

THE BEHAVIOR OF REINFORCED CONCRETE PIERS UNDER STRONG SEISMIC ACTIONS

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SUMMARY

A fibre column finite element based on Timoshenko beam theory has been developed to model the cyclical response of the end critical zones of bridge piers having low-to-intermediate shear slenderness. Shear resistance is obtained by modelling the principal resisting mechanisms; these are linked to the flexural behaviour by means of suitable kinematics assumptions. In flexure the element differs from standard fibre beam element since, to account for the contribution to shear resistance due to arch action and for the inclined trust-line which develops in squat elements subjected to shear, the principal direction of the compressive stress, which is normal to the cross-section in standard fibre elements, is here rotated. Non linear behaviour of material is taken into account by means of appropriate constitutive relations. The proposed element, implemented in a well established non linear computer code, has been tested by comparison with some experimental results.

INTRODUCTION

When subjected to strong cyclic horizontal loading, reinforced concrete (R.C.) bridge piers can suffer from many different possible failures. Among these, as it has been recently recognised, shear failure deserves particular care, not only for its brittle character, but also because shear resistance is due to the interaction of several different mechanisms: beam action (grouping aggregate interlocking, dowel action of the main longitudinal reinforcement and uncracked concrete resistance), arch action and truss action. Tracing back to the work of Leonhardt [Leonhardt, 1965], it appears that, for R.C. elements having an intermediate ratio (2 to 6) between the unsupported length and the cross-section depth, the failure can be of mixed type, i.e. involving both flexural and shear mechanisms; in this respect the interaction between the two mechanisms is made more evident by the contribution of the arch effect, this causing strong coupling of flexure and shear within the element end zones. Moreover, for bridge piers, as recognised in [Priestley and Verma and Xiao, 1994], even for moderately high values of shear slenderness, failure might still be in shear but due to the previous flexural ductility experienced by the element. Even though shear effects actually spread throughout the element, regions where shear-flexure interaction is more pronounced can still be limited to the end zones. This suggests to model bridge piers as an assemblage of three sub-elements: two non-linear end zones and a linear intermediate zone (Figure 1). Adopting this separation, the intermediate portion of the pier may still be modelled by means of an usual elastic beam finite element. The non-linear end zones are devoted to capture shear-flexure coupling both for strength and stiffness.

Following these considerations, the proposed finite element model for the pier end zones has been formulated to capture interaction between shear resistance and inelastic flexural behaviour in reinforced concrete bridge piers, subjected to seismic excitation. In the following, the element kinematics, the modelling of the shear resisting mechanisms together with material models and some comparison with experimental results will be presented.

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FINITE ELEMENT FORMULATION

In usual finite element formulation some volume integrals are necessary either to compute the stiffness matrix or the nodal reaction forces; usually these integrals are numerically computed following the material behaviour at selected points in the element spatial domain. For beam elements formulated according to the fibre element approach, a suitable way to split these volume integrals is the subdivision in some area integrals, defined over the beam cross section, and subsequent line integrals along the element length. The cross section is subdivided in small portions (fibres) to compute the cross section integrals and to model its geometry; in each of these the non-linear material behaviour is followed. Integrals over the cross section can be viewed as generalised quantities relative to the cross section; in this way line integrals of moments, shear and axial action have to be computed in calculating the nodal restoring forces.

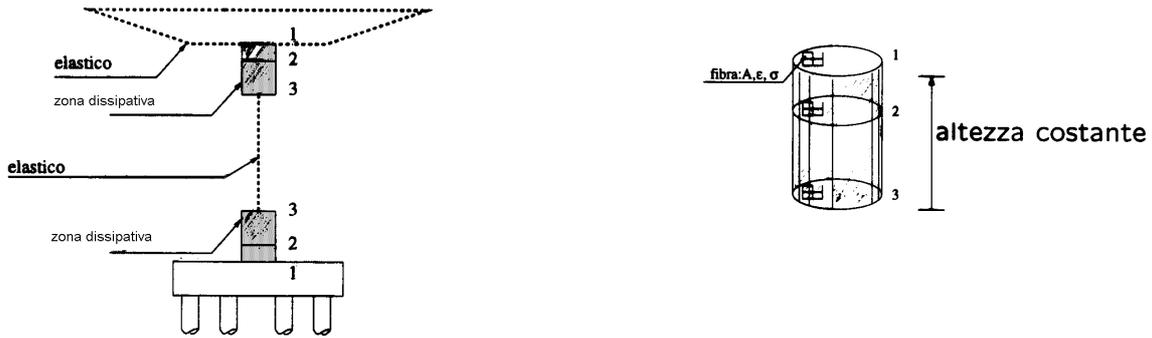


Figure 1: Discretisation of bridge pier

In this respect the proposed finite element follows the usual fibre finite element approach for beams. The shear resultant over the cross section, however, is derived by different resisting mechanisms: truss mechanism, which considers also in a simplified way aggregate interlock, and arch action; the latter accounts for the contribution to shear resistance in squat structural elements. The inclined trust line, which simulates arch action, is modelled by a rotation for the principal direction of the compressive stresses, which is usually taken as normal to the cross-section in standard fibre elements.

Kinematics assumptions

The end zones are modelled by means of stiffness based finite element theory and by assuming small displacements. Adopting the Bernoulli hypothesis of plane sections, displacements inside the finite element may be described using the following generalised displacement vector $\mathbf{u}(x)$:

$$\mathbf{u}(x) = \left\{ u_a(x) \quad v(x) \quad w(x) \quad \varphi_z(x) \quad \varphi_y(x) \right\}^T \quad (1)$$

where $u_a(x)$ is the displacement along the element axis x of a reference point in the element cross section, and transversal displacements v and w and rotations φ_y and φ_z are defined according to Figure 2. Engineering strain components are obtained from $\mathbf{u}(x)$ according to Timoshenko beam model. In a generic point P inside element volume, identified by a position vector $\mathbf{x}=(x,y,z)$, the only non zero components of the strain tensor $\boldsymbol{\varepsilon}$ are the normal strain ε_x , and the shear strains γ_{xy} and γ_{xz} . The following expressions relating γ_{xy} and γ_{xz} to transversal displacements u and w and rotations φ_y and φ_z are obtained as follows, where the prime denotes derivative with respect to x :

$$\begin{aligned} \gamma_{xy}(x) &= -\varphi_z(x) + v'(x) \\ \gamma_{xz}(x) &= \varphi_y(x) + w'(x) \end{aligned} \quad (2)$$

Strain fields inside the element volume may be described by the generalised strain components grouped in the strain vector $\boldsymbol{\varepsilon}_g = \{ \varepsilon_a, \chi_z, \chi_y, \gamma_{xy}, \gamma_{xz} \}^T$. As usual, $\mathbf{u}(x)$ is computed from nodal displacements $\mathbf{u}_{an}, \mathbf{v}_n, \mathbf{w}_n$ and rotations $\boldsymbol{\varphi}_{zn}, \boldsymbol{\varphi}_{yn}$ via use of shape functions; $\mathbf{u}_{an}, \mathbf{v}_n, \mathbf{w}_n, \boldsymbol{\varphi}_{zn}, \boldsymbol{\varphi}_{yn}$ are the vectors grouping nodal displacements and rotations. Since curvature variation along the element should be at least linear, a three node element (Figure 2) has been adopted (i.e. $\boldsymbol{\varphi}_{zn} = \{ \varphi_1, \varphi_2, \varphi_3 \}^T$).

In (2) the derivative order for rotations and displacements fields differs; to avoid locking phenomena, which might arise if rotation and transverse displacement are modelled using polynomials of equal degree, the ‘‘shear constraints’’ method [Crisfield, 1986] has been used. Locking becomes possible when pure strain states (bending

or shearing) are coupled. To avoid locking, in a Timoshenko beam, pure bending strain state associate with shear strain $\gamma=0$ has to be possible. This becomes possible if polynomia interpolating transversal displacements v or w is of one order greater than that interpolating rotations φ_y or φ_z .

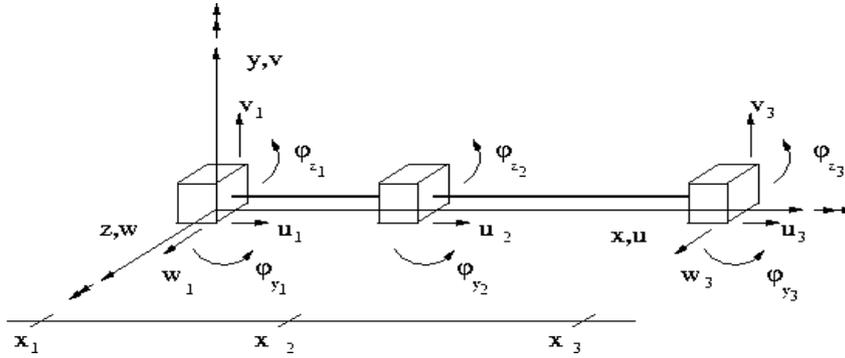


Figure 2: Element nodal displacements

To achieve a linearly varying curvature, polynomia of order 3 for transversal displacements and 2 for rotations are necessary, leaving 7 constants to be determined as functions of nodal displacements. Mapping x axis domain in $r \in [-1, 1]$ using an isoparametric formulation, shear strain γ can be written as: $\gamma = A_1 + A_2 r + A_3 r^2$. Assuming the γ strain to be constant, which is the case for an elastic beam having a linearly varying moment, two restraints equations are gained, which reduce independent nodal parameters to 5 for each one of the (2). In [Crisfield, 1986] it is shown how it is possible to choose such free nodal parameters as the nodal rotations φ_{zn} , φ_{yn} and transverse displacements of the end nodes only (w_1, w_3, v_1, v_3).

Stiffness Matrix computation and nodal Restoring forces computation

Stiffness matrix \mathbf{K} and nodal restoring forces \mathbf{R} are computed according to the usual procedure for stiffness based isoparametric finite elements:

$$\mathbf{K} = \int_V B_x^T D B_x dV = \int_{\hat{V}} B_r^T D B_r |J| d\hat{V} \quad (3)$$

$$\mathbf{R} = \int_{-1}^1 B_r^T R_s |J| dr \quad (4)$$

where $|J|$ is the Jacobian of the transformation $x(r)$. Volume integrals are decomposed in area integrals defined over the element cross-section and subsequent line integral along the element. Except for shear related quantities, area integrals are approximated by summation over cross section fibres; for line integrals a 5 point Gauss-Lobatto scheme has been used.

MODELLING OF SHEAR RESISTING MECHANISMS

As it is well known, shear resistance is due to many complex interacting mechanisms, see for example [Park and Paulay, 1975]. As a simplification, shear resisting force at the cross section level V_r is here modelled as a sum of different terms, thus considering these mechanisms acting in parallel: $V_r = V_t + V_c$. V_t is the Moersch's truss contribute and V_c the concrete one. Shear resistance V_t due to truss effect is accounted for by modelling a single truss portion, composed of the compression concrete diagonal, the transverse reinforcement and the tension diagonal; it is further assumed that, under shear, an equally spaced set of diagonal cracks of constant inclination develops, thus simplifying the truss model. Truss deformation is obtained from shear deformation computed via Timoshenko beam kinematics. The concrete contribution V_c to shear resistance groups many other mechanisms, as aggregate interlock (V_{IN}), compression concrete above the neutral axis (V_{cc}), including arch action, and dowel action (V_{dw}).

Inclined strut mechanism

As pointed out in [Priestley and Verma and Xiao, 1994] for squat bridge piers, and as already well known for squat shear walls, part of shear forces are transferred from one end of the structural element to the other by mean of an inclined strut, Figure 3, forming an angle α with the element longitudinal axis.

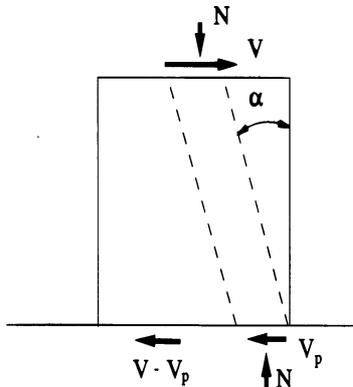


Figure 3: Inclined strut

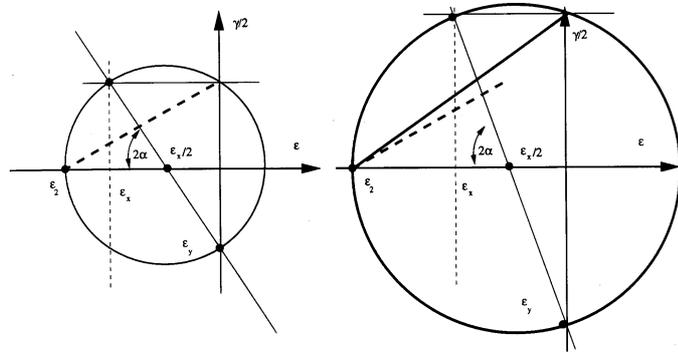


Figure 4: Strain corresponding to different α angle given ϵ_x

Experimental results [Garstka and Kraetzig and Stangenberg, 1993] point out that at the increase of angle α corresponds an increasing importance of this resisting mechanism in relation to the others present; however, at the same time, a more than proportional decrease in the available resistance and ductility is found. For very flat struts, collapse is due to crushing of the compression concrete. In [Priestley and Verma and Xiao, 1994], the portion of acting shear transferred with this mechanism is limited by the axial action value: $V_p = N \tan \alpha$. In the present model, it is assumed, in a simplified way, that such strut connects the compression centres at each end of the element.

Differently from usual fibre models where fibres direction is always normal to cross section, to simulate this mechanism, a rotation is applied, in the presented model, to the fibres, so that these become aligned with the strut. Two more assumptions are made: consistently with the well known "Compression Field Theory" (CFT) [Mitchell and Collins, 1974], principal stresses directions coincide with principal strains directions and the strut gives the direction of the principal compressive stress σ_2 . Given the element deformation fields, it is possible to compute the deformation ϵ_x at each fibre; using Mohr's circle and assuming that $\epsilon_y = 0$ it is possible to compute the principal compression strain ϵ_2 and the shear strain $\gamma/2$. Material model used in association with the strut mechanism are of uniaxial type and principal compressive strain ϵ_2 will be only used to compute σ_2 . It is worth noting that in this way the strain used to compute fibre stress is still depending on the truss inclination but not on shear strain γ ; consequently the model is applicable also to non Timoshenko beam kinematics. Furthermore, a similar result could be attained considering also shear deformation γ in computing ϵ_2 ; in this last case the analogy with CFT would have been complete. Once σ_2 is computed, it is possible to obtain σ_x and τ_{xy} using once more standard Mohr's circle relations under the hypothesis $\sigma_1 = 0$. Integration of σ_x and τ_{xy} over the cross section area A_c yield the generalised internal actions at the cross section: $N = \int_{A_c} \sigma_x dA$, $M_y = \int_{A_c} \sigma_x z dA$, $M_z = \int_{A_c} \sigma_{xy} dA$, $V_{pxz} = \int_{A_{cc}} \tau_{xz} dA$, $V_{pxy} = \int_{A_{cc}} \tau_{xy} dA$. It's worth noting that here A_{cc} is the area of compressed concrete in the cross section.

This process requires an a priori knowledge of the angle α , to compute it a further assumption has been adopted. Since axial action value is not usually high in squat bridge piers, it is possible to assume that the inclined strut connects the centre of compression of the more loaded end section with the centroid of the cross section having null moment. The value of α is computed according to the following steps:

- 1) Nodal reaction forces are computed, in particular moments at the initial and final node (M_{zi} M_{yi} M_{zf} M_{yf}).
- 2) Centroid of compressive stresses is computed at the element end cross sections.
- 3) For each position vector component (Y or Z) in the cross section the problem is reduced to a plane one; as described in Figure 4 for computation in x-y plane.
- 4) At the element end having the smaller moment absolute value, position of compressive stresses centroid is computed from a triangle similitude; i.e. for the Y component: $Y_{cpi} : Y_{cpi} = L_i : L_f$ where Y_{cpi} is Y component of compressive stresses centroid at the element initial section (or section having M_z maximum absolute value).

Given the position of the compression centres at the element end, α is finally computed. Figure 5 shows how, for a ε_x fixed value, the compressive strain ε_2 becomes greater as α increases; concrete degradation, depending on plastic compressive strain, also increases with α . It has been noted that, in a step-by-step dynamic analysis, α it is not sensitive to the stress level attained in the cross section fibres, especially when the cross section is heavily loaded in flexure with a small portion reacting in compression. Accordingly, in this case, α can be taken, in a simplified way, as the value at the end of previous step.

Truss mechanism

The resisting mechanisms should be coupled in order that equilibrium and deformations compatibility is preserved at a local level (i.e. considering just a portion of material around a shear crack) as well that at element level (i.e. considering a large chunk of the structural element). As it is well known, and as it has been modelled in some of the more refined micro models [Kupfer and Bulicek, 1992] [Di Prisco and Gambarova, 1995], truss mechanism and aggregate interlock are coupled to each other; this interaction influences relative displacements at the crack faces and, due to shear and normal stresses at the crack face, the principal stresses direction which deviates slightly from the crack direction. Separating in two parts the structural element along a shear crack, it is possible to consider the transmitted shear as the sum of two contribution; V_t , transmitted by the truss action and V_c , transmitted by other resisting actions. These are aggregate interlock V_{IN} , compression concrete above neutral axis V_{cc} , dowel action V_{dw} . Of these, only V_{IN} and V_{cc} terms have been modelled in this work. Moreover the aggregate interlock, compression concrete and truss contributions are considered in an independent way.

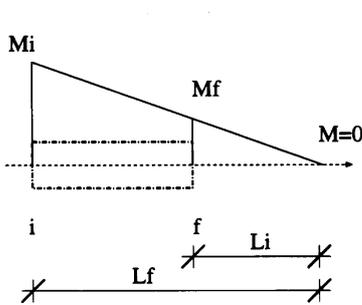


Figure 5: Computation of strut angle α

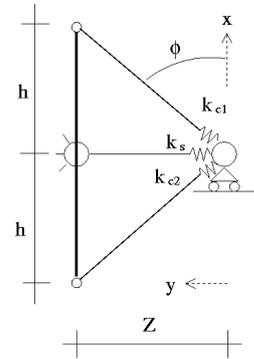


Figure 6: Assemblage used to model truss mechanism

Truss mechanism model is based on the planar structural assemblage of Figure 6; diagonals inclination ϕ , a parameter of the model, is assumed to be equal to crack inclination. The shear transferred by truss action is computed by integrating shear stress due to the truss mechanism over the tensioned concrete area in the cross section. Mohr's circle allows to determine the shear stress τ acting in the cross section given the inclination of both tension and compression diagonals and given the corresponding stresses σ_{d1} and σ_{d2} :

$$\tau_{xy} = \sigma_{d1} \alpha_{x,d1} \alpha_{y,d1} + \sigma_{d2} \alpha_{x,d2} \alpha_{y,d2} \quad (5)$$

here, and in the following, $\alpha_{i,j}$ means the cosine of angle between axes i and j . The stresses σ_{d1} and σ_{d2} are computed from ε_{d1} and ε_{d2} using concrete constitutive relationships. These are in turn computed, using Mohr's circle, as functions of shear strain γ and strain ε_y in transversal reinforcing steel:

$$\varepsilon_{d1} = \varepsilon_y \alpha_{y,d1} \alpha_{y,d1} + \gamma_{xy} \alpha_{y,d1} \alpha_{y,d1} \quad (6)$$

$$\varepsilon_{d2} = \varepsilon_y \alpha_{y,d2} \alpha_{y,d2} + \gamma_{xy} \alpha_{y,d2} \alpha_{y,d2}$$

The transversal strain ε_y is obtained imposing equilibrium in direction Y to the truss model, as in usual Moersch's truss approach:

$$\sigma_y = \sigma_s + \sigma_{d1} \alpha_{y,d1} \alpha_{y,d1} + \sigma_{d2} \alpha_{y,d2} \alpha_{y,d2} \quad (7)$$

The shear stiffness contribution due to the truss is derived from the model of figure 6, as well, once the stiffness of the transversal steel and of the concrete fibres are known. It is worth pointing out that longitudinal steel has no effect on this model. Even though truss analogies, like the one adopted, are simple in the usage also in the case of cyclic deformations, they are restricted to the case of shear acting in a fixed plane.

Interlocking mechanism

Effect of aggregate interlock on principal stresses directions rotation is well known; in this simplified model, however, this effect is disregarded. Thus, the relative angle ϕ of concrete fibres in truss mechanism remains fixed throughout the analysis. It is assumed that concrete in tension is characterised by a field of cracks having constant spacing s . Crack shape is linear and crack angle with element longitudinal axis ϕ is constant and assumed equal to that of the concrete fibres. For demonstration purposes, in the following cracks laying in the element x - y plane are considered.

Due to relative displacements at crack faces, normal stress σ_n and tangential stress τ_{nt} arise. Treating the cracked concrete as a continuum medium, the tangential component τ_{xyIN} on the cross section can be computed:

$$\tau_{xyIN} = \sigma_n \alpha_{n,x} \alpha_{n,y} + \tau_m \alpha_{t,x} \alpha_{n,y} + \tau_{nt} \alpha_{n,x} \alpha_{t,y} \quad (8)$$

Assuming τ_{xyIN} as an average value over the cross section concrete area A_t , the mechanism contribution to the generalised stress component then is: $V_{IN} = \tau_{xyIN} A_t$.

Stresses σ_n and τ_{nt} are computed using the interface constitutive law presented in [Li and Maekawa, 1987]. This is formulated in terms of crack faces relative displacements v (parallel to the crack) and w (normal to the crack); these are taken equal to the average values over the area A_t computed making use of the smeared crack concept. Crack opening w and relative slip v are computed from average strain values ε_{nAV} and γ_{ntAV} in crack frame of reference: $w = \varepsilon_{nAV} s$, $v = \gamma_{ntAV} s$; these are the weighted average on area A_t :

$$\varepsilon_{nAV} = \frac{\sum_i \varepsilon_{ni} A_{ci}}{A_t} \quad \gamma_{ntAV} = \frac{\sum_i \gamma_{nti} A_{ci}}{A_t} \quad (9)$$

In these equations ε_{ni} and γ_{nti} are the strain components, in the crack reference system, for fibre i and A_{ci} is the fibre area. It is supposed that strain components depend on section shear strain γ_{xy} and transversal steel strain ε_s :

$$\begin{aligned} \varepsilon_{ni} &= \varepsilon_{x_i} \alpha_{n,x} \alpha_{n,x} + \varepsilon_y \alpha_{n,y} \alpha_{n,y} + \gamma_{xy} \alpha_{n,x} \alpha_{n,y} \\ \gamma_{nti} &= \varepsilon_{x_i} \alpha_{t,x} \alpha_{n,x} + \varepsilon_y \alpha_{t,y} \alpha_{n,y} + \frac{\gamma_{xy}}{2} (\alpha_{t,y} \alpha_{n,x} + \alpha_{t,x} \alpha_{n,y}) \end{aligned} \quad (10)$$

Implicitly (10) disregards strain in concrete between cracks. Considering the usually small shear steel ratios (0.0015-0.0030) the largest part of the strain will be due to yielding of transversal steel; from this point of view (10) appears acceptable at least for the simplified model at hand. In (10) ε_{xi} is the fibre strain adopted to compute the cross section flexural behaviour. The aggregate interlock mechanism is coupled with flexure using this kinematics assumption. One of the model parameters is the crack spacing s ; in the numerical tests ran it has been estimated from specimens photographs but some theoretical or semi empirical formula are available, see for example [Dei Poli and Gambarova and Karakoc, 1987] [Dei Poli and Di Prisco and Gambarova, 1990] [Kupfer and Bulicek, 1992].

Materials models

Steel and concrete material constitutive laws are required both for flexural behaviour at cross section level and for shear resisting mechanisms. Since these are all basically composed of fibres, essentially uni-axial material models are required. Concrete and steel in both cross-section fibres and truss mechanism follow the same constitutive relations.

Many uniaxial material model for steel are available today; among them the one proposed in [Monti and Nuti, 1992] has been adopted in this work due to its explicit formulation and because it appears capable to account for the post-elastic buckling behaviour of the reinforcing bar.

Of course many uniaxial constitutive models for concrete are available as well; however many of them lack what were felt as key factors in successfully simulating cyclic concrete behaviour, especially in the truss mechanism: capability of simply reproducing the complex phenomena of crack faces misalignment (crack bridging) and concrete fibre compressive strain softening due to coexisting principal tensile strain. For these reasons the model proposed in [Stevens and Uzumeri and Collins, 1987], restricted to fixed principal strain directions, has been assumed to describe the concrete behaviour; this model is able to account for the early reloading in compression following a large tensile strain.

Interface behaviour is simulated by means of the Contact Density Model [Li and Maekawa, 1987]. According to this model, crack faces are decomposed in a set of contact surfaces of unit area; the probability density function of the relative angle between the contact surface and the crack global direction is given. Given crack relative displacements w and v and face-to-crack angle, the normal displacement component is computed on each contact face. At this location an elastic perfectly plastic material behaviour, in terms of a stress-displacement law, is assumed normal to the face. Interface forces are computed integrating contact faces normal stress, considering the probability density function for faces position. Faces behaviour is treated in a way similar to finite element discretisation of a displacement field to reduce computation time; the local elastic perfectly plastic material behaviour is followed on a small number of faces and a linear interpolation is assumed for the plastic displacement and the yielding displacement.

NUMERICAL TESTS

The column model has been implemented in an established computer code [Martinelli and Mulas and Perotti, 1996] and it has been able to correctly follow experimental results for squat and almost slender elements whose behaviour was severely controlled by shear: in particular it has been able for monotone loading to account for the brittle concrete collapse induced by excessive strain which develops in squat members due to the inclined truss-line transferring shear [Garstka, 1993] and, for cyclic loading, to correctly reproduce pinching and stiffness degradation in more slender elements principally governed by the truss response: element R5 in [Ma and Bertero and Popov, 1976].

As an example of the accuracy obtainable using the proposed truss model in Figure 7 is reported the response for specimen SE8 as studied in [Stevens and Uzumeri and Collins, 1987]; this is a 1524x1524x285 mm reinforced concrete panel; two layers of reinforcing bars are present, each one formed by bars at 90 degrees angle. Geometric reinforcement content is, respectively, 1% and 3% in the two bars directions. Specimen has been tested in pure shear.

In the modelling, reported material properties have been used; truss transversal direction has been assumed coincident with specimen direction having 1% reinforcement ratio and truss model diagonals angle has been assumed equal to 45 degrees.

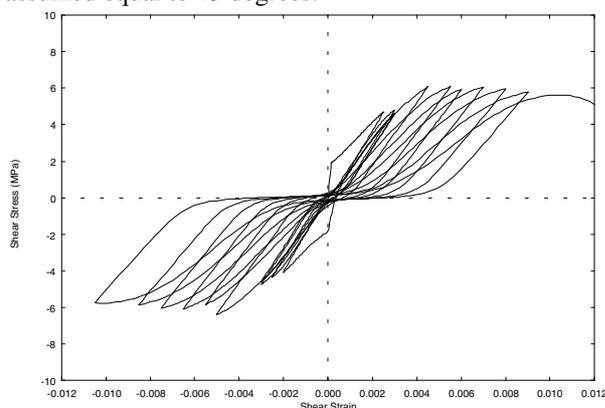


Figure 7: Numerical result for specimen SE8

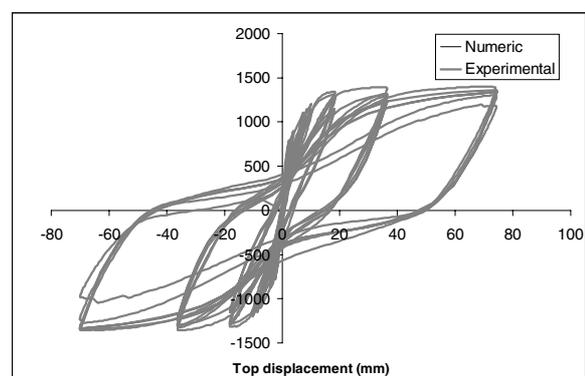


Figure 8: Numerical result for squat pier

Panel SE8 has been chosen for its different reinforcement ratios as is the case in beam or column structural elements; transversal direction reinforcement ratio is sufficient to induce a large inelastic strain in concrete diagonals thus helping to show the limits for assumptions made in the truss model. Numerical results are in good agreement with experimental one showing the satisfactory performance of the proposed model.

The model has then been used to simulate a full scale element, a short bridge pier with aspect ratio = 1.75 subjected to imposed displacement history at its top [Pinto and Verzeletti and Negro and Guedes, 1996]. The specimen had a 1.6x0.80 m boxed cross section with thickness=0.16 m with flexural steel ratio equal to 0.919% and 5 mm stirrups at a 60 mm spacing (Type 3 section). The displacement history was based on some increasing amplitude cycles up to yielding of flexural steel, followed by three cycles for each displacement ductility 1.5, 3.0 and 6.0. During the test an axial load corresponding to a normalised axial load $\nu=0.1$ was imposed on the specimen. The test was ended at ductility =6 after rupture of flexural bars which had previously buckled. The specimen was chosen since more than 30% of top displacement was due to shear and because of axial load contributing to the strut mechanisms. The specimen was modelled, with equally satisfactory results, using both one or two of the proposed elements; in Figure 8 numerical and experimental results are compared for specimen

modelled with one element. Apparently the model lack the capacity to capture specimen collapse, this was partly due to having not activated post buckling bars behaviour in the Monti and Nuti bar model.

CONCLUSIONS

The proposed model has been able to reproduce the behaviour of structural members strongly influenced by shear; simplifications of adopted material models appear possible. In this respect a simpler uniaxial constitutive law for concrete able to model crack bridging is highly desirable. Among the possible applications, the model will be used to study the dynamic behaviour of complete 3-D bridges having non slender piers.

REFERENCES

- Crisfield, M. A. (1986), *Finite Elements and Solution Procedures for Structural Analysis- Vol.1: Linear Analysis*, Pineridge Press, Swansea.
- Dei Poli, S. and Gambarova, P.G. and Karakoç, C. (1987), «Aggregate Interlock Role in RC Thin Webbed Beams in Shear», *ASCE Journal of Structural Engineering*, 113, 1, pp 1-19.
- Dei Poli, S. and Di Prisco, M. and Gambarova, P.G. (1990), «Stress Field in Web of RC Thin-Webbed Beams Failing in Shear», *ASCE Journal of Structural Engineering*, 116, 9, pp 2496-2515.
- Di Prisco, M. and Gambarova, P.G. (1995), «Comprehensive Model for Study of Shear in Thin-Webbed RC and PC beams», *ASCE Journal of Structural Engineering*, 121, 12, pp 1822-1831.
- Garstka, B. and Krätzig, W.B. and Stangenberg, F. (1993), «Damage Assessment in Cyclically Loaded Reinforce Concrete Members», *Proceedings Struct. Dyn. EURO DYN'93*, Balkema, Rotterdam, pp 121-128.
- Garstka, B. (1993), «Investigation on Resistance and Damage Behaviour of Reinforced Concrete Linear Elements Considering Shear Effects under Cyclic Nonlinear Loading», *Tech. Rep. TWM 93-2*, Institut für Konstruktiven Ingenieurbau, Rhur University, Bochum.
- Kupfer, H. and Bulicek, H. (1992), «A Consistent Model for the Design of Shear Reinforcement in Slender Beams with I of Box Shaped Cross Section», *Proceedings Concrete Shear in Earthquake*, Elsevier Science Publisher, Barking, pp 256-265.
- Leonhardt, F. (1965), «Reducing Shear Reinforcement in Reinforced Concrete Beam and Slabs», *Magazine of Concrete Research*, 17, 53, pp 187-198.
- Li, B. and Maekawa, K. (1987), «Contact Density Model for Cracks in Concrete», *Transactions of IABSE Colloquium Deft 87 on Computational mechanics of Concrete Structures*, pp 51-62.
- Martinelli, L. and Mulas, M.G. and Perotti, F. (1996), «The Seismic Response of Concentrically Braced Moment-Resisting Steel Frames», *Earthquake Engineering and Structural Dynamics*, 25, pp 1275-1299.
- Mitchell, D. and Collins, M.P. (1974), «Diagonal compression field theory –a rational model for structural concrete in pure torsion», *ACI Journal*, 71, pp 396-408.
- Monti, G. and Nuti, C. (1992), «Nonlinear Cyclic Behavior of Reinforcing Bars Including Buckling», *ASCE Journal of Structural Engineering*, 118, 12, pp 3268-3284.
- Park, R. and Paulay, T. (1975), *Reinforced Concrete Structures*, John Wiley & Sons, New York.
- Priestley, M.J.N. and Verma, R. and Xiao, Y. (1994) «Seismic Shear Strength of Reinforced Concrete Columns», *ASCE Journal of Structural Engineering*, 120, 8, pp 2310-2329.
- Stevens, N.J. and Uzumeri, S.M. and Collins, M.P., «Analytical modelling of Reinforced Concrete Subjected to Monotonic and Reversed Loadings», *Tech. Rep., Department of Civil Engineering, University of Toronto*.
- Ma, S.M. and Bertero, V.V. and Popov, E.P. (1976), «Experimental and Analytical Studies on Hysteretic Behaviour of Reinforced Concrete in Rectangular and T-Beams» *Tech. Rep. 02/76 Earthquake Engineering Research Center*, University of California, Berkeley.
- Pinto, A.V. and Verzeletti, G. and Negro, P. and Guedes, J. (1995), «Cycling Testing of a Squat Bridge Pier» *Report EUR 16247 EN, EC, JRC, Ispra*.