SUPER-FEM FOR ANALYSIS OF INELASTIC RESPONSE OF TALL STEEL BUILDINGS SUBJECTED TO EARTHQUAKES

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SUMMARY

To provide a practical method for structural engineers to conduct inelastic analysis for seismic-resistant design of tall steel frames, a Super-FEM model is proposed in this paper. The advantage of this model is that the structural degrees of freedom can be reduced. Comparisons are made through a number of examples between Super-FEM and normal FEM. It is shown that satisfactory results with appropriate accuracy can be achieved by Super-FEM.

INTRODUCTION

Braced and unbraced steel frames are frequently used for tall buildings. To prevent this kind of buildings from collapse in earthquake events, the analysis of structural inelastic response to severe seismic ground motions is required and the checking of the maximum storey drift is demanded in the Specification of China for Design of Tall Steel Buildings. However, implementation of inelastic analysis for seismic response of tall steel buildings by using normal FEM is very time-consuming, which hinders the structural engineers to follow the specification strictly when conducting seismic-resistant design of tall steel buildings and leads the omission of inelastic seismic analysis in practice.

In order to make the analysis of inelastic response of tall steel buildings subjected to earthquakes practical, a Super-FEM is proposed. In this method, the normal FEM model is used for individual girders, columns and braces consisting of the framework for a building. Then, a deformation pattern represented by s series of deformation functions is assumed for the joints from bottom to top of the building aligned in the same column axis. By this assumption, the structural degrees of freedom are greatly reduced, so as that the time for time-history analysis of structural seismic response can remarkably be saved.

Comparisons are made through a number of examples between Super-FEM proposed in this paper and normal FEM encased in the computer software, Drain-2D. It is shown that satisfactory results with appropriate accuracy can be achieved by Super-FEM.

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Assumptions of Super-FEM

The idea of Super-FEM is to prescribe a displacement pattern represented by a series of deformation functions for the joints from bottom to top of the structure aligned in the same column axis. The orthogonal polynomial is adopted in this paper for its good stability as well as rapid convergency. Thus, the displacement function with variable of the height along the structure can be expressed as:

$$f_m(w) = \sum_{j=1}^{m} (-1)^{j-1} \frac{(m+1)!}{(j-1)!(j+)!(m-j)!} (\frac{w}{H})^j \quad (1)$$

where, w is the height of joint in consideration
H is the total height of structure
m is the number of polynomial items

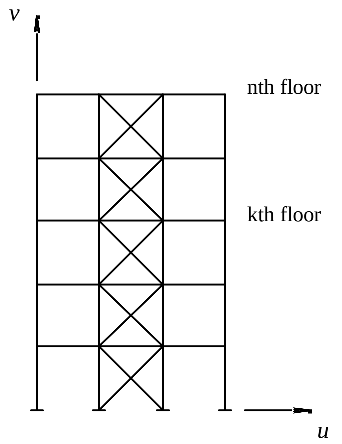


fig.1 plane braced frame

For plane braced framework as shown in fig.1, with consideration of a rigid slab, the fundamental structural degrees of freedom at *k*th floor includes:
horizontal displacement of the floor, U_{ok};
vertical displacement of joint at column axis i of the floor, V_{ik};
rotational displacement of joint at column axis i of the floor, θ_{ik};
panel shear deformation of joint at column axis i of the floor, γ_{ik};
which can be expressed in form of the displacement function as:

$$U_{ok} = \sum_{m=1}^{r} a_{om} f_m(w_k) \quad (2)$$

$$V_{ik} = \sum_{m=1}^{r} b_{im} f_m(w_k) \quad (3)$$

$$\theta_{ik} = \sum_{m=1}^{r} c_{im} f_m(w_k) \quad (4)$$

$$\gamma_{ik} = \sum_{m=1}^{r} d_{im} f_m(w_k) \quad (5)$$

where, w_k represents the height of kth floor
Thus, the joint displacement at *k*th floor and column axis *i* can be expressed as:

$$\left\{D_{ik}\right\} = \sum_{m=1}^{r} \left[N_k\right]_{im} \{e\}_{im} = \left[N_k\right]_i \{e\}_i \quad (6)$$

where:

$$\left\{D_{ik}\right\} = \left\{U_{ik}, V_{ik}, \theta_{ik}, \gamma_{ik}\right\}^T \quad (7)$$

$$\{e\}_i = \left\{\{e\}_{i1}^T, \{e\}_{i2}^T, \cdots, \{e\}_{ir}^T\right\}^T \quad (8)$$

$$\{e\}_{im} = \left\{a_{om}, b_{im}, c_{im}, d_{im}\right\}^T \quad (9)$$

$$\left[N_k\right]_i = \left[\left[N_k\right]_{i1}, \left[N_k\right]_{i2}, \cdots, \left[N_k\right]_{ir}\right] \tag{10}$$

$$\left[N_k\right]_{im} = diag\left[f_m(w_k), f_m(w_k), f_m(w_k), f_m(w_k)\right] \tag{11}$$

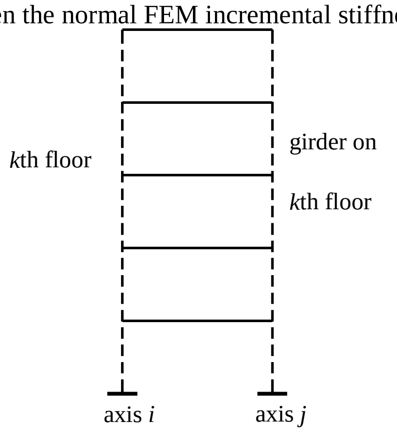
in which $\left\{D_{ik}\right\}$ represents joint displacement vector; $\left\{e_i\right\}$ is a parameter vector to be resolved, and may be denoted as a generalized displacement; $\left[N_i\right]_i$ is a conversion matrix .

Generalized Member Stiffness Equation

For structural members, i.e., girders, columns, braces and joint panels, by substitution of global displacement with generalized displacement in member stiffness equation, the generalized stiffness matrix can be derived.

Generalized Girder

A generalized girder is made up of all the girders between any two adjacent column axis, as shown in fig.2. Given the normal FEM incremental stiffness equation of a girder between axis *i* and *j* at *k*th floor:



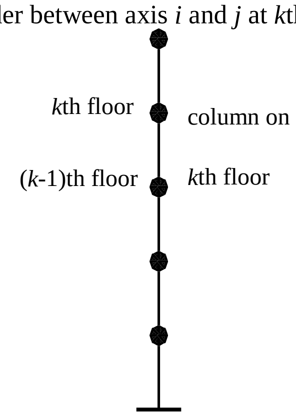


fig. 2 generalized girder member **fig. 3 generalized column member**

$$\left\{dF_{gij}\right\}_k = \left[K_{gij}\right]_k \left\{d\delta_{gij}\right\}_k \tag{12}$$

With eq.(6) and eq.(12) , the equation between generalized force increment and generalized displacement increment of the girder between axis *i* and *j* and on *k*th floor can be derived:

$$\left\{df_{gij}\right\}_k = \left[k_{gij}\right]_k \left\{de\right\}_{ij} \tag{13}$$

where

$$\left\{df_{gij}\right\}_k = \left[N_k\right]_{ij}^T \left[R_g\right]^T \left\{dF_{gij}\right\}_k \tag{14}$$

$$\left\{k_{gij}\right\}_k = \left[N_k\right]_{ij}^T \left[R_g\right]^T \left\{K_{gij}\right\}_k \left[R_g\right]\left[N_k\right]_{ij} \tag{15}$$

$$\left[N_k\right]_{ij} = \begin{bmatrix} \left[N_k\right]_i & [0] \\ [0] & \left[N_k\right]_j \end{bmatrix} \tag{16}$$

$$\left[R_g\right] = \begin{bmatrix} \left[r_g\right] & [0] \\ [0] & \left[r_g\right] \end{bmatrix} \tag{17}$$

$$\left[r_g\right] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

And the generalized girder's incremental stiffness equation is as follow:

$$\left\{df_{gij}\right\} = \left[k_{gij}\right]\left\{de\right\}_{ij} \tag{19}$$

where

$$\left\{df_{gij}\right\} = \sum_{k=1}^{n} \left\{df_{gij}\right\}_k \tag{20}$$

$$\left[k_{gij}\right]=\sum_{k=1}^{n}\left[k_{gij}\right]_k \tag{21}$$

where $\{de\}_{ij}$ is the generalized displacement increment of a generalized girder
$\{df_{gij}\}$ is the generalized force increment of a generalized girder
$[k_{gij}]$ is the generalized stiffness matrix of a generalized girder

Generalized Column

A generalized column is made up of all the columns aligned in the same column axis from bottom to top of a structure, as shown in fig.3. Given the normal FEM incremental stiffness equation of a column on axis *i* and *k*th floor:

$$\left\{dF_{ci}\right\}_k=\left[K_{ci}\right]_k\left\{d\delta_{ci}\right\}_k \tag{22}$$

With eq.(6) and eq.(21) , the equation between generalized force increment and generalized displacement increment of the column on axis *i* and *k*th floor can be derived:

$$\left\{df_{ci}\right\}_k=\left[k_{ci}\right]_k\left\{de\right\}_i \tag{23}$$

where

$$\left\{df_{ci}\right\}_k=\left[N_{k-1,k}\right]_i^T\left\{dF_{ci}\right\}_k \tag{24}$$

$$\left\{k_{ci}\right\}_k=\left[N_{k-1,k}\right]_i^T\left\{K_{ci}\right\}_k\left[N_{k-1,k}\right]_i \tag{25}$$

$$\left[N_{k-1,K}\right]_i=\begin{bmatrix}\left[N_{k-1}\right]_i\\ \left[N_k\right]_i\end{bmatrix} \tag{26}$$

And the generalized column’s incremental stiffness equation is as follow:

$$\left\{df_{ci}\right\}=\left[k_{ci}\right]\left\{de\right\}_i \tag{27}$$

where

$$\left\{df_{ci}\right\}=\sum_{k=1}^{n}\left\{df_{ci}\right\}_k \tag{19}$$

$$\left[k_{ci}\right]=\sum_{k=1}^{n}\left[k_{ci}\right]_k \tag{20}$$

where $\{de\}_i$ is the generalized displacement increment of a generalized column
$\{df_{ci}\}$ is the generalized force increment of a generalized column
$[k_{ci}]$ is the generalized stiffness matrix of a generalized column

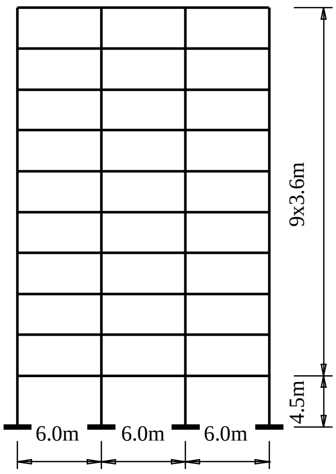
Similarly, the stiffness matrix equation of generalized brace and generalized joint can be also derived. By assembling all the member stiffness matrix, the generalized structure stiffness matrix may be obtained.
When conducting analysis of inelastic response of tall steel buildings subjected to earthquakes, the motion equation based on the generalized structure stiffness matrix may be established, which can also be solved by employing Wilson-θ method with step by step strategy.

EXAMPLES

To verify the efficiency and accuracy of Super-FEM, two examples with comparison to FEM results are given below. For Super-FEM, the polynomial item number, named r, is set as 3, 6 and 9 respectively. The FEM results are obtained by Drain-2D, the widely used structural inelastic analysis program. El-Centro NS component is chosen as the seismic excitation. A parameter ζ is imported to indicate the degree of structural inelasticity development, which is defined as the ratio of elastic maximum inter-storey shear force created by earthquakes to the limit elastic shear force of that storey.

Example A

A 3 bay 10 storey pure frame with structural dimension is shown in fig. 4. The input seismic record is scaled to a peak ground acceleration of 6.22m/s^2, which is correspond to a ζ=2.0. Fig. 6 is the computed maximum inter-storey displacement of Super-FEM vs. FEM results. It can be seen that when r is set as 6, the result is close enough to that of FEM.



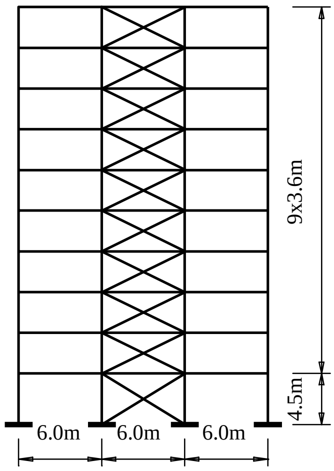
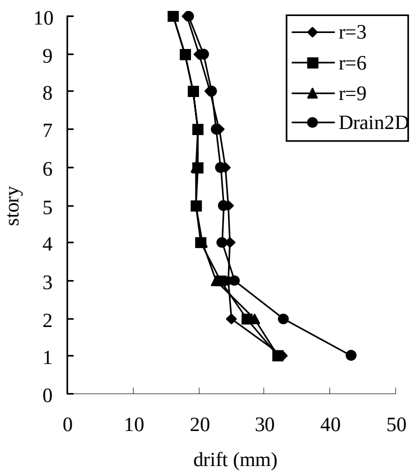


fig. 4 structure of example A **fig. 5 structure of example B**

Example B

A 3 bay 10 storey braced frame with structural dimension is shown in fig. 5. The input seismic record is scaled to a peak ground acceleration of $4.55 m/s^2$, which is correspond to a ζ=1.5. Fig 7 is the computed maximum inter-storey displacement of Super-FEM vs. FEM results. The results of two method is not as close as that of example A. This is mainly due to the different brace model adopted by the two programs. The program of Super-FEM use a more complicated brace model which takes into account abrupt strength degradation and continual stiff fluctuation of brace member after initial buckling.



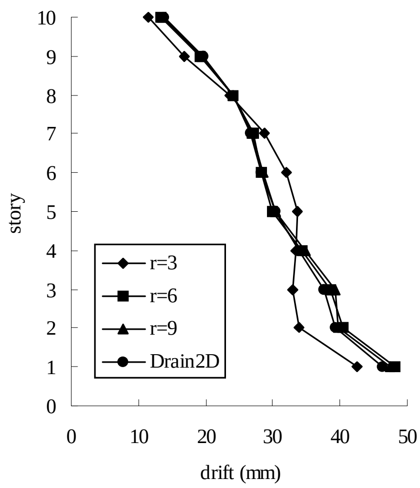


fig.7 maximum inter-story drift of example A **fig.8 maximum inter-story drift of example B**

CONCLUSIONS

To perform maximum inter-storey deformation check under severe seismic ground motion, which is required in the China National Code for Design of Tall Steel Building, a practical approach for inelastic seismic response analysis is provided. Comparisons are made between that Super-FEM method and FEM through two examples. It can be seen from examples that with the increment of polynomial item number, the results of Super-FEM converge to that of FEM's. Hence with a adequately chosen polynomial item number, the results of Super-FEM is a close enough approximation of FEM, while the computer time may be greatly reduced. Hence the adoption of Super-FEM is especially advisable for huge structures.

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