BOUNDARY EFFECTS IN STEEL MOMENT CONNECTIONS

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SUMMARY

Fully restrained beam-to-column connections in steel moment resisting frames have been traditionally designed by the assumption that classical beam theory applies. Accordingly, the beam flanges are assumed to transfer the moment, while the beam web is assumed to carry the shear force. Thus, welded flanges and the bolted web connections are assumed to be adequate to develop the plastic moment capacity of the beam.

This study shows that, due to boundary effects, the stress distribution in the beam near its connection to the column differs drastically from the traditional assumptions. The stress flow, represented by the principal stress resultant vectors, tends toward the flanges, forming a distinct pair of diagonal struts. Thus, both normal and shear stresses concentrate in the flanges, leaving most of the beam web practically devoid of shear stress. As a result, most of the bending moment and shear force are carried by the flanges, producing a condition of gross overload.

This paper presents the principal findings of the study of boundary effects in steel moment connections. The study is based on a comprehensive finite element investigation of the stress and strain distributions in a representative fully restrained beam-to-column connection. The factors responsible for the stress redistribution and deformation restraint are explained using St. Venant's principle. It was found that: 1) warping of the beam cross section caused by shear deformation; 2) three-dimensional volumetric deformation due to Poisson's effect; and 3) flexural and shear deformations of the column, are restrained in the traditional connection configuration and produce self-equilibrated stress fields that alter the stress field given by the classical beam theory. The superposition of these stress fields explains the stress overload condition found in beam flange and the welds.

Understanding of the actual stress flow in the moment connection leads to a simple truss analogy for stress flow model and rational design procedure for steel moment connections. Introduction of a new design method based on the truss analogy concludes this paper.

INTRODUCTION

Boundary conditions of structural members can be divided into two categories: traction boundary condition and displacement boundary condition. For a beam member, significant changes of stress distribution occur near the boundary where Poisson effect or warping due to shear deformation is fully or partially restrained, or where tractions are not compatible with the stress distribution given by classical beam theory. According to Saint's Venant's principle, boundary conditions of an elastic member can be replaced by statically equivalent systems of forces, which maintain self-equilibrium.

Statically equivalent systems of forces due to the displacement boundary conditions can be induced in a member by two different mechanisms. One is self-straining due to restraint of deformations of the member itself. For example, at a fixed end of a beam, the restraints of Poisson effect and shear deformation at the support produce additional elastic strain energy due to self-straining in the beam cross section near the support region. The other

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mechanism is additional displacements imposed by deformation of the adjacent members. For example, in a beam-to-column moment connection case, additional displacements imposed on the beam by flexural and shear deformation of the column panel zone or the displacement imposed on the beam flange due to local bending of the column flange, amplify the stress redistribution in the beam.

DEEP RECTANGULAR BEAM WITH A TIP DOWN LOAD

Fig. 1 shows a cantilever rectangular beam with a tip down load. Finite element analysis results show that the distribution of normal stress in the x-direction is no longer linear, the shear stress distribution is not quadratic, and the normal stress in the y-direction does not vanish over the cross section near the fixed-end boundary. These stress redistributions are due to self-straining induced by displacement restraints of the beam cross section at the support.

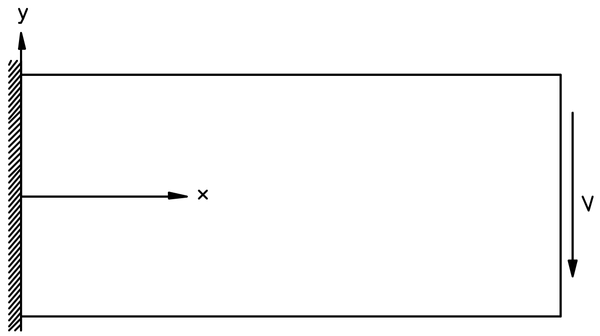


FIG. 1. Rectangular Beam with a Tip Down Load

In order to examine the effect of the displacement restraints of the beam end cross section, the self-straining induced by the displacement restraint is divided into two types: 1)“restraint of Poisson effect” and 2)“restraint of warping due to shear deformation”. Two different finite element analysis results with Poisson ratio equal to 0.3 and 0 were compared with the stress patterns given by classical beam theory. The difference in stresses between the finite element analysis results with Poisson ratio equal to 0.3 and 0 is herein called “restraint of Poisson effect”. The difference between the finite element analysis results with zero Poisson ratio and the results given by using the classical beam theory is called “restraint of warping due to shear deformation”.

Restraint of Poisson Effect

If a beam can freely deform in the y-direction and if there is no traction on the top and bottom surfaces of the beam, the normal stress in the transverse direction of the beam, σ_y, can be assumed to be zero along the cross section at a considerable distance away from the support. Thus, the transverse direction normal strain, ε_y, away from the support depends only on the Poisson effect in a plane stress condition. Consequently, the depth of the beam in the tensile region (upper half) tends to become shorter than the undeformed depth, while that of the beam in compressive region (lower half) tends to become longer depending on the magnitude of ε_x (Fig. 2). However, because the vertical deformation of the beam cross section is restrained at the fixed support, ε_y must be zero along the support line of the beam. Thus, additional normal stress in the transverse direction, σ_y^{sp}, due to the “restraint of Poisson effect” is induced near the support. The strain ε_y rapidly changes from zero at the support to $-\nu\varepsilon_x$ away from the support. The maximum and minimum values of ε_y occur at the top and bottom surfaces of the beam, respectively, at a small distance away from the support. This strain distribution creates “kink” regions at the top and bottom surfaces of the beam near the support, forming “depressed” and “extruded” regions (Fig. 2).

Thus, the “restraint of Poisson effect” creates additional shear strain near the support. This shear strain induces additional shear stress, τ_{xy}^{sp}, which has the same direction as the shear stress induced by the applied load in the top and bottom regions and opposite direction in the middle in order to maintain self-equilibrium along the cross section. As a result, additional downward shear forces are induced in the top and bottom regions and an upward shear force is induced in the middle region of the beam. In order to maintain force equilibrium with t_{xy}^{sp} in the x-direction, additional normal stress, s_x^{sp}, and its gradient in the x-direction are required. The resultant moment couple produced by s_x^{sp} should also be zero at any cross section of the beam. Therefore in the tensile stress region, the positive area of s_x^{sp} in the top portion of the beam is smaller than the negative area in the middle above the centerline of the beam. Thus, the magnitude of normal stress in the x-direction increases near the top and bottom of the beam, while it decreases in the middle of the beam, as compared with that obtained from the classical beam theory.

Restraint of Warping due to Shear Deformation

Fig. 3 shows the shape of longitudinal warping due to shear deformation of the rectangular beam. The stress redistribution due to restraint of this warping of the cross section can also be replaced by statically equivalent systems of forces in the x-direction along the support line A-A, such that the work done by self-straining is minimum [6]. These additional forces induce additional normal stress, s_x^{sw}, in the x-direction along the support line. The sign and magnitude of s_x^{sw} are related to the shape of warping. Thus, the sign of s_x^{sw} in the middle of the beam is opposite to the sign of the normal stress in the x-direction obtained from the classical beam theory, while s_x^{sw} has the same sign in the top and bottom regions of the beam.

The gradient of s_x^{sw} in the x-direction creates additional shear stress, t_{xy}^{sw}, to maintain equilibrium in the x-direction. Similar to the case of the restraint of Poisson effect, additional downward shear forces in the top and bottom regions, and additional upward shear force in the middle are induced. The magnitude of t_{xy}^{sw} is smaller than that of t_{xy}^{sp} because of slower decay rate of s_x^{sw}.

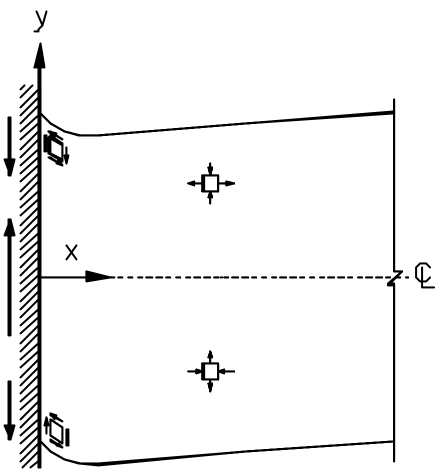


FIG. 2. Restraint of Poisson Effect

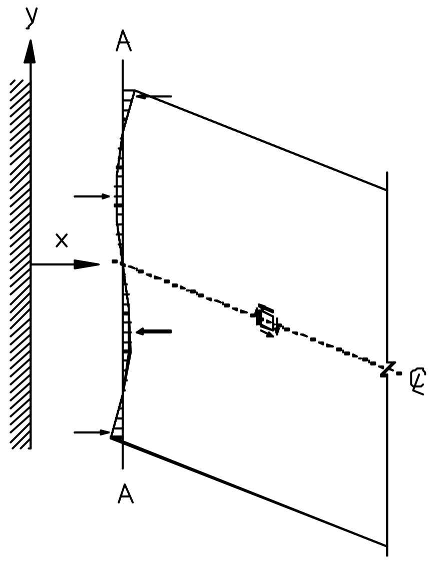
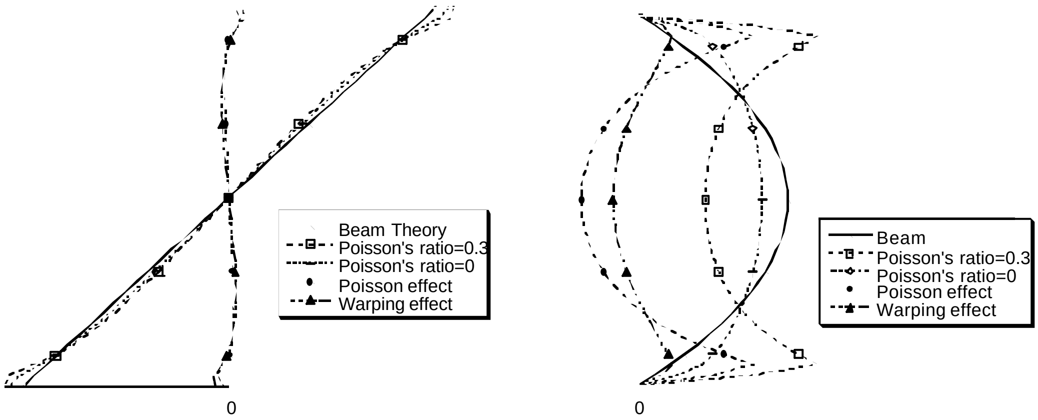


FIG. 3. Restraint of Shear Deformation

Stress Comparisons

For an elastic member, the stress distributions in the support region can be obtained by superposition of the stress values from the classical beam theory, the additional stresses due to the restraint of Poisson effect, and the additional stresses due to the restraint of warping due to shear deformation. Fig. 4 shows the stress contributions of each effect near the support line. The magnitude of the normal stress in the x-direction is larger than that given by the classical beam theory in the top and bottom regions, while it is smaller in the middle due to the self-straining of the cross section (Fig. 4(a)). The shear stress distribution pattern near the support is completely different from the parabolic distribution given by the classical beam theory. The shear stress obtained from the beam theory, additional shear stress due to the restraint of Poisson effect, t_{xy}^{sp}, and that due to the restraint of warping due to shear deformation, t_{xy}^{sw}, offset each other in the middle, but add up in the top and bottom regions

(Fig. 4(b)). The non-vanishing s_y and the redistribution of t_{xy} and s_x change the directions and magnitudes of the principal stresses near the support. The directions of principal stress vectors are directed towards the corners of the beam in the support region.



(a) Normal Stress **(b) Shear Stress**

FIG. 4. Stress Contributions of Boundary Effects Near the Support of the Rectangular Beam

CANTILEVER I-BEAM WITH A TIP DOWN LOAD AND MOMENT

Typical wide flange section beams which have a thin web and thick flanges show significant changes of stress distribution and “load path” in the vicinity of the boundary. Fig. 5 shows a cantilever I-beam with a tip down load and moment. Boundary effects for the I-beam are more complex than those for the rectangular beam because of interaction between the flanges and the web and the non-uniform stress distribution along the width of the flanges.

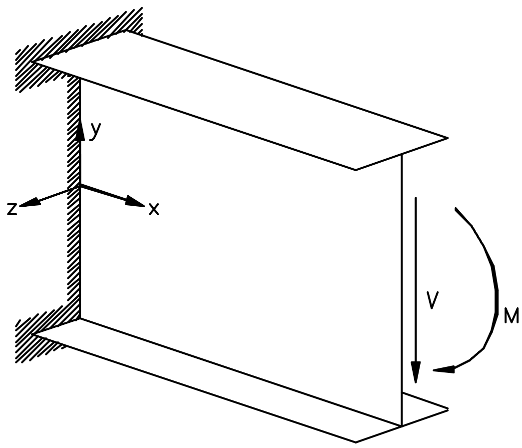


FIG. 5. Cantilever I-beam with a Tip Down Load and Moment

Restraint of Poisson Effect

The I-beam in the finite element model is a W36x150 (U.S.) wide flange section, whose web is the same as the rectangular beam examined previously. The moment-to-shear ratio at the support is 134 inches (3.4 m). Poisson effect in the web of the I-beam is restrained in a similar manner as in the rectangular beam. As the beam flanges are attached to the web, the flanges constrain the vertical “kinks” in the web caused by the restraint of Poisson effect near the support region, as shown in Fig. 2. Consequently, concentrated local bending occurs in the middle of the top and bottom flanges near the support (Fig. 6). This local bending of the flanges produces additional shear stresses in the flanges. The directions of local bending in the top and the bottom flange are identical. The local bending moments in the flanges also induce an additional moment couple of opposite direction in the web in order to maintain zero resultant moment over the beam cross section. Therefore, the additional normal stresses in the x-direction, s_x^{sp}, due to the restraint of Poisson effect for the I-beam are created by a superposition of the stresses caused by the restraint of Poisson effect in the web and the interaction between the flanges and the web.

Flange local bending induces additional shear forces in the flanges, which are concentrated near the junction of the flanges and the web. The resultant shear force in the web due to the restraint of Poisson effect is directed upwards to equilibrate the downward resultant shear forces in the flanges in the positive local bending regions. Similarly, the resultant shear force in the web is directed downwards in the negative flange local bending regions. It was assumed that the sign of local bending in the flanges is positive if the direction of the flange local bending is the same as that of global bending of the beam. Therefore, a portion of the shear force from the web transfers to the flanges in the positive flange local bending region near the support.

Restraint of Warping Due to Shear Deformation

Fig. 7(a) shows shear stress distribution in the I-beam from the classical beam theory assuming that the vertical shear stress is uniformly distributed along the width of the flanges. The corresponding longitudinal warping of the beam cross section due to shear deformation is shown in Fig. 7(b). Even though the shear deformation in the web is large, the slope of the warping in the web is quite flat when compared with that for the rectangular beam because the gradient of the shear stress in the y-direction is small for the I-beam. The shear deformation in the flanges is negligible, converging to zero slope at the free surface of the flanges. Thus, the slope of the warping abruptly changes at the junction of the flanges and the web. The location of the support line (zero work line of self-straining) will vary according to the proportions of the cross section. For example, for a thin web and thick flange cross section, the support line would pass through the lower portion of the flanges at the support. Restraint of this shear deformation creates local bending in the flanges. Thus, additional downward shear forces are induced in the flanges and an upward shear force is induced in the web.

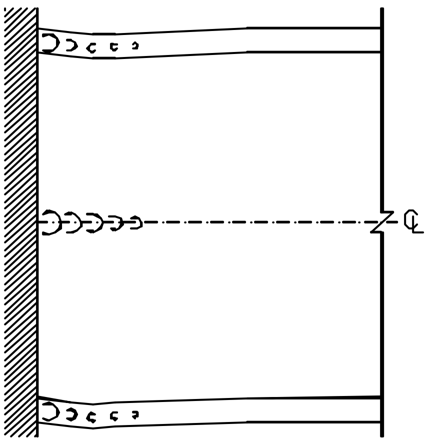


FIG. 6. Restraint of Poisson Effect

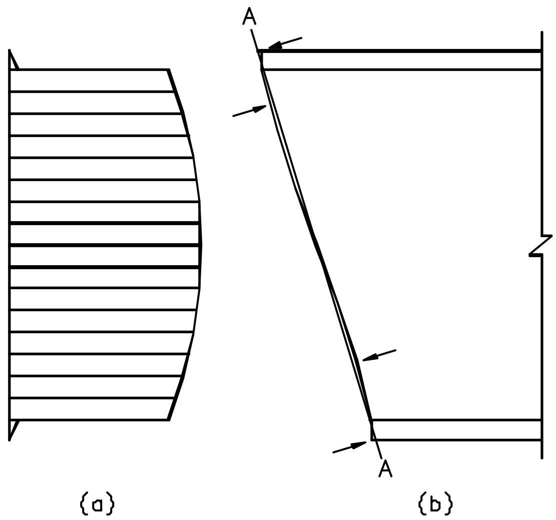
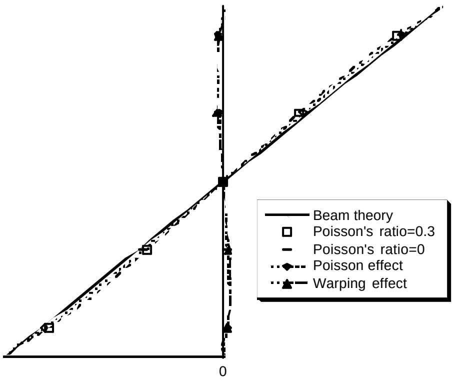


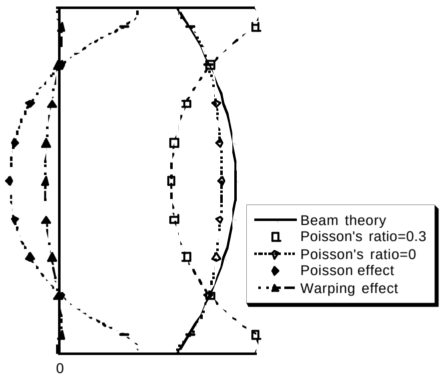
FIG. 7. Restraint of Shear Deformation

Stress Comparisons

Similar to the rectangular beam case, the stress distributions in the I-beam in the support region can be obtained by superposition. Fig. 8(a) shows the stress contributions of each effect for the normal stress in the x-direction, s_x, in the web of the I-beam near the support, based on the results from the finite element analysis using shell elements. Fig. 8(b) shows a similar comparison for the shear stress, t_{xy}. The resultant shear forces in the web due to the boundary effects have an upward direction. Thus, additional downward shear forces are induced in the flanges. Therefore, portions of the normal force and the shear force in the web transfer to the flanges near the support. In finite element analysis using shell elements, the shear force transferred through the beam flanges was approximately 12% of the total beam shear force, which is 6 times larger than that given by the classical beam theory. High shear stresses in the middle of the beam web away from the support diverge into two paths and are directed towards the corners of the web in the vicinity of the support. Consequently, near the support, the shear stresses in the middle of the web become small.



(a) Normal Stress



(b) Shear stress

FIG. 8. Stress Contributions of Boundary Effects in the Web Near the Support of the I-beam

BEAM-TO-COLUMN MOMENT CONNECTION

The beam in beam-to-column moment connections sustains even more complex boundary effects than the fixed end I-beam case. In addition to the effects of self-straining due to the deformation restraints of the beam, more displacements are imposed on the beam by column deformation.

Column Flexural and Shear Deformation

There are two main sources of column deformation in the moment connection region. One is column flexural deformation and the other is shear deformation of the column panel zone. For a thin web-thick flange wide flange section beam, most forces due to the moment couple are transferred through the beam flanges to the column. Thus, on the column side, equal magnitude and opposite direction normal forces are applied to the column flange. The column deforms with opposite curvatures near the top and bottom beam flanges.

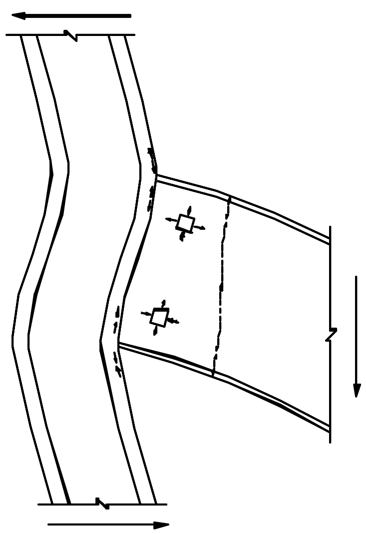
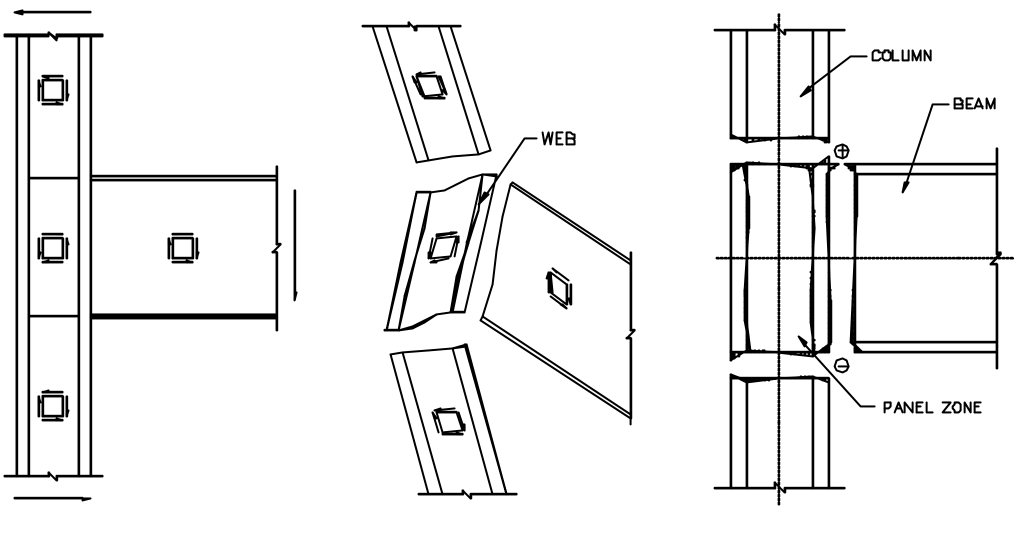


FIG. 9. Effect of Column Deformation

Column bending induces normal strains in the y-direction (direction of the column axis), e_y, in the column. The column flange facing the beam has positive e_y in the upper half of the beam (tensile region) and negative e_y in the lower half of the beam (compressive region) as shown in Fig. 9. The direction of e_y in the column flange is opposite to the direction of e_y in the beam web caused by the restraint of Poisson effect. In other words, negative e_y is induced in the beam tensile region while column bending produces positive e_y at the column face. Column deformation induces statically equivalent systems of forces in the beam, which have the same direction as those from beam self-straining due to the restraint of Poisson effect. Therefore, these additional forces add together, amplifying the stress redistribution in the beam. The resultant forces and moment couples induced by the column deformation should also be zero along any cross section of the beam.

The deformed shape of the column is opposite to the warped shape of the beam cross section caused by its shear deformation (dashed line in Fig. 9). Thus, column deformation induces additional statically equivalent systems of forces in the beam, which have the same direction as those due to the restraint of shear deformation of the beam. Note that the same direction forces imply that the signs of the additional forces in critical regions (corners of the beam) at the column face are the same. In the beam tensile region, compressive force is induced in most portions of the beam web, according to the deformed shape of the column flange. Thus, additional tensile force would be induced in the outer portions of the beam flange to achieve self-equilibrium. These geometric effects caused by the column deformation increase local bending of the beam flanges. Consequently, more shear and normal forces are transmitted from the web to the outer portions of the beam flanges near the column face than in the fixed-end I-beam case.

Another effect on the force flow in the moment connection region is restraint of the panel zone shear deformation. The direction of shear strain in the column panel zone is opposite to that in the column web away from the panel zone. Fig. 10(a) shows the directions of shear strains at each cross section of the moment connection region. The corresponding deformed shapes due to the shear deformations are shown in Fig. 10(b). As can be seen from these figures, the directions of the shear strains and the deformed shapes due to the shear deformations are opposing each other at all sections at the boundary of the panel zone. Thus, restraints of these shear deformations at all boundaries of the panel zone can be replaced by statically equivalent systems of forces, which maintain self-equilibrium at any cross section of the beam and the column. Statically equivalent systems of forces due to the restraint of shear deformations in the column are induced at the top and bottom of the panel zone because the shapes of warping are opposite. Fig. 10(c) shows approximate distributions and directions of the statically equivalent systems of forces around the panel zone due to the restraint of shear deformation.



(a) Shear Strains **(b) Shear Deformations** **(c) Additional Forces**

FIG. 10. Effects of Shear Deformation in the Connection Region.

Stress Comparisons

Finite element analyses were performed on a model of a W36x150 beam with the same moment-to-shear ratio as the fixed-end I-beam studied in the previous section, connected to a W14x455 column. The results obtained using a shell-element model show that the shear force transferred to the beam flanges was approximately 25% of the total beam shear force. That is more than two times larger than in the fixed-end I-beam case and 12 times larger than predicted by classical beam theory. This increase is due to the interaction between the beam and the column.

Fig. 11 shows the principal stress vectors in the beam and the column webs for the beam-to-column connection region. The stress resultants tend to flow towards the flanges as they approach the connection, thereby causing heavy normal and shear stress concentration in the beam flanges and leaving most of the beam web devoid of stress. The force flow in the beam web forms two diagonal struts in the moment connection region. A new design approach using this realistic load path has been proposed by Goel, et al. [1, 2, and 3]. This approach uses the reaction forces at the corners of the truss model to design steel moment connections.

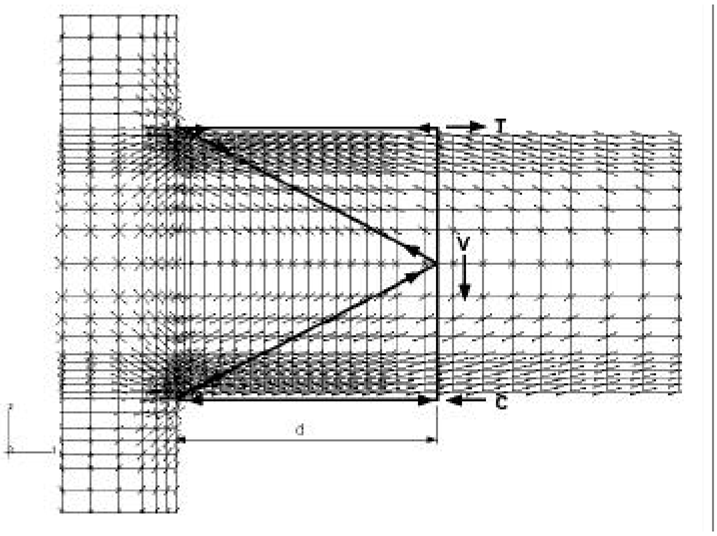


FIG. 11. Principal Stress Vectors and the Statically Admissible Truss Model

CONCLUSIONS

The "load path" in steel moment connections postulated by the classical beam theory is such that the flanges primarily transfer the moment while the web resists the shear force. Designing steel moment connections by using this assumption results in substantial overloading of the beam flanges.

The results of this study show that deformation restraints lead to drastic redistribution of stresses, caused by the two boundary effects, restraint of Poisson effect and restraint of warping due to shear deformation. Additional deformation of the column in the connection region further amplifies the stress redistribution in beam-to-column moment connections. A truss analogy model gives more realistic picture of the true stress flow in steel moment connections.

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