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Assessment of earthquake forces in low-rise structures from lateral and torsional coupling

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ABSTRACT: A correction procedure is developed that enables designers to modify the predictions from a planar lateral force procedure to more accurately account for the additional lateral response from torsion without doing a rigorous 3-dimensional analysis. This correction procedure was developed by utilizing a 3-dimensional structural system response from a ground motion record and the corresponding planar lateral force procedure. The applicability of the correction procedure in predicting earthquake demands inclusive of the torsional contribution is evaluated by comparing its results with those from a 3-dimensional response spectrum analysis.

1 INTRODUCTION

It is well known that structural systems subjected to earthquake ground motion can exhibit torsional response in addition to lateral motion. The presence of torsional response imposes greater demands on columns, particularly corners, and must be accounted for in design to avoid excessive damage and/or collapse. The problem is that determining the torsional contributions to the overall demand experienced by columns is complicated and ideally requires a rigorous 3-dimensional dynamic analysis. Alternatively, present building codes provide approximate methods of determining the torsional contribution to the overall system response. Traditionally, the minimum effects of torsion are accounted for by building codes through the application of an accidental eccentricity. Then the static torsional moment caused by this eccentricity is calculated and the induced shear forces distributed to each column according to stiffness.

There is evidence, however, that the torsional predictions in present building codes may not be accurate in light of the severe damage sustained by structures designed using such an approach. Numerous studies and earthquake reconnaissance reports have observed torsion in the structural response and its contributions to damage as well, Lin (1989), Rosenblueth (1989), and Chopra et al. (1989). This coupled motion has been shown to be significant in some cases and can lead to excessive damage and collapse. These observations have generated a significant amount of research effort to correct the shortcomings of present methods. Furthermore, it has been noted by Chopra et. al. (1977, 1989) that the code procedures neglect important parameters influencing the torsional response. Thus, the need for the adoption and implementation of improved methods to predict the torsional response of structures in present building codes is unquestionably needed.

The intent of this study was to provide a correction procedure for a standard building code application. The Uniform Building Code (UBC) was selected because of its wide application across the United States, however, the general concept developed here can be incorporated into other building codes as well.

The study progressed in the following conceptual phases to meet the objectives of this research. First, a simple yet accurate means to obtain the additional lateral response due to the torsional motion of a single story structure under multi-component lateral excitation was developed by correcting the planar statically-calculated value to a pseudo-dynamic value. Secondly, once this additional lateral response from torsion was found, it was then superimposed onto the lateral response as predicted by the UBC static lateral force procedure. Finally, a means of amplifying the static lateral displacement, inclusive of torsion, through a dynamic factor and the UBC seismic zone coefficient was developed. The end result provides a means for the designer, utilizing the UBC planar lateral force procedure, to obtain a more accurate lateral response, inclusive of the torsional response, under both wind and earthquake ground motions with minimal computational effort.

The first step in developing a correction procedure to account for the additional response due to torsion requires a means of evaluating the response. Analytically, this was accomplished by determining the equations of motion for a three degree of freedom (DOF) continuous parameter model where the structural elements are assumed to consist of a rigid diaphragm supported by inextensible columns.

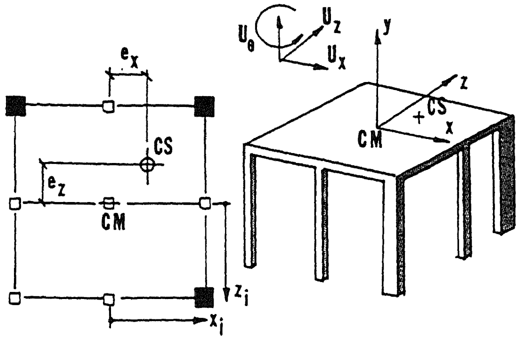

Fig. 1 Single story structure model

2 EQUATIONS OF MOTION

For the model in Fig. 1, the equations of motion can be derived using rigid body dynamics as demonstrated by Kan and Chopra (1977). The equations of motion can be obtained as follows: let K_{ix} and K_{iz} represent the total translational stiffness of the i^{th} resisting element along the principal major and minor axes of resistance, x and z , respectively, Fig.1. Once the lateral stiffness is found, the effective torsional stiffness must be determined. Here, let x_i and z_i be the distances of the i^{th} resisting element from the center of mass along the x and z axes, respectively. The effective torsional stiffness is defined with respect to the center of mass. The torsional stiffness of the individual resisting elements will not be included since it is assumed to have a negligible effect on the overall response. Thus, the corresponding effective torsional stiffness is defined as:

$$K_\theta = \sum_{i=1}^{n} k_{ix} \cdot z_i^2 + \sum_{i=1}^{n} k_{iz} \cdot x_i^2 \tag{1}$$

For a system of discrete resisting elements, the center of resistance can be statically computed as being the distances e_x and e_z measured from the center of mass along the x and z axes, Fig.1.

$$e_x = \frac{1}{K_z}\sum_{i=1}^{n} x_i \cdot k_{iz} \quad \text{and} \quad e_z = \frac{1}{K_x}\sum_{i=1}^{n} z_i \cdot k_{ix} \tag{2}$$

Based on these stiffness terms, the resulting undamped equations of motion, applicable for linear behavior of the continuous parameter model, can be expressed in matrix form as: (3)

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{z}_{cg} \\ r\ddot{\theta}_{cg} \\ \ddot{x}_{cg} \end{Bmatrix} + \begin{bmatrix} K_z & \frac{e_x}{r}K_z & 0 \\ \frac{e_x}{r}K_z & \frac{K_\theta}{r^2} & -\frac{e_z}{r}K_x \\ 0 & -\frac{e_z}{r}K_x & K_x \end{bmatrix} \begin{Bmatrix} z_{cg} \\ r\theta_{cg} \\ x_{cg} \end{Bmatrix} = \begin{Bmatrix} -m\ddot{u}_{gz} \\ 0 \\ -m\ddot{u}_{gx} \end{Bmatrix}$$

Damping is not included in the equations of motion since it will be included in the response spectrum. The ground acceleration $\ddot{u}_{gx}(t)$ and $\ddot{u}_{gz}(t)$ along the x and z axes, respectively, is assumed to be uniform across the base of the structure. The torsional ground acceleration, $\ddot{\theta}_{cg}$, will be assumed to be zero so that the lower bound of the torsional response is obtained.

According to Newmark and Rosenblueth (1971) an accurate estimate of the maximum value of response displacement, U_i, can be determined by combining u_1, u_2, u_3, the maximum values in the three natural modes of vibration:

$$U_i^2 = \sum_{n=1}^{3} u_n^2 + \sum_{n=1}^{3} \sum_{m=1}^{3} \frac{u_n u_m}{1+\epsilon_{nm}^2} \tag{4}$$

where m and n designate the modes of vibration and u_n is to be taken with the sign that its unit impulse response function has when it attains its maximum numerical value. The first term in Eq. 4 represents the square root of the sum of the squares of the modal maxima. The second term is needed in this study since it modifies the system responses to account for coupling action between closely spaced natural modes of vibration.

3 CORRECTION PROCEDURE

The correction procedure was developed by applying the definitions provided in Eqs. 1 to 4. Here a range of structures were studied to obtain a basis for predicting the torsional response and a static-to-dynamic amplification process. This development is described in the following discussion.

In order to cover a wide range of single story structures, four common column classifications were studied. This approach provided four, single story steel structures differing only in the column cross sectional properties, Fig. 2. The four column classifications are numerically designated as the following: (1) columns consisting of various wide flange sections; (2) columns consisting of various wide flange sections with additional stiffness in corner columns; (3) columns consisting of one wide flange section size used throughout the structure; and (4) columns consisting of one square tube steel column size used throughout the structure.

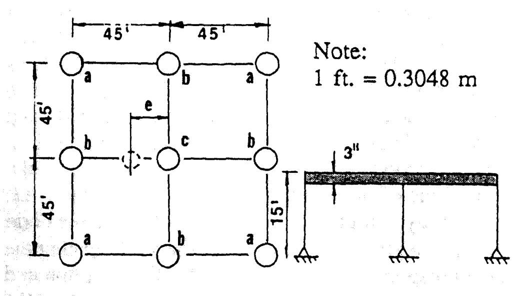

Fig. 2 Column plan of study models

Table 1. Column cross sectional properties at 0.5% story drift.

Model number	Column	$I_x \cdot 10^6$ mm^4	I_x / I_z
1	a	673	9.73
	b	1348	9.73
	c	2695	9.73
2	a	1348	9.73
	b	673	9.73
	c	2695	9.73
3	all	1177	9.73
4	all	123	1.0

The four single story structures were subjected to two types of multi-component lateral loads; (1) governing UBC lateral forces, and (2) 1940 El Centro ground motion. The governing lateral loads obtained from UBC, either wind or seismic, were used as the statically applied lateral loads. The multi-component lateral loads were applied in accordance with orthogonal considerations per UBC where 100% of the design load is applied along the axis of interest and 30% of the design load is applied perpendicular to this axis. In this study, the axis of interest corresponds to the lateral response coupled to the torsional motion. Therefore, 100% of the wind or earthquake ground motion was applied along the z (weak) axis of the structure and 30% was applied along the x axis.

Four different column drifts were studied for the four single story structures: 0.167%, 0.5%, 1.5%, and 4.5% of the story height. These column drifts were effectively achieved by applying the controlling UBC lateral load and sizing the columns accordingly to obtain these drifts. Table 1 provides the cross sectional properties of the .5% story drift only. However, the remaining story drifts maintained the same I_x / I_z ratio while I_x and I_z differed to meet the drift requirement. The UBC lateral loads applicable in seismic zone 2A and corresponding wind loads of 129 km/h were evaluated and compared. The controlling lateral loads were wind and thus became the design lateral loads for the study models.

In each column drift, twelve single component eccentricities ranging from 0 to 36% of the maximum building dimension were imposed on the system by relocating the center column mass along the x axis of the structure, effectively coupling the torsional response to the lateral response along the weak axis of the structure. The resulting lateral and torsional responses were computed for each of these cases.

3.1 *Static and dynamic analysis*

A 3-dimensional dynamic analysis was completed for each structure for the column drift and eccentricity cases defined above. Furthermore, a static planar analysis of the structures utilizing the governing UBC lateral wind loads was completed for each analytical case. Knowing the dynamic torsional and coupled lateral responses provided a means to develop a generic procedure capable of estimating the additional lateral response caused by torsion for both statically- and dynamically- applied loads. In addition, by evaluating the discrepancy between static and dynamic procedures, a method can be obtained to extend and correct the static predictions to match more closely dynamic demands on the structure. Therefore, the correction procedure is accomplished in two parts, torsional and dynamic corrections.

Torsional response correction: The additional lateral response due to torsion and the corresponding coupled lateral response, both defined at the center of gravity, were obtained for each column drift and eccentricity, e, combination as a function of e/r, where r is the radius of gyration of the slab. The torsional response was obtained in the four building types for twelve different eccentricities.

Knowing the two system responses at the center of gravity, the torsional response, $r\,\theta$, could then be quantified as a percentage of the coupled translational response, Δ, denoted as $r\,\theta/\Delta$ for each e/r ratio. A best fit curve for each column drift case, utilizing the least squares method, was determined for the twelve eccentricities as a function of e/r for each single story structure. It was found that since the ratio of the lateral stiffness, k_x / k_z, was the same for each column location, the coefficients determined for the best fit curves were identical for the four column drifts examined for each single story structure. Thus, this procedure provided one design curve per single story structure applicable for the range of column drifts studied. Since the to coupled lateral response as a function of normalized eccentricity torsional to lateral response of the square tube steel column configuration is approximately eight times higher than the other column configurations, the corresponding design curve is not presented in Fig. 3 so that the graph can be more legible. It can be seen by comparing the torsional response plotted in Fig. 3, the torsional response differs among the different column configurations. Therefore, for brevity, the correction procedure will be developed for the various wide-flange column configuration designated as number 1. However, the correction procedure can be developed for other column configurations as well.

Furthermore, an important assumption in this study is that the ratio of the torsional response to the coupled lateral response, $r\,\theta/\Delta$, is constant regardless if the lateral loads are due to seismic ground motion or design wind. Since $r\,\theta/\Delta$ is generic with respect to the type of applied lateral load, it is convenient to use the UBC static predictions as a basis for the correction procedure to obtain a closer prediction of dynamic response. In order to predict seismic demands based on the UBC static predictions a means of scaling the torsional and lateral responses to represent such demands is needed.

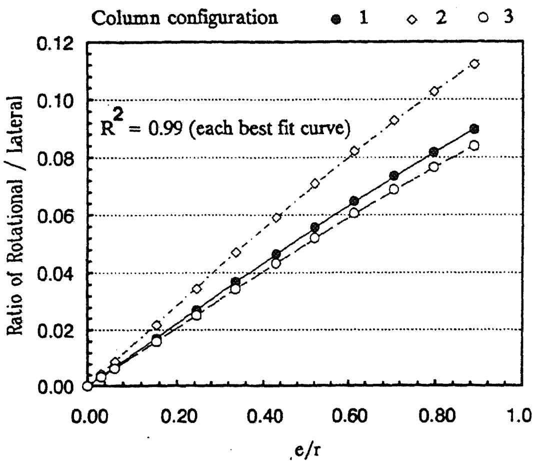

Fig. 3 Best fit curve for torsional to coupled lateral response as a function of normalized eccentricity

Dynamic factor correction: The dynamic factor (DF) was based on the ratio of the dynamic to static lateral predictions for each imposed eccentricity and column drift case. This resulted in twelve dynamic factors per column drift considered which were plotted as a function of e/r. Thus, four DF design curves per single story structure were obtained. The DF curves for the various wide-flange column configuration is presented in Fig. 4.

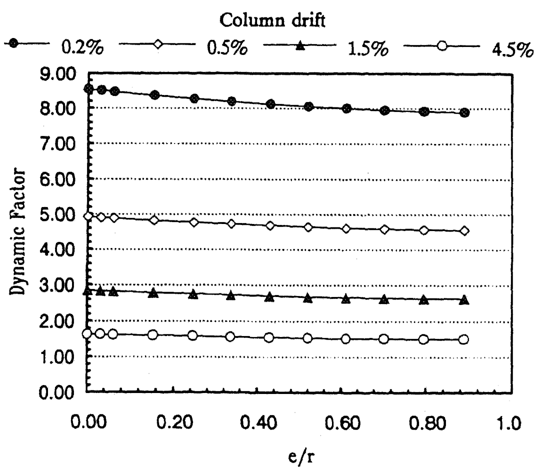

Fig. 4 Dynamic factors for column configuration

It was assumed that since the 1940 El Centro spectra corresponds to ground motion experienced in seismic zone 4, the DF for other seismic zones is approximated by taking the design curve value and multiplying by the ratio of Z/0.4. Here Z is the UBC seismic zone coefficient for the structure in question and 0.4 is the seismic zone coefficient for seismic zone 4. This procedure will reduce the DF to be more representative of the discrepancy that might be seen between the governing UBC lateral loads and earthquake ground motion in less severe seismic regions.

Generalizing application of design curves: In order to generalize the application of the design curves for $r\,\theta/\Delta$ and DF in Figs. 3 and 4, a modification factor must be used to adjust the curves so that they are applicable for other single story structures. Previous research by Hejal and Chopra (1989) has shown that the coupled lateral-torsional response of single story structures depends on e/r, the uncoupled torsional to lateral frequency ratio ω_θ/ω_z, the fundamental lateral vibration period, and damping ratio ξ for the structure. Accordingly, the design curves are based on a damping ratio of 5% and are expressed as a function of e/r ratios. However, the influence of the uncoupled torsional to lateral frequency ratio, ω_θ/ω_z, needs to be accounted for in order to generalize the application of the design curves.

Therefore, in order to generalize the system response, a modification factor is defined below:

$$\alpha = \frac{r_{actual}}{r_{ref.}} \cdot \frac{\omega^2_{z-actual}}{\omega^2_{\theta-actual}} \cdot \frac{\omega^2_{\theta-ref.}}{\omega^2_{z-ref.}} \cdot \frac{\%\ drift_{actual}}{\%\ drift_{ref.}} \qquad (5)$$

The subscript, *ref.*, refers to numerical values obtained from the study models and the subscript, *actual*, designates values that must be computed from the properties of the actual structure in question. Furthermore, the UBC utilizes a numerical coefficient, R_w, to allow for an assumed level of nonlinear behavior under strong shaking by reducing the design forces. However, since the study is restricted to linear elastic behavior, R_w will be equal to one. The modification factor, α, is used as a multiplier on the final system response and will be illustrated in the example problem to follow.

4 SUMMARY OF ANALYSIS PROCEDURE

1. Compute the centers of mass and stiffness.

2. Compute the eccentricity. Compute the distance between the centers of mass and stiffness.

3. Compute the distance of each column from the center of mass. Compute the distance x_i and z_i (along the x and z axis, respectively) from the center of mass to each column to obtain r_i, defined as:

$$r_i = \sqrt{x_i^2 + z_i^2} \qquad (6)$$

4. Compute the radius of gyration of the rigid diaphragm.

5. Compute the design story drift.

6. Compute $\omega_{\theta\text{-actual}}$, $\omega_{z\text{-actual}}$ and α. Compute the modification factor, α, according to Eq. 5, where $r_{ref.}$ = 1120.14 cm and the corresponding % drift $_{ref.}$ is the closest % drift to the actual value in question.

7. Compute the β value to obtain $r\,\theta/\Delta$ and DF. Using the appropriate design curves for the column classification being considered with the abscissa value, β, obtain $r\,\theta/\Delta$ and the dynamic factor, DF.

$$\beta = \frac{e}{r} \tag{7}$$

8. Modify planar static predictions for the column displacements to account for torsion. To estimate the coupled lateral-torsional response of a structure subjected to planar lateral design loads, use the lateral displacement obtained from the UBC planar code procedure and modify as follows. Note, the column position dictates as to whether a sine or cosine function is used and that the net lateral global displacement differs per column location. When $x_i < z_i$: (If $x_i > z_i$: the sine is replaced by a cosine function.) (8)

$$\Delta_{(\delta+\theta)_z} = \delta_{code-z} + [\frac{r_i}{r_{actual}} \cdot \frac{r\theta}{\Delta} \cdot \delta_{code-z}] \sin\{\frac{r\theta}{\Delta} \cdot \frac{\delta_{code-z}}{r_{actual}}\}$$

$$\Delta_{(\delta+\theta)_x} = \delta_{code-x} + [\frac{r_i}{r_{actual}} \cdot \frac{r\theta}{\Delta} \cdot \delta_{code-z}] \cos\{\frac{r\theta}{\Delta} \cdot \frac{\delta_{code-z}}{r_{actual}}\}$$

where δ_{code-z} is the lateral displacement excluding torsional motion due to UBC lateral design loads applied along the weak axis of the structure, $r\,\theta/\Delta$ is obtained from the design curves, r_i is determined in Step 3, and r_{actual} is the radius of gyration of the structure in question.

9. Compute the dynamic factor and modify static displacements for the design seismic zone. Compute Eq. 9 or 10 and apply the appropriate seismic zone factor, Z, obtained from the UBC code to scale the response appropriately. The ratio of the design seismic zone factor, Z, to 0.4 is used to scale the response for the appropriate seismic zone. The R_w appears in the equations as a reminder that an assumed level of nonlinear behavior is not already addressed in the correction procedure but can be applied on the final result.

$$\frac{Z}{0.4} \cdot \frac{\alpha}{R_w} \cdot [DF] \cdot \Delta_{(\delta+\theta)} \quad \text{wind governs} \tag{9}$$

$$\delta_{code} + \frac{Z}{0.4} \cdot \frac{\alpha}{R_w} \cdot [DF] \cdot \Delta_{(\theta)} \quad \text{seismic governs} \tag{10}$$

4.1 *Example problem*

The structure in Fig. 5, meets column classification number 1 with pinned/fixed column end restraints and will be analyzed using the correction procedure. The structure is analyzed for UBC wind speeds of 129km/h and a seismic zone 2A in the USA.

1. Compute the centers of mass and stiffness: The center of gravity is at the center of the slab. The center of stiffness is located 10.3 m from the north edge of the building.

2. Compute the eccentricities:

e_z = 10.32 - 9.14 = 1.16 m and e_x = 0.0

3. Compute the distance of each column from the center of mass: Consider the displacements in z for columns 1, 8, and 16.

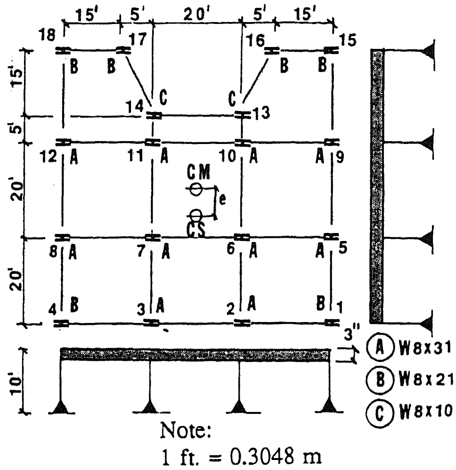

Fig. 5 Example structure

$r_1 = \sqrt{914.4^2 + 914.4^2}$ = 1293.16 cm

$r_8 = \sqrt{914.4^2 + 304.8^2}$ = 963.86 cm

$r_{16} = \sqrt{457.2^2 + 914.4^2}$ = 1022.33 cm

4. Compute the radius of gyration:

$r = \sqrt{(1828.8^2 + 1828.8^2)/12}$ = 746.6 cm

5. Compute % column drift

UBC wind loads = 1.27 kN/m ^{2} % drift = 0.61%

6. Compute ω_θ, ω_z and α (Eq.5): Let R_w = 1.0

$\{\omega_\theta/\omega_z\}^2_{actual}$ = 3.609 , $\{\omega_\theta/\omega_z\}^2_{base}$ = 8.085

$$\alpha = \frac{746.6}{1120.14} \cdot \frac{8.085}{3.609} \cdot \frac{0.005}{0.0061} \cdot \frac{1}{1} = 1.22$$

7. Compute the ß value and obtain $r\,\theta/\Delta$:

ß = (116)/(746.6) = 0.156 → Fig. 3: $r\,\theta/\Delta$ = 0.024

8. Modify static predictions for column displacements to account for torsion. For the three columns (1, 8, 16), the static lateral displacements in z inclusive of torsion become: (δ_{code-z} = 1.85 cm excluding torsion)

$$\Delta_{(\delta+\theta)_{1z}} = 1.85 + \left[\frac{1293.16}{746.6} \cdot 0.024 \cdot (1.85)\right] \cos\frac{0.024(1.85)}{746.6} = 1.93 \text{ cm}$$

similarly: $\Delta_{(\delta+\theta)_{8z}}$ = 1.88cm $\Delta_{(\delta+\theta)_{16z}}$ = 1.85cm

9. Compute the dynamic factor and modify static displacements for a seismic zone 2A. With e/r= 0.156 in Fig. 4 Interpolate: DF= 4.83.

$$\Delta'_{(\delta+\theta)_{1z}} = \frac{0.15}{0.4} 4.83 \cdot 1.22 \cdot [1.93] = 4.26 \text{ cm}$$

$\Delta'_{(\delta+\theta)_{8z}}$ = 4.15 cm , $\Delta'_{(\delta+\theta)_{16z}}$ = 4.09 cm

In the example problem the lateral displacement was obtained from the governing UBC lateral loads and designated as δ_{code-z}. The UBC lateral displacement was then modified in step 8 to include the additional lateral response from torsion by using the suggested correction procedure and was designated as $\Delta_{\delta+\theta}$.

Table 2. Comparison of correction procedure with results of 3-dimensional dynamic analysis.

Column Number	Correction Procedure	Response Spectrum	
	$\Delta'_{\delta+\theta}$ cm	$\Delta'_{\delta+\theta}$ cm	$\%\Delta'_{\delta+\theta}$
1	4.26	4.26	0.0%
8	4.15	4.11	1.0%
16	4.09	4.09	5.6%

The more accurate static lateral demands, $\Delta_{\delta+\theta}$, were then scaled by the DF, α, and design seismic zone coefficient to earthquake displacements that might be seen in a 2A seismic zone in the USA. In Table 2, a comparison of the predicted earthquake demands using the correction procedure to predictions obtained from a 3-dimensional dynamic response spectrum analysis accounting for the additional lateral response due to torsion is done. The percent difference between the correction procedure and the more rigorous 3-dimensional analysis are found in the last column of Table 2. As can be seen the correction procedure is slightly conservative and well within 10% of the earthquake predictions obtained from a 3-dimensional dynamic analysis.

It should be noted again that application of the design curves developed here are limited to single story structures meeting column configuration number 1 with a single component eccentricity effectively coupling the torsional response to the weak axis of the structure.

5 CONCLUSIONS

It has been shown that a generic ratio of the torsional to coupled lateral response, $r\,\theta/\Delta$, can be expressed as a function of the eccentricity to radius of gyration, e/r. By plotting $r\,\theta/\Delta$ as a function of e/r, design curves were developed which enable the rotational response, $r\,\theta$, of a single story structure for a known e/r to be obtained. The additional lateral response due to torsion can then be scaled from $r\,\theta/\Delta$ for each column location and superimposed upon the lateral response obtained from the planar code lateral loads. Furthermore, a dynamic factor to correct the static behavior to represent dynamic behavior can be computed by comparing lateral displacements obtained from planar code procedures to a regional design ground motion record. Similarly, the DF can be expressed as a function of e/r and column drift. By knowing the lateral response, inclusive of the torsional contribution, the designer can amplify the static response with the DF obtained from the design curves, a correction factor, α, and the regionally designated seismic zone coefficients to that of an expected earthquake demand within any selected seismic zone.

The correction procedure enables a planar lateral force procedure to be extended to accurately predict the lateral dynamic demands experienced by columns in coupled lateral-torsional structures under wind and earthquake excitations. To demonstration the development of the correction procedure with a planar lateral force procedure in the study, the Uniform Building Code utilized in the U.S.A. was used. Verification of the applicability of the correction procedure and design curves can be seen by comparing the results of the enclosed example problem to the results of a 3-dimensional response spectrum analysis. The correction procedure produces results well within 10% of the lateral predictions obtained from the 3-dimensional dynamic analysis and is slightly conservative. Therefore, the correction procedure is a simple yet accurate means of improving a planar lateral force procedure to more accurately account for the effects of torsion on lateral resisting elements for both wind and seismic considerations.

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