( $K_4 = K_5 = 0$ ,  $F_4 = F_5 = 0$ ). In Fig. 3 PDD/DD<sub>sym</sub> is plotted vs the force reduction factor R for the three models as designed by the CEB code. Note that for rectangular buildings loaded normal to their longer dimension  $b = 2.5\rho \rightarrow 3.5\rho$ . It is seen

that the SME model is the most vulnerable, and that the PDD ratio increases with increasing R only for the Mexico record (also for Bucharest - not shown). Similar behaviour is obtained when  $T_y = 1.25$  sec. The PDD ratios of similar systems when designed by the NBCC are on the whole smaller and do not increase with R even for the Bucharest and Mexico records (not shown). The effect of  $T_y$  and  $\Omega_0$  on PDD is shown in Fig. 4 for the CR and SME models excited by the El Centro record. The contrast between the CEB and ATC on the one hand and the SEAOC and NBCC on the other is clear. The inability of the former codes to control the PDD of the SME model where  $\Omega_0 = 0.8$  is also evident. Results for the CM model are similar to those of the CR model and are not shown. Also, the responses to the three other records are similar. The effect of  $e$  is presented in Fig. 5. Again the high vulnerability of the CEB and ATC designed SME model at larger eccentricities is evident.

Since the computed total strengths were normalized to that of the associated symmetric system, the PDD ratios of low and intermediate period systems designed by codes leading to larger total strength, such as the NBCC, SEAOC and CEB are in reality somewhat lower than shown in the Figures.

The DDs of the three elements are different, and it is interesting to see whether element 3 is indeed the most vulnerable as observed in earlier studies. Fig. 6 shows some of the results for the CR model designed by the CEB and NBCC for the Taft record. The pattern is quite clear for the CEB design (and ATC - not shown). For the NBCC design it is seen that at some periods elements 1 and 2 have somewhat larger DD than element 3. Also interesting are the NBCC results for the SME model excited by the Bucharest record (not shown): element 2 (the central) is the one sustaining the PDD, although the differences in DD among the elements are marginal. Note also the lower PDD of the NBCC design. These examples show that the NBCC (and SEAOC) distributes the total strength among the three elements more efficiently than the other two codes.

2. Systems with normal elements. In Fig. 7 PDD is plotted vs R for the CEB designed CR model with  $\lambda = \Sigma K_x / \Sigma K_y = 0.5$ , and the Mexico record. It is seen that the presence of normal elements lowers PDD to some extent. Note, however, that increasing the strength ratio  $\gamma = \Sigma F_x / \Sigma F_y$  from 0.5 to 1.5 has practically no effect on PDD. The results for the other three records show similar or smaller differences. The effect of the normal elements on PDD as affected by  $T_y$  is shown in Fig. 8 with  $\gamma = 0.5$  for the SEAOC designed CR model under the El Centro record. It is seen that the stiffness of the normal

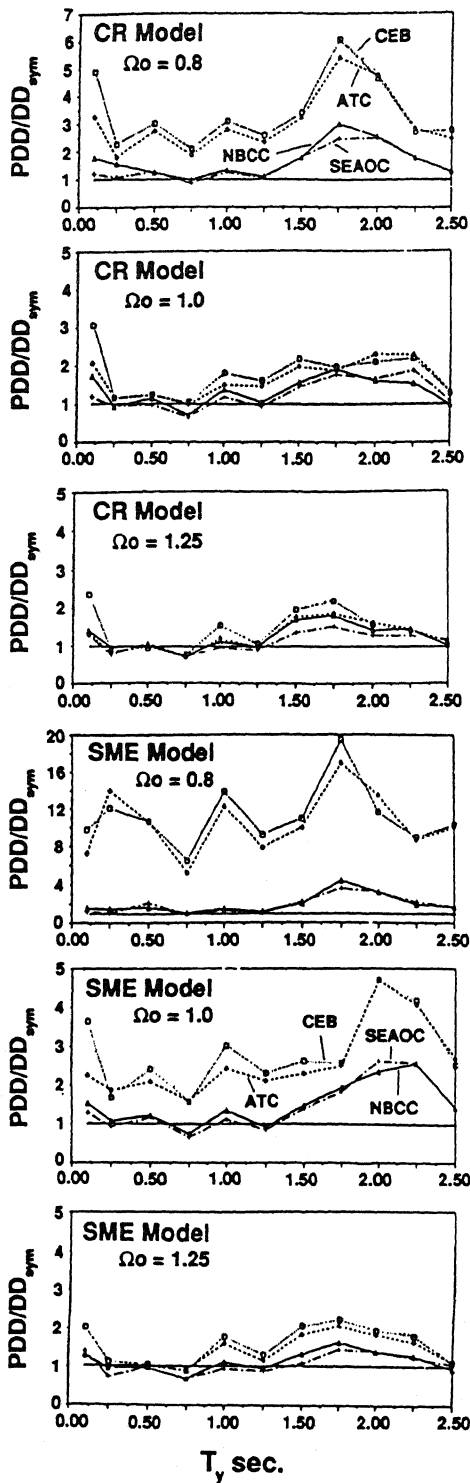


Figure 4. PDD/DD<sub>sym</sub> vs T<sub>y</sub>: Effect of  $\Omega_0$ , Comparison of codes, CR & SME models, El Centro,  $e^* = 0.5$ ,  $R = 4.0$ .

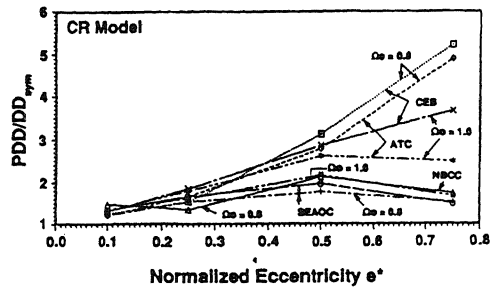


Figure 5. PDD/DD<sub>sym</sub> vs  $e^*$ : Comparison of codes, CR model, Taft, T<sub>y</sub> = 0.5, R = 4.0.

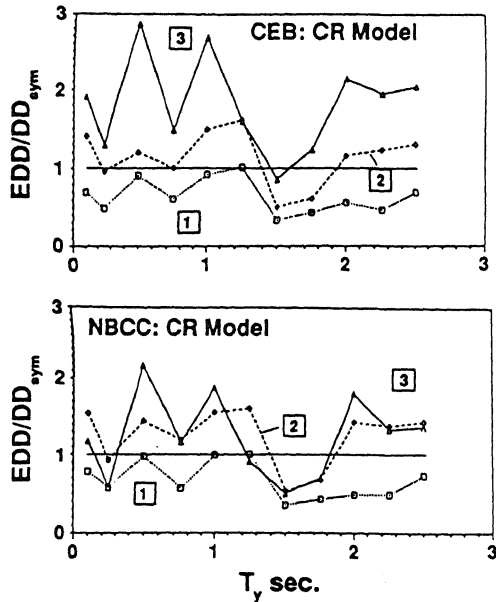


Figure 6. EDD/DD<sub>sym</sub> vs T<sub>y</sub>: CEB & NBCC codes, Taft,  $e^* = 0.5$ ,  $\Omega_0 = 1.0$ .

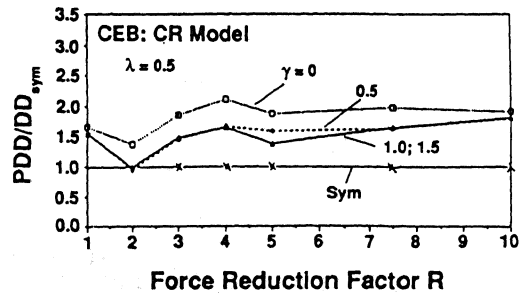


Figure 7. PDD/DD<sub>sym</sub> vs R: Effect of normal elements CEB code, Mexico, T<sub>y</sub> = 1.25 sec.,  $e^* = 0.5$ ,  $\Omega_0 = 1.0$ ,  $\lambda = 0.5$ .

elements has practically no effect. Note that since the contribution of the normal elements to the torsional stiffness is constant, changes in  $\lambda$  are accompanied by changes in their distance from CR.

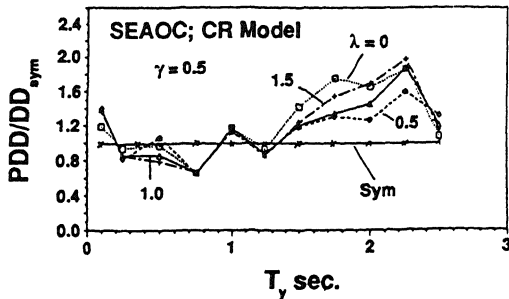


Figure 8.  $PDD/DD_{sym}$  vs  $T_y$ : effect of normal elements' stiffness, SEAOC code, CR model, El Centro,  $e^*=0.5$ ,  $\Omega_0=1.0$ ,  $R=4.0$ ,  $\gamma=0.5$ .

## 2.2 Maximum displacement ( $y_{max}$ )

Upper bounds on lateral displacements and interstorey drift are usually imposed by codes in order to limit nonstructural damage and guard against excessive P-Delta effects.

1. Systems without normal elements. Typical plots of  $y_{max}/y_{max,sym}$  vs  $T_y$  for the SME model designed by the four codes with the Bucharest record are shown in Fig. 9. It is seen that there is

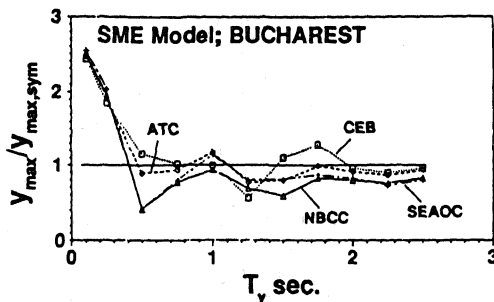


Figure 9.  $y_{max}/y_{max,sym}$  vs  $T_y$ : Comparison of Codes, SME model, Bucharest,  $e^*=0.5$ ,  $\Omega_0=1.0$ ,  $R=4.0$ .

not much difference among the models but, on the whole, their  $y_{max}$  are larger than those of their symmetric counterparts. Note, however, that for low period systems  $y_{max}/y_{max,sym}$  falls with increasing period for the Bucharest and Mexico records (the latter is not shown). There are no substantial differences among the codes with respect to  $y_{max}$  (other codes are not shown).

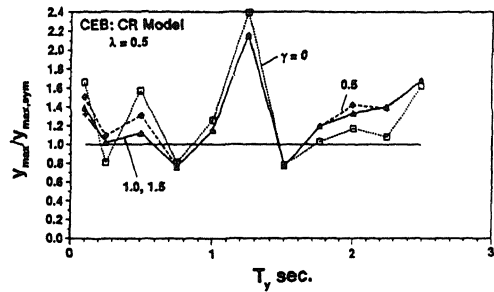


Figure 10.  $y_{max}/y_{max,sym}$  vs  $T_y$ : Effect of normal elements' strength, CR model, CEB Code, Taft,  $e^*=0.5$ ,  $\Omega_0=1.0$ ,  $R=4.0$ ;  $\lambda=0.5$ .

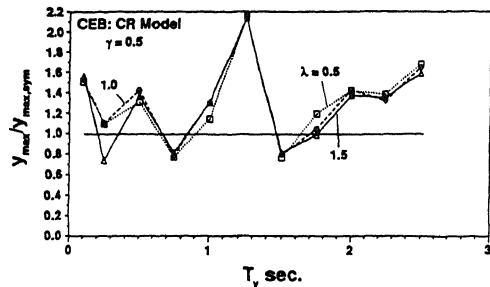


Figure 11.  $y_{max}/y_{max,sym}$  vs  $T_y$ : Effect of normal element stiffness, CR model, CEB code, Taft,  $e^*=0.5$ ,  $\Omega_0=1.0$ ,  $R=4.0$ ,  $\gamma=0.5$ .

2. Systems with normal elements. The effect of normal elements on  $y_{max}$  is shown for the CR model in Fig. 10 for  $\lambda=0.5$  and in Fig. 11 for  $\gamma=0.5$ . It is seen that the normal elements have usually a minor effect on  $y_{max}$  and their strength or stiffness ratios hardly affect the response. However, for the  $\gamma=0.5$  case increasing the stiffness of these elements (i.e. reducing their CR distance to keep  $\Omega_0$  intact) may sometimes tend to slightly increase  $y_{max}$ .

## 3 CONCLUSIONS

### 3.1 Ductility demand

1. Response is affected by choice of model. The SME model (Fig. 2c) sustains the largest peak ductility demand (PDD) whereas the CR and CM models (Figs. 2a and 2b) have similar but usually lower PDD.

2. The ratio  $PDD/DD_{sym}$  is practically not affected by variation in the lateral natural period  $T_y$  within the range studied.

3. PDD is strongly affected by the code formula for  $e_d$ . It is shown that the strength distributions of the SEAOC and the NBCC lead to lower DD than the CEB and ATC, even when the total strength is lowered (normalized) to the level of the associated symmetric system. This is because the latter codes do not increase the strengths of rigid side elements (3 & 2) sufficiently (i.e.  $\alpha=1.0$ , whereas  $\alpha=0.5$  or 0 in the former codes). In their "non-normalized" strength, structures designed by the former codes sustain even lower DD, in fact NBCC designs often have lower PDD than the symmetric counterparts due to their higher total strength. Note also that the CEB and ATC type codes are much less able to control DD with increasing load reduction factor and eccentricity.

4. Increasing the torsional to lateral frequency ratio  $\Omega_o$  tends to lower PDD.

5. The presence of normal or torque resisting elements (4 & 5) tends to lower PDD, but the effect is not large. The strength of these elements is not a very important parameter within the range studied so that increasing the rotational yield strength of the system does not appear to affect PDD appreciably, i.e. it does not significantly lower the peak angle of rotation.

### 3.2 Displacements

1. The maximum displacements ( $y_{max}$ ) of asymmetric systems increase with period, as in symmetric systems. However, for low period systems  $y_{max}/y_{max,sym}$  falls with period for the Bucharest and Mexico records. On the whole  $y_{max}$  are larger than in similar but symmetric systems and a ratio of two is not uncommon.

2. The presence of normal elements has a minor effect on  $y_{max}$ . For a given strength level of these element ( $\gamma=0.5$ ), increasing their stiffness (i.e. reducing their distance to CR) may lead to larger  $y_{max}$  than for systems without normal elements.

### 3.3 General

PDD and  $y_{max}$  do not usually occur in the same element. It appears that the strength of the flexible side element (No. 1) may be somewhat lowered without increasing PDD and  $y_{max}$ , but the extent to which  $\alpha$  in Eqn. 1 can be reduced depends, of course, on the seismic code under consideration.

## 4 REFERENCES

- Applied Technology Council 1978. Tentative provisions for the development of seismic regulations for buildings. ATC 3-06, National Bureau of Standards, Washington, D.C.
- Associate Committee on the National Building Code 1990. National Building Code of Canada (NBCC) 1990 and Supplement. National Research Council of Canada, Ottawa.
- Benbenishti, A. & A. Rutenberg 1991. Asymmetric structures and seismic codes: Ductility demand. Publ. 305, Fac. Civ. Engng. Technion, Haifa.
- Building Seismic Safety Council 1988. NEHRP recommended provisions for the development of seismic regulations for new buildings. Earthquake Hazard Reduction Series 17. FEMA 96, Federal Emergency Management Agency.
- Chandler, A.M. & X.M. Duan 1991. Evaluation of Factors influencing the inelastic seismic performance of torsionally asymmetric buildings. Earthquake Engng. Struct. Dyn. 20: 87-95.
- Chopra, A.K. & R.K. Goel 1991. Evaluation of torsional provisions in seismic codes. J. Struct. Engng., ASCE 117: 3762-3782.
- Comite Euro International du Beton 1987. CEB Model Code for Concrete Structures, Technical Press, Aldershot, England.
- Diaz-Molina, I. 1988. Dynamic torsional behavior of inelastic systems. M.Sc. Rept. Dept., Civ. Engng. Carnegie-Mellon University, Pittsburgh, PA.
- ICBO 1979 and 1988. Uniform Building Code (UBC). International Conference of Building Officials. Whittier, CA.
- Rutenberg, A., G. Shohet & M. Eisenberger, 1989, Inelastic seismic response of code-designed asymmetric structures. Publ. 303 Fac. Civ. Engng., Technion, Haifa (revised version of summer 1987 manuscript).
- Rosenblueth, E. & R. Meli 1986. The 1985 earthquake causes and effects in Mexico City. Concrete International, May: 23-35.
- Seismology Committee, SEAOC, 1975 and 1988. Recommended lateral force requirement and tentative commentary. Structural Engineers Association of California.
- Tso, W.K. & H. Ying 1990. Additional seismic deformation caused by structural asymmetry. Earthquake Engng. Struct. Dyn. 19: 243-258.