INELASTIC DISPLACEMENT RATIOS FOR DISPLACEMENT-BASED EARTHQUAKE RESISTANT DESIGN

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SUMMARY

A summary of results of a comprehensive statistical study of inelastic displacement ratios is presented. An inelastic displacement ratio is defined as the ratio of the maximum lateral inelastic displacement demand of a structure to the maximum lateral elastic displacement demand. These ratios were computed for single-degree-of-freedom systems undergoing different levels of inelastic deformation when subjected to a relatively large number of recorded earthquake ground motions. The study is based on 264 acceleration time histories recorded on firm sites during various earthquakes in California. Three types of soil conditions with shear wave velocities higher than 180 m/s in accordance with NEHRP recommendations are considered. More than 90,000 inelastic displacement ratios were computed and classified according to the type of soil conditions at the recording station, the level of ductility demand and the period of vibration. The influence of these parameters is carefully evaluated and discussed. A special emphasis is given to the dispersion of the results. Inelastic displacement ratios associated to mean values and those associated with various levels of probability of exceedance are presented. It is concluded that for sites with average shear wave velocities higher than 180 m/s the influence of soil conditions is relatively small and can be neglected for design purposes. Similarly the influence of period of vibration and of level of inelastic deformation is negligible for systems with periods of vibration longer than 1.0s. Finally results from nonlinear regression analyses are presented that provide a simplified expression to be used in design that permit an estimation of maximum inelastic displacements from maximum elastic displacements.

INTRODUCTION

Seismic design provisions typically allow structures to undergo inelastic deformations in the event of strong earthquake ground motions. However, in most practical design situations only linear elastic analyses are used to estimate the maximum response of the structure. Even if the availability and selection of adequate earthquake accelerations time histories for design purposes was not a problem, at present it is still considered impractical to carry out nonlinear time history analyses for most practical design situations. Thus, it is still necessary to use simplified analysis techniques to estimate the maximum inelastic response of a structure during severe earthquake ground motions.

A particularly appealing approach is to try to estimate the maximum inelastic response, and particularly the maximum lateral inelastic displacement demand, using the results from a linear elastic analysis. Efforts in this direction began many years ago. One of the first studies was done by Veletsos [9, 10] who pointed out that "it is instructive to relate the maximum deformation of the elasto-plastic system to that of an elastic system having the same stiffness as the initial stiffness of the inelastic system". Using single-degree-of-freedom (SDOF) systems when subjected to simple pulses and to three earthquake ground motions he noticed that in the low frequency range (frequencies smaller than 0.38 Hz) the maximum deformation of the inelastic and the associated elastic systems may be considered the same. This observation gave rise to what is now called *the "equal displacement rule*" which is the basis for estimating maximum deformations in most building codes. This study also concluded

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that in the high frequency and moderately high frequency regions, the inelastic displacements are significantly higher than their elastic counterparts.

For many years emphasis was placed on force-based methods, thus, little research was done after the 70's to improve approximate methods to estimate maximum inelastic displacements from maximum elastic displacements. Concerned with the possible limitations of the equal displacement rule in the short period region and also when applied to soft soils, Miranda [5, 6, 7] more recently studied the ratio of maximum inelastic displacement to that of elastic systems of SDOF systems with the same period and same damping ratio as the inelastic systems when subjected to 124 earthquake ground motions recorded on different types of ground motions. This study gave a special insight to this ratio in the short period range and to the limiting periods of the regions where the equal displacement rule is applicable. Based on the results from this and other studies several design recommendations now explicitly take into account the inelastic displacement ratio as a tool to estimate maximum inelastic deformation demands [1, 3].

On the other hand, in recent years and as a result of various earthquakes, there has been an important emphasis on the effect of soil conditions on the response of structures to earthquake ground motions. Based on improved information on soil conditions at accelerographic recording stations, there have been important modifications to site classifications included in design recommendations [3, 4].

The objective of this paper is to present a summary of the results of a comprehensive statistical study on inelastic displacement ratios for firm sites. The study presented here is limited to rock and relatively firm soil sites with shear wave velocities higher than 180 m/s. One of the main answers that this research was trying to respond to was if the inelastic displacement ratio to be used to estimate maximum inelastic deformations from maximum elastic deformations needed to be modified for various firm site classes for which seismic codes recommend different elastic design spectra.

INELASTIC DISPLACEMENT RATIOS

The inelastic displacement ratio, C_μ, is defined as the maximum lateral inelastic displacement demand, $\Delta_{inelastic}$, divided by the maximum lateral elastic displacement demand, $\Delta_{elastic}$, on a system with the same mass and initial stiffness (i.e., same period of vibration) when subjected to the same earthquake ground motion. Mathematically this is expressed as

$$C_\mu = \frac{\Delta_{inelastic}}{\Delta_{elastic}} \tag{1}$$

Thus, if information on this ratio is available, an estimation of the maximum inelastic displacement can be obtained from the maximum elastic displacement. In the study presented herein inelastic displacement ratios were computed for SDOF systems having viscous damping ratio of 5% and a nonlinear elasto-plastic hysteretic behaviour.

For earthquake design purposes it is useful to compute inelastic displacement ratios corresponding to different values of displacement ductility ratios, μ, in accordance to the inelastic deformation capacity of a structure. In this study, maximum inelastic displacement demands $\Delta_{inelastic}$ corresponding to a specific values of μ were computed by iteration on the lateral strength of the system until the displacement ductility demand was, within a tolerance, equal to the specified ductility ratio. The tolerance was chosen such that $\Delta_{inelastic}$ was considered satisfactory if the computed ductility demand was within 1% of the specified displacement ductility ratio.

Inelastic displacement ratios were computed for SDOF systems subjected to six different levels of inelastic deformation corresponding to the following ductility ratios: 1.5, 2, 3, 4, 5 and 6. For each earthquake record and each target displacement ductility ratio, the inelastic displacement ratios were computed for a set of 50 periods of vibration between 0.05 and 3.0 s.

EARTHQUAKE GROUND MOTIONS CONSIDERED

For the selection of earthquake ground motions to be considered in this study specific criteria were followed, namely that selected ground motions needed to comply with the following characteristics: (1) recorded on rock or firm sites with average shear wave velocities higher than 180 m/s; (2) recorded on accelerographic stations where detailed information exists on the geological and geotechnical conditions at the site that enables the classification of the recording site in accordance to recent code recommendations; (3) recorded on free field stations or in the first floor of low-rise buildings with negligible soil-structure interaction effects; (4) recorded in earthquakes with surface wave magnitudes (M_s) larger than 5.7; and (5) records in which at least one of the two horizontal components had a peak ground acceleration larger than 40 cm/s2.

Based on these criteria 264 earthquake acceleration time histories recorded in the state of California in the United States of America in 12 different earthquake were selected. A particularly large number of earthquake ground motions was selected in this study in order to assess the dispersion on the inelastic displacement ratios. The earthquake ground motions were divided into three groups according to the soil conditions at the recording stations. The first group consisted of ground motions recorded on stations located on rock with average shear wave velocities higher than 760 m/s (2,500 ft/s). The second group consisted of records obtained on stations on very dense soil or soft rock with average shear wave velocities between 360 m/s (1,200 ft/s) and 760 m/s while the third group consisted of ground motions recorded on stations on stiff soil with average shear wave velocities between 180 m/s (600 ft/s) and 360 m/s. Recording stations on the first group correspond to site classes A and B according to recent NEHRP recommendations [3, 4] while recording stations on the second and third group correspond to site classes C and D, respectively. Table 1 presents a summary of the number of records on each soil group selected in the different earthquakes considered herein. It can be seen that the smallest earthquake magnitude corresponds to the 1991 Sierra Madre earthquake (Ms=5.8) and the largest magnitude corresponds to the 1957 Kern County earthquake (Ms=7.7). A complete list of the earthquake records considered in this study can be found on Ref. 8.

Figure 1 shows the distribution of magnitudes and epicentral distances of the earthquake ground motions selected in the study. It can be seen that with the exception of one recording station, all other stations were closer than 120 km (75 miles) from the epicentre. Similarly, most of the records selected are from earthquakes with magnitudes between 6 and 7.5. Figure 2 shows peak ground accelerations and corresponding epicentral distances for the 264 records considered in this study. It can be seen that the set of ground motions selected, with exception of a few records, follows the overall trend that one would expect from known attenuation relationships in the sense that high levels of maximum ground acceleration correspond to recording sites with small epicentral distances and as the epicentral distance increases the maximum ground acceleration decreases.

Table 1. Summary of earthquake ground motions considered in this study.

Earthquake	Date	Magnitude	Site Classification			
		(Ms)	A & B	C	D	**TOTAL**
Kern County	21 Jul 57	7.7	0	4	2	**6**
Borrego Mountain	9 Abr 68	6.7	0	0	2	**2**
San Fernando	9 Feb 71	6.5	8	10	6	**24**
Imperial Valley	15 Oct 79	6.8	2	2	36	**40**
Morgan Hill	24 Apr 84	6.1	2	4	6	**12**
Palm Springs	8 Jul 86	6.0	8	0	0	**8**
Whittier	1 Oct 87	6.1	4	18	22	**44**
Whittier Aftershock	4 Oct 87	6.1	2	0	0	**2**
Loma Prieta	17 Oct 89	7.1	26	22	16	**64**
Sierra Madre	28 Jun 91	5.8	4	0	0	**4**
Landers	28 Jun 92	7.5	4	2	16	**22**
Northridge	17 Jan 94	6.8	18	14	4	**36**
		TOTAL	**78**	**76**	**110**	**264**

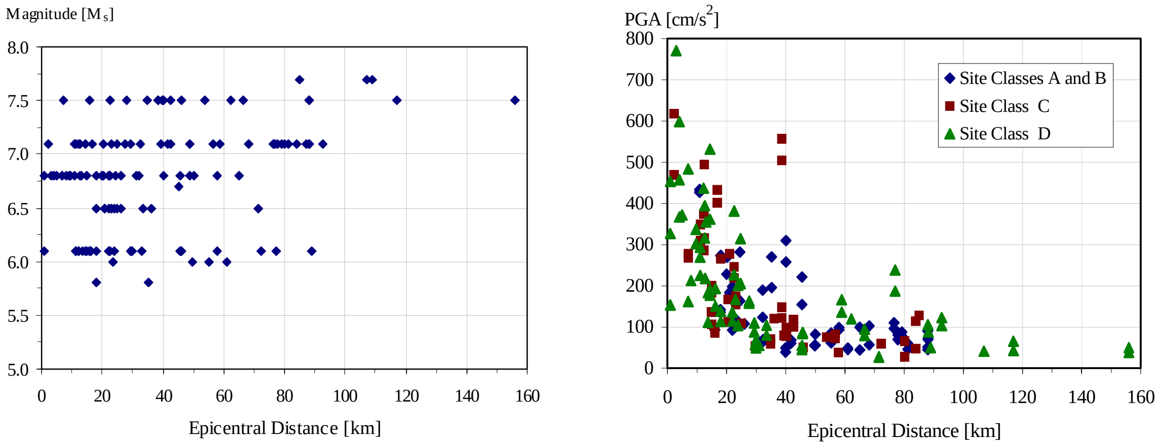

Figure 1.- Distribution of magnitudes, epcientral distances and maximum ground accelerations of earthquake ground motions considered in this study.

PRESENTATION OF RESULTS

Inelastic displacement ratios were organised for each soil group, each level of inelastic displacement demand and each period of vibration. The left hand side of figure 2 shows constant-ductility mean inelastic displacement ratios corresponding to the sites with average shear wave velocities higher than 760 m/s (site classes A and B). It can be seen that inelastic displacement ratios are characterised by being larger than one in the short period spectral region (maximum inelastic displacements are larger than maximum elastic displacements) and approximately equal to one (i.e. maximum inelastic displacements approximately equal to maximum elastic displacement) for periods longer than about 1.0 s. For periods smaller than 1.0 s inelastic displacement region are strongly dependent on the period of vibration and on the level of inelastic deformation. In general, in this spectral region maximum inelastic displacements become much larger than maximum elastic displacements as the ductility demand increases and as the period decreases. Furthermore, inelastic displacement ratios tend towards μ as the period of vibration tends to zero. It is important to notice that the limiting period that divides the spectral regions where the equal displacement rule is unconservative from the region where this approximation is applicable depends on the level of ductility. For example for a ductility ratio of 2 the equal displacement rule is applicable for periods longer than 0.4s while for a ductility ratio of 6 is applicable for periods longer than 1.0 s.

Mean inelastic displacement ratios for records obtained on sites with average shear wave velocities between 360 and 760 m/s (site class C) are shown in the right side of figure 2. It can be seen that the results are very similar to

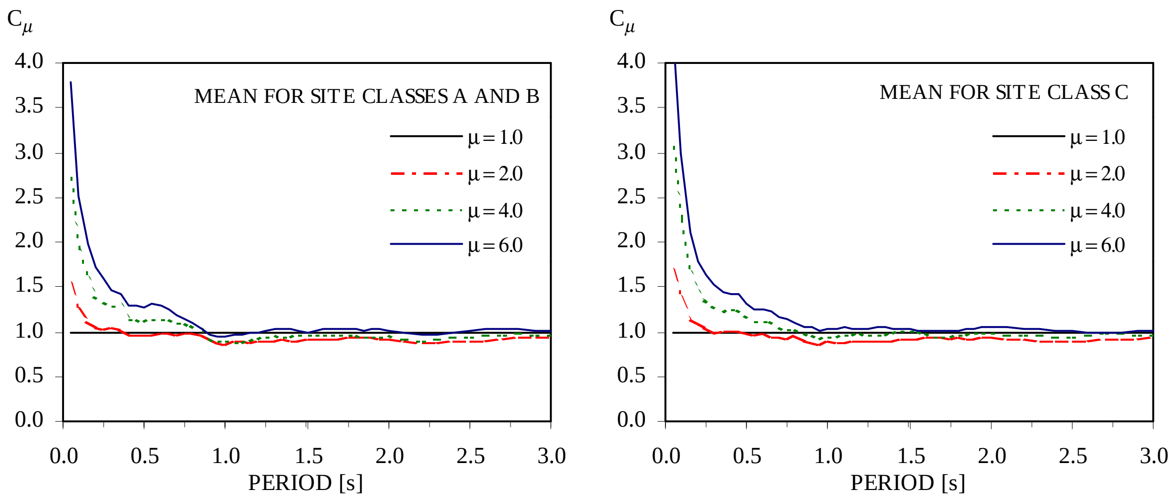

Figure 2. Mean inelastic displacement ratios for sites classes A and B and for site class C.

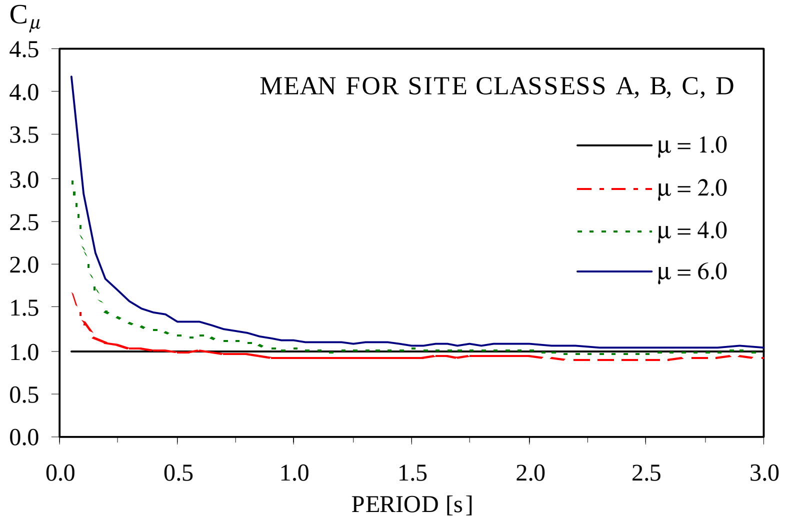

Figure 3. Mean inelastic displacement ratios for sites classes A, B, C and D (average of 264 records).

those for sites classes A and B. Although not shown here because of space limitations results, mean inelastic displacement ratios for site class D are also very similar to those for site classes A and B or those for site class C. Results for displacement ductility ratios equal to 1.5, 3, and 5 and for site class D are given in Ref. 8.

After observing that there are not important differences between the inelastic displacement ratio of these site classes it was decided to obtain mean values considering the results of all 264 records which correspond to sites with average shear wave velocities higher than 180 m/s. Results are shown in figure 3. In order to evaluate the errors that could be introduced if one uses the results of figure 3 as representative of any site with average shear wave velocity higher than 180 m/s, ratios of mean inelastic displacement ratios of each soil group to mean values of all sites classes considered here (A, B, C and D) were computed. Mean inelastic displacement ratios of sites classes A and B normalised by mean inelastic displacement ratios of all site classes are shown on the left-hand side of figure 4. This ratio is a measure of the error that would be produced if the effect of soil conditions on inelastic displacement ratios is neglected for these site classes. It can be seen that with the exception of periods around 1.0 s and ductilities higher than 4, the error is less than 10%. Similar results but for site class C are shown on the right hand side of figure 4. It can be observed that the errors are relatively small and always smaller than 10%. Ratios shown in figure 4 are typically smaller than 1.0 which means that mean inelastic displacement ratios for site classes A and B or for site class C are, in general, smaller than mean inelastic displacement ratio for all records. Thus for site classes A, B and C the use of figure 3, that neglects the influence of soil conditions,

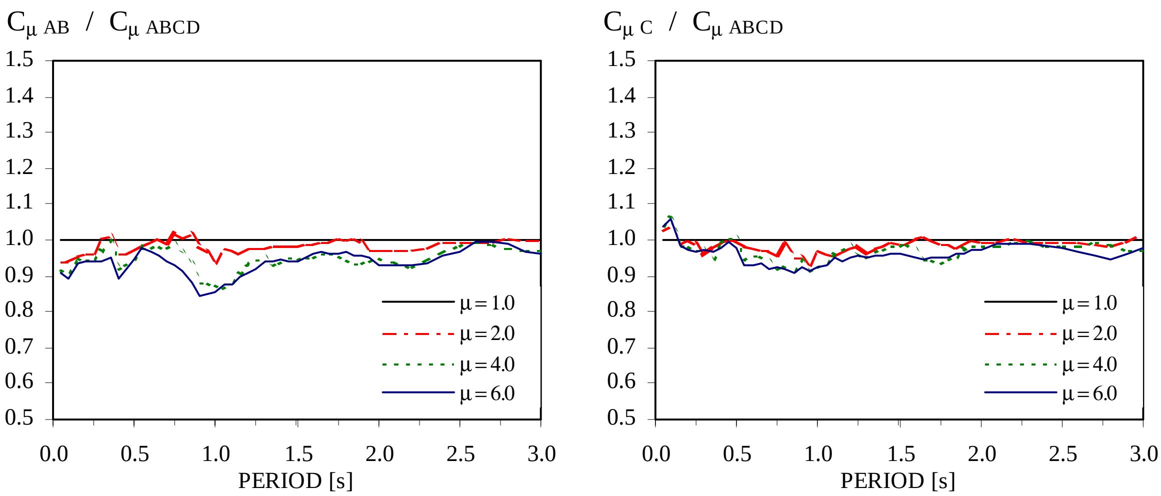

Figure 4. Mean inelastic displacement ratios for sites classes A and B (on the left) and for site class C (on the right) normalized by mean inelastic displacement ratios of site classes A, B, C and D.

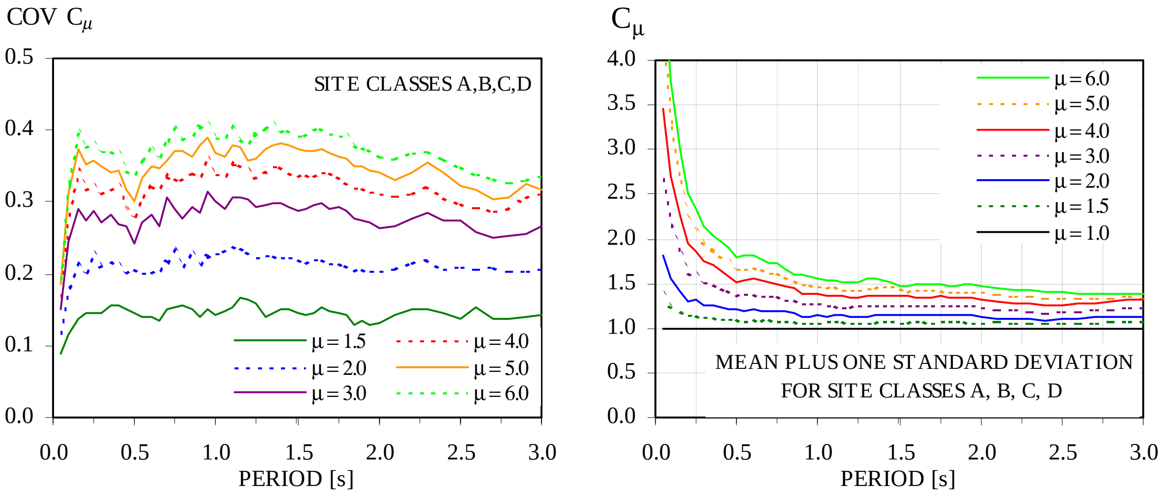

Figure 5. Coefficients of variation (on the left) and mean plus one standard deviation (on the right) of inelastic displacement ratios corresponding to site classes A, B, C and D.

would result on a relatively small overestimation of the inelastic displacement ratios. Although not shown here mean ratios for site class D show that with the exception of periods around 1.0 s and for ductilities higher than 4 neglecting the influence on soil conditions would only result on small underestimations (smaller than 10%) of mean inelastic displacement ratios.

While mean results are very important, equally important is to know the dispersion that exist in the results. Figure 5 shows coefficients of variation of inelastic displacement ratios corresponding to all site classes considered here. It can be seen that dispersion increases as the level of inelastic deformation increases. Furthermore, with the exception of very short periods (smaller than 0.2 s), for a given level of ductilty demand the coeficients of variation are approximately independent of the period of vibration. Thus, since mean inelastic displacement ratios are approximately constant and equal to one for periods of vibration larger than 1.0 s, then inelastic displacement ratios associated to mean plus one standard deviation will also be approximately period independent in this spectral region. This can be confirmed from observation of the right-hand side of figure 5 where inelastic displacement ratios corresponding to mean plus one standard deviation are show. It can be seen that for small levels of ductility maximum inelastic displacements associated to mean plus one standard deviation and for periods larger than 1.0 s are less than 20% larger than maximum elastic displacements.

Miranda [8] has shown that with the exception of near-fault records inelastic displacement ratios are not affected by the magnitude of the earthquake event, by the level of ground acceleration experienced at the site nor by the distance to the epicentre, thus making them particularly valuable in earthquake resistant design. For a detailed discussion of inelastic displacement ratios in near-field motions the reader is referred to Baez and Miranda [2].

NONLINEAR REGRESSION ANALYSES

For displacement-based design it is desirable to have a simplified expression to estimate mean inelastic displacment ratios which would then allow the estimation of maximum inelastic displacement demands from from maximum elastic displacement demands. Using the Levenberg-Marquardt method nonlinear regression analyses were conducted to derive a simplified expression for estimating mean inelastic displacement ratios. This method combines the steepest-descent method and a Taylor series based method to obtain a fast and reliable technique for nonlinear optimization. The resulting equation is given by

$$C_\mu = \left[1+\left(\frac{1}{\mu}-1\right)exp\left(-12T\mu^{-0.8}\right)\right]^{-1} \quad (2)$$

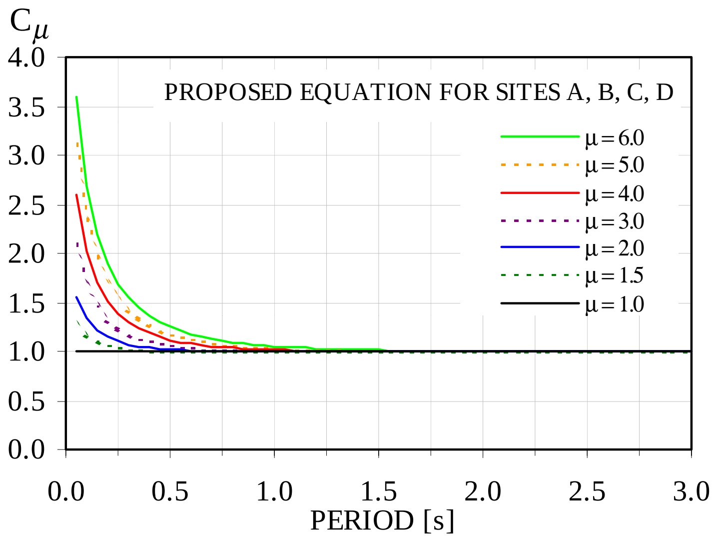

Figure 6. Mean inelastic displacement ratios computed with equation 2.

where μ is the displacement ductility ratio and T is the period of vibration. Equation (2) represents a surface and provides estimates of mean inelastic displacement ratios as a function of μ and T. Figure 5 shows inelastic displacement ratios computed with equation 2. Additionally, this equation yields correct limits to C_μ, namely $C_\mu = \mu$ as $T \rightarrow 0$ and $C_\mu = 1$ as $T \rightarrow \infty$. Figure 6 shows a comparison of mean inelastic displacement ratio in the short period spectral region from the statistical study to those computed using equation 2 where it can be seen that, despite its simplicity, this equation provides very good results and is able to capture both the effect of μ and T on inelastic displacement ratios.

It is important to notice that equation 2 correctly addresses the fact that the period limiting the spectral region where the equal displacement rule is applicable changes with the level of inelastic deformation. Furthermore, the proposed equation does not depend on any characteristic site period and thus it can be used for all sites with average shear wave velocities larger than 180 m/s without the need of estimating such parameter which for these types of soil conditions is not uniquely defined and in particular it may vary for a given site from one earthquake to another.

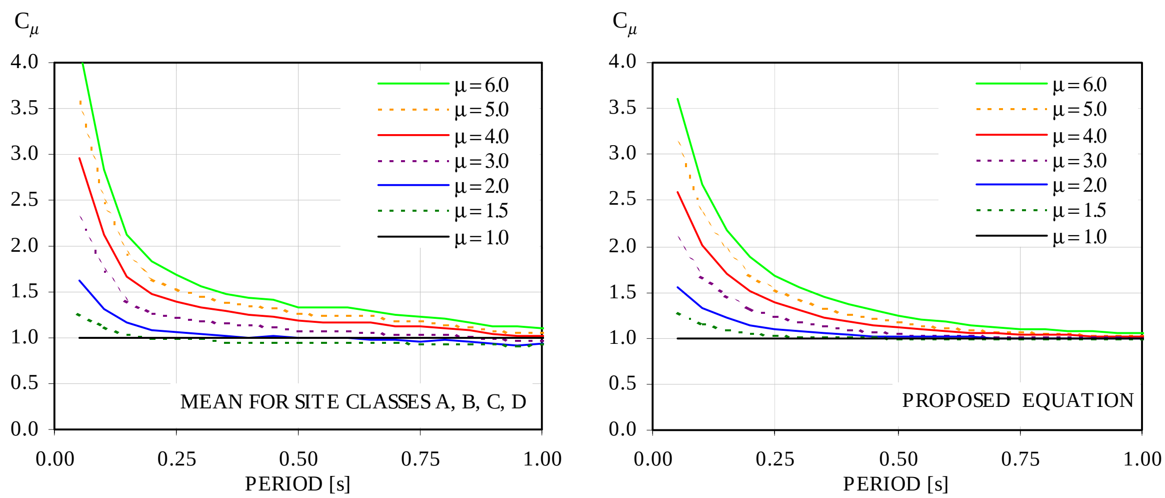

Figure 7. Comparison of mean inelastic displacement ratios in the short period spectral region from the statistical study (shown on the left) and those computed with equation 2 (shown on the right).

CONCLUSIONS

Inelastic displacement ratios are attractive tools in displacement-based design that permit the estimation of maximum inelastic displacements from maximum elastic displacements demands. These ratios are characterised by being larger than one and both period- and ductility-dependent in the short period spectral region and period independent for periods larger than about 1.0 s. In this latter spectral region maximum inelastic displacements are on average equal to maximum elastic displacements. Periods limiting the spectral region where the equal displacement rule is applicable depend on the level of inelastic deformation. This limiting period increases with increasing displacement ductility ratios.

For sites with average shear wave velocities higher than 180 m/s, the effect of changes in site conditions is, in general, relatively small. For most rock or stiff soil sites neglecting specific details of the site conditions will introduce errors smaller than 10% on mean inelastic displacement ratios. Thus, in these cases the effects of site conditions on C_μ may be neglected for practical situations.

Dispersion of inelastic displacement ratios increases as the level of inelastic deformation increases. However, the increase in dispersion is not linearly proportional to increases in the level of inelastic deformation. Furthermore, with the exception of periods smaller than 0.2s coefficients of variation of inelastic displacement ratios are not significantly affected by changes in the period of vibration.

A simplified expression derived from nonlinear regression analyses has been proposed to estimate mean inelastic displacement ratios of sites with average shear wave velocities higher than 180 m/s. This expression is relatively simple and does not require the estimation of a site characteristic period.

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