Evaluation of seismic code provisions for asymmetric-plan systems

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ABSTRACT: The effects of plan asymmetry on the earthquake response of code-designed, one-story systems are identified with the objective of evaluating how well these effects are represented by torsional provisions in building codes. The earthquake-induced deformations and ductility demands on resisting elements of asymmetric-plan systems, are compared with their values if the system plan were symmetric. The presented results demonstrate that the design eccentricity in building codes should be modified in order to achieve the desirable goal of similar ductility demands on asymmetric-plan and symmetric-plan systems. The design eccentricity should be defined differently depending on the design value of the reduction factor R.

1 INTRODUCTION

The evaluation of torsional provisions in building codes based on computed responses of elastic as well as inelastic, asymmetric-plan systems has been the subject of numerous studies in the past. However, the conclusions of these studies may not be generally applicable to code-designed buildings because the assumed plan-wise distribution of stiffness and strength is not representative of code-designed buildings and the strength distribution can significantly influence the inelastic structural response (Chopra and Goel 1991). Thus, the main objective of this work is to investigate the effects of plan-asymmetry on the earthquake response of code-designed, one-story systems and to determine how well these effects are represented by torsional provisions in building codes. For this purpose, the deformation and ductility demands on resisting elements of asymmetric-plan systems are compared with their values if the system plan were symmetric. Based on these results, deficiencies in code provisions are identified and improvements suggested.

2 TORSIONAL PROVISIONS IN SEISMIC CODES

2.1 Method for Computing Design Forces

The design force \( V \) specified in building codes is usually much smaller than the strength \( V_o \) required for the system to remain elastic during intense ground shaking. Instead of computing the base shear from code formulas which would result in different values according to different codes, the base shear is defined as

\[
V = \frac{1}{R} V_o
\]  

where \( R \) is a reduction factor depending on the capacity of the system to safely undergo inelastic deformation during intense ground shaking.

In a one-story, symmetric-plan system, the design force \( V \) is applied at the center of stiffness (CS). If the floor diaphragm is rigid, all resisting elements along the direction of ground motion undergo the same lateral displacement \( u \), the lateral resisting force in the elements is \( k_p u \), and the total resisting force is \( V = k_p u \); \( k_p \) and \( K_p \) are the lateral stiffness of the \( j \)th element and the total system, respectively. Thus, the design force in the \( j \)th resisting element is \( (k_p K_p) V \) and the forces are distributed to the elements in proportion to their lateral stiffnesses or rigidities.

In asymmetric-plan systems, the design force \( V \) is applied eccentric from the CS at a distance equal to design eccentricity, \( e \), which is defined in the next section. Under the action of the resulting torque, \( e V \), the rigid roof diaphragm will undergo rotation of \( e V K_{be} \), where \( K_{be} = \sum_{i} K_i (x_i')^2 + \sum_{j} y_j^2 \) is the torsional stiffness about the CS, in which \( x_i' = x_i - e \); \( x_i \) is the distance of the \( i \)th element oriented in the Y-direction from the center of mass (CM); \( e \) is the stiffness eccentricity, i.e., the distance between the CM and the CS; and \( y_j \) defines the location of the \( j \)th element oriented in the X-direction. Thus the design force in the \( i \)th element along the direction of ground motion is

\[
V_j = \frac{k_p}{K_p} V + \frac{e V}{K_{be}} (-x_j') k_p
\]  

The second term represents the element force associated with its deformation resulting from deck rotation and thus the change in element force due to plan-asymmetry. Obviously, the torsion induced forces are distributed to the various resisting elements in proportion to their torsional
stiffnesses or rigidities.

2.2 Design Eccentricity

Most building codes require that the lateral earthquake force at each floor level of an asymmetric-plan building be applied eccentrically relative to the CS. The design eccentricity $e_d$ specified in most seismic codes is of the form (International 1988)

$$ e_d = \alpha e_s + \beta b $$

(3a)

$$ e_d = \delta e_s - \beta b $$

(3b)

where $b$ is the plan dimension of the building perpendicular to the direction of ground motion; and $\alpha$, $\beta$, and $\delta$ are specified coefficients. For each element the $e_d$ value leading to the larger design force is to be used. Consequently, Eq. 3a is the design eccentricity for elements located within the flexible-side of the building and Eq. 3b for the stiff-side elements (Fig. 1).

The coefficients, $\alpha$, $\beta$, and $\delta$ vary among building codes (International 1988, Tentative 1978). For example, the Uniform Building Code (UBC-88) and Applied Technology Council (ATC-3) provisions specify $\beta=0.05$ and $\alpha=1$, with $\alpha=1$ implying no dynamic amplification of torsional response; the Mexico Federal District Code (MFDC-77) specifies $\beta=0.1$, $\alpha=1$, and $\alpha=1.5$, which implies dynamic amplification; the National Building Code of Canada (NBCC-85) specifies $\beta=0.1$, $\alpha=1.5$, and $\delta=0.5$; and the New Zealand Code (NZC-84) specifies $\beta=0.1$ and $\alpha=1$.

The first term in Eq. 3 involving $e_s$ is intended to account for the coupled lateral-torsional response of the building arising from lack of symmetry in plan, whereas the second term is included to consider torsional effects due to factors not explicitly considered, such as the rotational component of ground motion about a vertical axis; differences between computed and actual values of stiffnesses, yield strengths, and dead-load masses; and unforeseeable unfavorable distribution of live-load masses. This accidental eccentricity, $\beta b$, which is a fraction of the plan dimension, $b$, is obviously considered in design to be on either side of the CS.

3 INELASTIC RESPONSE

From a design point of view, it would be useful to know how the deformations and ductility demands of resisting elements in an asymmetric-plan system differ from those in the corresponding symmetric-plan system. For this purpose, presented in this investigation are the deformations $u_i$ and ductility demands $\mu_i$ of resisting elements in the asymmetric-plan system, normalized by $u_o$ and $\mu_o$, the respective response quantities of the corresponding symmetric-plan system -- a system with $e_s=0$ but mass $m$, lateral stiffness $K_s$, torsional stiffness $K_{to}$ about the CS, and element stiffnesses $k_{ip}$ same as in the asymmetric-plan system (Goel and Chopra 1990). The normalized response quantities, $u_i/u_o$ and $\mu_i/\mu_o$, for the system of Fig. 1 are presented in the form of response spectra for the first 6.3 secs. of the S00E component of the 1940 El Centro ground motion applied in the Y-direction. This excitation and its response spectra with various frequency regions identified are available elsewhere (Goel and Chopra 1990, Veletzos and Vann 1971). The yield force for the system is defined by Eq. 1 and the element yield forces are determined in accordance with the torsional provisions of UBC-88. In order to focus on effects of plan asymmetry, the accidental eccentricity is not included in computing the design forces for the resisting elements of the asymmetric-plan system and its corresponding symmetric-plan system. Two types of asymmetric-plan systems are considered: in the first system, the code design force for the stiff-side element can be smaller than the design force of the same element in the corresponding symmetric-plan system; and in the second type, such a reduction is precluded. Each resisting element oriented along the ground motion direction is idealized as elastic-perfectly-plastic with its yield force defined by the design force; the perpendicular elements are taken as elastic, an assumption which has little influence on the response (Goel and Chopra 1990). Several parameters of the system are fixed at: stiffness eccentricity normalized by the radius of gyration, $e_s/r = 0.5$, ratio of the uncoupled torsional and lateral frequencies, $\Omega_0 = 1$, and damping ratio, $\xi = 0.05$.

The deformations of resisting elements in the system designed according to UBC-88 may be significantly affected by plan-asymmetry, as indicated by the deviation of $u_i/u_o$ or $\mu_i/\mu_o$ from unity (Fig. 2). Plan-asymmetry tends to reduce the deformation of the stiff-side element in medium-period, velocity-sensitive systems and increase the deformation of the flexible-side element compared to their respective deformations in the corresponding symmetric-plan system. However, the effects of plan-asymmetry on element deformations are small for short-period, acceleration-sensitive systems, and negligible for long-period, displacement-sensitive systems. The increased strength of the system resulting from the restriction that the stiff-side element design force must not fall below its symmetric-plan value affects the response ratio, $u_i/u_o$, in a manner consistent with the effects of strength increase on the response of SDF systems (Veletzos and Vann 1971).

The ratio $u_i/u_o$ of the element ductility demands in an asymmetric-plan system and the corresponding symmetric-plan system are also presented in Fig. 2. If the design force for the stiff-side element is permitted to be smaller than its value in the corresponding symmetric-plan system, over a wide range of periods the element ductility demand is significantly larger due to plan-asymmetry, primarily because the yield deformation of the element is smaller in asymmetric-plan systems if reduction in its design force is permitted (Chopra and Goel 1991). However, if reduction in the element design force is precluded, $\mu_i/\mu_o = u_i/u_o$ because the yield deformations of this element are identical in the symmetric-plan and asymmetric-plan systems, and the above observations on how deformations are affected by plan-asymmetry also apply to ductility demand. The ductility demand on the flexible-side element is significantly reduced because of plan-
asymmetry, with exceptions at few periods (Fig. 2), because the yield deformation of this element in the code-designed asymmetric-plan system is significantly larger than in the symmetric-plan system (Chopra and Goel 1991). These trends are unaffected whether the design force reduction for the stiff-side elements is permitted or not (Fig. 2), primarily because the yield deformation of the flexible-side element is unaffected by such reduction (Chopra and Goel 1991).

The preceding results have demonstrated that the response of systems with and without reduction in the stiff-side element design force, arising from plan-asymmetry, may differ significantly. In particular, the ductility demand on the stiff-side element may increase significantly because of plan-asymmetry when reduction in the stiff-side element design force is permitted. Since it is desirable that the element ductility demands be similar whether the plan is symmetric or not, the presented results suggest that seismic codes should preclude reduction in the design forces of the stiff-side elements below their values for symmetric-plan systems.

Several earlier investigations (e.g., Goel and Chopra 1990, Tso and Hongshan 1990) of the earthquake response of asymmetric-plan systems with equal stiffness and strength eccentricities, i.e., \( e_p = e_g \), indicate that the largest deformation as well as the largest ductility demand generally occurs in the flexible-side elements, which were therefore interpreted as the most critical elements for design purposes. However, the preceding results for the system of Fig. 1 indicate that, although the largest deformation among all the resisting elements of the code-designed asymmetric-plan systems for which \( e_p << e_g \) occurs in the flexible-side element, the largest ductility demand may occur in the stiff-side element. Thus, additional care is required not only in the design of flexible-side elements for deformation demand, but also in the design of stiff-side elements for ductility demand.

4 'ELASTIC' RESPONSE

It is the intent of most seismic codes that buildings suffer no damage during some, usually unspecified, level of moderate ground shaking. Thus, the elastic response of asymmetric-plan systems designed according to UBC-88 is examined next. The normalized deformation, \( \mu_1 / \mu_r \), and ductility demand \( \mu_d \), are presented in the form of response spectra for the El Centro ground motion; values for other parameters are fixed: \( e_p / r = 0.5 \), \( \Omega_g = 1 \), \( R = 1 \), and \( \Xi = 0.05 \). \( R = 1 \) implies that the design strength \( V \) of the corresponding symmetric-plan system is just sufficient for it to remain elastic during the selected excitation. However, as will be shown in subsequent sections, asymmetric-plan systems designed for the same base shear may not remain elastic.

The deformation of resisting elements may be significantly affected by plan-asymmetry. The deformation of the stiff-side element is reduced because of plan-asymmetry for most short-period, acceleration-sensitive and medium-period, velocity-sensitive systems whereas deformation of the flexible-side element in such systems is considerably increased (Fig. 3). The element deformations of long-period, displacement-sensitive systems are essentially unaffected by plan-asymmetry (Fig. 3).

The ductility demand for stiff-side and flexible-side elements in the asymmetric-plan system exceeds one in some period ranges (Fig. 3) indicating yielding in these elements, which were designed to remain elastic if the building plan were symmetric. The stiff-side element yields more if its design force is permitted to fall below its symmetric-plan value because this results in smaller yield deformation (Chopra and Goel 1991). As a corollary, this element yields less if reduction in its strength is not permitted. The flexible-side element yields primarily because of its significantly larger deformation (Fig. 3) compared to the symmetric-plan system, although its yield deformation is also larger (Chopra and Goel 1991). However, its ductility demand is unaffected whether reduction in the stiff-side element design force is permitted or not because the peak deformation as well as the yield deformation of the flexible-side element is unaffected by such reduction.

5 RESPONSE OF SYSTEMS DESIGNED BY OTHER CODES

The response results presented in this paper are for systems designed by the UBC-88. Similar results generated for systems designed according to other codes -- NBCC-85, MFDC-77, MFDC-87, and NCZ-84 -- are available in Chopra and Goel (1991). These results indicated that the element deformations in systems designed by various codes are essentially identical; however, the ductility demands may differ significantly among these systems. Among these codes, systems designed by UBC-88 experience the largest ductility demand, whereas systems designed by MFDC-87 undergo the smallest ductility demand; response of systems designed by other codes fall in between these two extremes. It was also found that element ductility demands in the MFDC-87-designed systems tend to be significantly reduced because of plan-asymmetry, suggesting that the additional requirements imposed in this code to restrict the strength eccentricity may not be necessary.

6 MODIFICATIONS IN DESIGN ECCENTRICITY

The results of preceding sections indicate that deformations and ductility demands on resisting elements in a code-designed asymmetric-plan system differ from those for the corresponding symmetric-plan system. However, it would be desirable that the responses of the two systems be similar so that the earthquake performance of the asymmetric-plan system would be similar to, and specifically no worse than, that of the symmetric-plan system. In order to investigate this issue further, the responses of asymmetric-plan systems with their element yield forces computed with three different values of \( \delta = 1, \ldots, 5737 \)
In all cases, $\alpha = 1$ and four different values of $R = 1, 2, 4, \infty$ were considered in Eq. 1. The ductility demand on the stiff-side element is the only response quantity presented because other responses are affected very little by $\delta$. It is apparent that the ductility demand $\mu_1$ on the stiff-side element in the asymmetric-plan systems designed with $\delta = 0$ is generally below the element ductility demand, $\mu_p$, if the system plan were symmetric. However, for some period values, precluding reduction of stiff-side element design force ($\delta = 0$) is not sufficient to keep $\mu_1$ below $\mu_p$. In order to achieve this objective, perhaps this design force should be increased relative to its symmetric-plan value, which implies a negative value of $\delta$ in Eq. 3b.

Even if such a reduction in the stiff-side element design force is precluded, earlier inelastic response results for systems designed with $R = 4$ have demonstrated that the ductility demand on the flexible-side element may be reduced because of plan-asymmetry (Fig. 2). Thus, the ductility capacity of the flexible-side element is underutilized in an asymmetric-plan system if it is designed for the ductility demand in a symmetric-plan system. In order to better utilize the element ductility capacity, the design eccentricity, $e_d$, in Eq. 3a should be modified by decreasing $\alpha$ to reduce the strength of the element. On the other hand for systems with $R = 1$, i.e., systems designed to remain elastic if their plan were symmetric and no accidental eccentricity were considered, the ductility demand on the flexible-side element in an asymmetric-plan system may exceed one indicating yielding of the element because of torsional motions (Fig. 3). Thus the strength of this element should be increased by increasing $\alpha$ in Eq. 3a to compute the design eccentricity, $e_d$.

In order to further investigate these concepts, the responses of asymmetric-plan systems with their element yield forces computed with three different values of $\alpha$ are compared in Fig. 5. In addition to $\alpha = 1$, two larger values are considered for systems designed with $R = 1$ or 2; two smaller values are considered when $R = 8$; and one smaller and another larger value is selected when $R = 4$. The ductility demand on the flexible-side element is the only response quantity presented because other response quantities are affected very little by $\alpha$. These results demonstrate that, in order to keep the ductility demand on the flexible-side element in the asymmetric-plan system below its symmetric-plan value, $\alpha$ should be selected as follows: $\alpha = 1$ if $R = 8$; $\alpha = 1.5$ if $R = 2$ and 4; and $\alpha = 2$ if $R = 1$. However, the optimal $\alpha$ values may differ with the ground motion. Thus, response results should be generated for several ground motions to determine for code use the coefficient $\alpha$ which should depend on the design value of the reduction factor $R$.

Even if the asymmetric-plan system can be designed for significant yielding in such a way that the ductility demand on the flexible-side element does not exceed the symmetric-plan value, the element deformation may still be larger because of plan-asymmetry. It may not be possible to reduce this deformation by increasing the strength of the system because, as shown by the responses of SDF systems (Veletasos and Vann 1971), the deformation of a medium-period, velocity-sensitive system is not strongly affected by its strength and it is for such systems that the additional deformation due to plan-asymmetry is most significant (Figs. 2 and 3). Because increasing the strength of a system beyond that required for it to remain elastic would not influence its response if it is within the elastic range, the additional deformations of elastic systems resulting from plan-asymmetry also can not be reduced. Thus, these larger deformations should be provided for in the design of asymmetric-plan structures.

7 CONCLUSIONS

This investigation of the effects of plan-asymmetry on the earthquake response of one-story systems designed by seismic codes and how well these effects are represented by the torsional provisions in building codes has led to the following conclusions:

A stiff-side resisting element with design force smaller than its symmetric-plan value, which is permitted by some codes, experiences increased ductility demand because of plan-asymmetry. However, if the force reduction is precluded, as in some codes, the ductility demand on this element is roughly unaffected by plan-asymmetry. The ductility demand on the flexible-side element is significantly smaller than in the symmetric-plan system, with exceptions at few periods, regardless of whether the design force reduction for the stiff-side element is permitted or not.

Although, symmetric-plan systems designed with $R = 1$ would be expected to remain elastic during the design ground motion, similarly designed asymmetric-plan systems may deform into the inelastic range. Also because of torsional motions, the element deformation may significantly exceed the deformation of the corresponding symmetric-plan system. Thus, asymmetric-plan systems designed with $R = 1$ may experience structural damage due to yielding and nonstructural damage resulting from increased deformations.

Building code provisions do not ensure that the deformation and ductility demands on an asymmetric-plan system are similar to those on a similarly-designed symmetric-plan system. This suggests that the design eccentricity should be modified. This goal can usually be achieved for stiff-side elements by precluding any reduction in their design forces below their symmetric-plan values; $\delta = 0$ in the design eccentricity, $e_d$, is equivalent to this requirement. However, for some period values, this requirement is not sufficient and the design force for this element should be increased relative to its symmetric-plan value, which implies a negative value of $\delta$.

Similarly, the ductility demand on the flexible-side element can be kept below and close to its symmetric-plan value by modifying the coefficient $\alpha$ in the design eccentricity, $e_d$. The optimal value of $\alpha$ in Eq. 3 depends on the design value of the reduction factor $R$ -- being larger

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for smaller $R$ -- and may differ with the ground motion. Thus, response results should be generated for several ground motions to determine the coefficient $\alpha$ appropriate for use in building codes. However, it does not appear possible to reduce the additional element deformations due to plan-asymmetry by modifying the design eccentricity; these larger deformations should be provided for in building design.

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REFERENCES


Figure 1. Idealized one-story system.

Figure 2. Ratio of element deformations, $\mu_e/\mu_p$, and ductility demands, $\mu_e/\mu_p$, for asymmetric-plan and corresponding symmetric-plan systems designed by UBC-88; $R = 4$. 

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Figure 3. Ratio of element deformations, $u_i/u_o$, for asymmetric-plan and corresponding symmetric-plan systems, and element ductility demands, $\mu_i$, for asymmetric-plan systems designed by UBC-88; $R = 1$.

Figure 4. Ratio of stiff-side element ductility demands, $\mu_i/\mu_o$, for asymmetric-plan and corresponding symmetric-plan systems; $\alpha = 1$ and $\beta = 0$.

Figure 5. Ratio of flexible-side element ductility demands, $\mu_i/\mu_o$, for asymmetric-plan and corresponding symmetric-plan systems; $\delta = 0$ and $\beta = 0$. 