PREDICTING THE EARTHQUAKE RESPONSE OF BUILDINGS USING EQUIVALENT SINGLE DEGREE OF FREEDOM SYSTEM

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SUMMARY

The building standard law of Japan was largely revised in June 1998. With the revision, the adoption of the capacity spectrum method is being considered as a new seismic design procedure, which will be enforced by June 2000. In the method, a base shear versus horizontal displacement relation referred to as the capacity spectrum that represents the structural performance of a building is used. The capacity spectrum is usually expressed as what represents the responses of the equivalent single degree of freedom system (SDOF) for the building. Accordingly, the conversion of a building into SDOF should appropriately be carried out especially in estimating the seismic performance of irregular shaped buildings. This paper describes a procedure for the conversion. In order to examine the validity and applicable extent of the converting procedure, the earthquake responses of the multi degree of freedom system (MDOF) and SDOF for several irregular shaped buildings are compared. The analyzed are three reinforced concrete buildings that are 6, 10 and 19 stories and three steel buildings that are 5, 10 and 20 stories, respectively. For each building, the analyses of four cases which are one regular and three irregular shaped building models, the soft first story, the stiff first story and the soft middle story types, are then executed. For a total of 24 cases, earthquake response analyses of both SDOF and MDOF are executed by using a simulated earthquake with random path of which the maximum acceleration and velocity are 355.7 cm/s^2 and 57.4 cm/s, respectively. This paper shows that the earthquake responses of not only regular shaped buildings but also irregular ones can be predicted by using SDOF converted by the proposed method. For relatively high-rise buildings, however, the higher mode effect should appropriately be considered to the response of SDOF.

INTRODUCTION

The building standard law of Japan was largely revised in June 1998 after an interval of about 50 years. The adoption of new seismic design procedures is being considered with the revision. The likeliest procedure for the adoption is the capacity spectrum method [Freeman 1978] that is one of nonlinear static analysis procedures. In the method, a base shear-versus-horizontal displacement relation referred to as the capacity spectrum that represents the structural performance of a building is used. The capacity spectrum is usually expressed as what represents the responses of the equivalent single degree of freedom system (SDOF) for the building. Accordingly, the conversion of a building into SDOF should appropriately be carried out especially in estimating the seismic performance of irregular shaped buildings. This paper describes a procedure for the conversion. Through comparison of the responses of the multi degree of freedom system (MDOF) and SDOF for irregular shaped buildings of various types, the validity and applicable scopes of the converting procedure are also examined.

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CONVERSION OF THE WHOLE STRUCTURAL SYSTEM OF BUILDING INTO *SDOF* SYSTEM

The capacity spectrum method was proposed by Freeman [1978] as an approximate way of determining whether or not a building will survive an earthquake and, if it does survive, how damaged the building will be. This method also uses a nonlinear static analysis to estimate the seismic performance of a building. As shown in Fig. 1, the performance is estimated as the maximum earthquake response (displacement) of a building by the intersection of the capacity spectrum, which represents the whole structural performance of the building, and a reduced response spectrum (demand spectrum) for the assumed earthquake. The advantage of this method is that it estimates positively the response of a building for an assumed earthquake, in other words, provides more information on the degree of inelastic deformation (damage) which is expected to occur. This point is largely different from the common used seismic design procedures using equivalent static analyses. In using this method, however, it is one of important issues how to convert appropriately the whole structural system of a building into an equivalent SDOF system. So, a method of converting into SDOF is shown below.

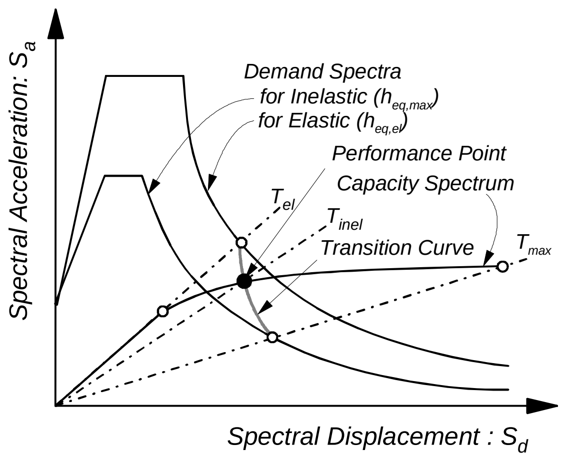

Fig. 1 Capacity and Demand Spectra

Considering the multi degree of freedom (MDOF) system for a *N*-story building, the maximum displacement response of the *i*-th story, $\delta_{i,\max}$, can generally be approximated by the following equation:

$$\delta_{i,\max} \approx \sqrt{\sum_{S=1}^{N} \left| {}_s\beta \cdot {}_s u_i \cdot {}_s\overline{M} \cdot {}_s S_a \big/ {}_s\overline{K} \right|^2} \qquad (1)$$

Similarly, the base shear at the maximum response, Q_B, can be given by

$$Q_B \approx \sqrt{\sum_{s=1}^{N} \left\{ \sum_{i=1}^{N} m_i \cdot {}_s\beta \cdot {}_s u_i \cdot {}_s S_a \right\}^2} = \sqrt{\sum_{s=1}^{N} \left\{ {}_s\overline{M} \cdot {}_s S_a \right\}^2} \qquad (2)$$

in which, m_i = lumped mass in the *i*-th story

${}_s u_i$ = normal mode of the s-th mode in the *i*-th story

${}_s\beta$ = participation factor of the s-th mode

${}_s\overline{K}$ = equivalent stiffness corresponding to the s-th mode ($= {}_s\beta\{{}_s u\}^T [K] {}_s\beta\{{}_s u\}$)

${}_s\overline{M}$ = equivalent mass corresponding to the s-th mode ($= {}_s\beta\{{}_s u\}^T [M] {}_s\beta\{{}_s u\}$)

${}_s S_a$ = spectral acceleration for the s-th mode

${}_s S_d$ = spectral displacement for the s-th mode

Under an assumption that the maximum response under dynamic vibration can be represented by that in static analysis, then, considering MDOF system subjected to statically horizontal forces at each mass, of which the distribution is proportional to the first mode of vibration, Eq.(2) can be rewritten as

$${}_1Q_B = {}_1\overline{M} \cdot {}_1S_a \qquad (3)$$

and from Eq.(1), the displacement in each story, $\{{}_1\delta\}$, is given by

$$\{{}_1\delta\} = {}_1\beta\{{}_1u\} {}_1\overline{M} \cdot {}_1S_a \big/ {}_1\overline{K} = {}_1\beta\{{}_1u\} {}_1Q_B \big/ {}_1\overline{K} \qquad (4)$$

As clarified in Eq.(4), MDOF system can be converted to the equivalent SDOF system with the equivalent lumped mass and stiffness, ${}_1\overline{M}$ and ${}_1\overline{K}$, because the external force distribution for MDOF system is corresponding to the first mode. In this case, the horizontal displacement of the equivalent SDOF system (the representative displacement), ${}_1\Delta$, is corresponding to displacement at the height that the participation vector of the first mode, ${}_1\beta\{{}_1u\}$, is equal to 1.0 in MDOF system. From Eq.(4), namely, the relation between shear (the representative shear) and the representative displacement in the equivalent SDOF system is given as

$${}_1\Delta = {}_1Q_B \big/ {}_1\overline{K} \qquad (5)$$

Accordingly, the representative shear is corresponding to the base shear, ${}_1Q_B$. Then, using Eq.(3) and (5), the representative displacement can be rewritten as

$$ {}_1\Delta = {}_1S_a \cdot {}_1\overline{M} \Big/ {}_1\overline{K} = {}_1S_d \tag{6} $$

Eq.(6) shows that the relation between the representative shear and displacement in the equivalent SDOF system is expressed using the spectral acceleration and displacement, ${}_1S_a$ and ${}_1S_d$. Eq.(1) also gives the horizontal displacement of the *i*-th story in MDOF system, ${}_1\delta_i$, as

$$ {}_1\delta_i = {}_1\beta \cdot {}_1u_i \cdot {}_1S_d = {}_1\beta \cdot {}_1u_i \cdot {}_1\Delta \tag{7} $$

Using Eq.(2) and (7), the external force applied at the i-th story, ${}_1P_i$, is given as follows:

$$ {}_1P_i = m_i \cdot {}_1\beta \cdot {}_1u_i \cdot {}_1S_a = m_i \cdot {}_1\delta_i \cdot {}_1S_a \big/ {}_1\Delta \tag{8} $$

and using Eq.(3) and (8) gives the relation between ${}_1\overline{M}$ and ${}_1\delta_i$ as

$$ {}_1\overline{M} = {}_1Q_B \big/ {}_1S_a = \sum_{i=1}^{N} {}_1P_i \Big/ {}_1S_a = \sum_{i=1}^{N} m_i \cdot {}_1\delta_i \Big/ {}_1\Delta \tag{9} $$

Similarly, using Eq.(7) forms the following relations:

$$ {}_1\overline{M} = {}_1\beta\{{}_1u\}^T[M]{}_1\beta\{{}_1u\} = \{{}_1\delta\}^T[M]\{{}_1\delta\} \Big/ {}_1\Delta^2 = \sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2 \Big/ {}_1\Delta^2 \tag{10} $$

$$ {}_1\overline{K} = {}_1\beta\{{}_1u\}^T[K]{}_1\beta\{{}_1u\} = \{{}_1\delta\}^T[K]\{{}_1\delta\} \Big/ {}_1\Delta^2 = \{{}_1\delta\}^T\{{}_1P\} \Big/ {}_1\Delta^2 = \sum_{i=1}^{N} {}_1P_i \cdot {}_1\delta_i \Big/ {}_1\Delta^2 \tag{11} $$

Therefore, these relations yield the natural circular frequency or the natural period of the first mode, ${}_1\omega$ or ${}_1T$,

$$ {}_1\omega = \sqrt{\frac{{}_1\overline{K}}{{}_1\overline{M}}} = \sqrt{\frac{\sum_{i=1}^{N} {}_1P_i \cdot {}_1\delta_i}{\sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2}} \qquad \left(\therefore\ {}_1T = 2\pi \sqrt{\frac{\sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2}{\sum_{i=1}^{N} {}_1P_i \cdot {}_1\delta_i}} \right) \tag{12} $$

Also, using Eq.(9) and (10) gives the equivalent mass of the first mode, ${}_1\overline{M}$, as

$$ {}_1\overline{M} = \frac{\left(\sum_{i=1}^{N} m_i \cdot {}_1\delta_i \right)^2}{\sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2} \tag{13} $$

Namely, the spectral acceleration and displacement corresponding to the first mode, ${}_1S_a$ and ${}_1S_d$, can be given as follows using Eq. (3) and (13), and Eq. (6) and (12), respectively.

$$ {}_1S_a = \frac{\sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2}{\left(\sum_{i=1}^{N} m_i \cdot {}_1\delta_i \right)^2} \cdot {}_1Q_B \tag{14} $$

$$ {}_1S_d = \frac{\sum_{i=1}^{N} m_i \cdot {}_1\delta_i^2}{\sum_{i=1}^{N} {}_1P_i \cdot {}_1\delta_i} \cdot {}_1S_a \tag{15} $$

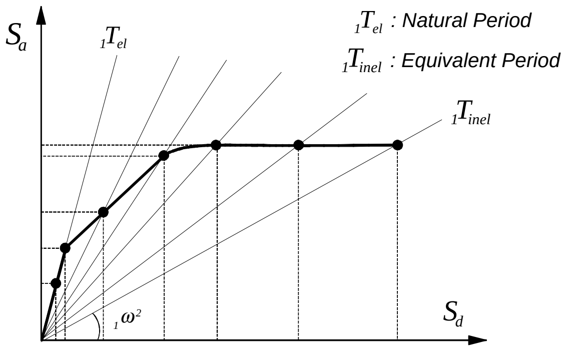

Fig. 2 Capacity Spectrum

Using Eq.(14) and (15) and the information on the external forces and displacements of each story and the base shear in each loading step obtained from non-linear push-over analysis with the external force distribution proportioned to the first mode, a $S_a - S_d$ curve (Capacity Spectrum) can be drown as shown in Fig. 2.

COMPARISON BETWEEN RESPONSES OF *SDOF* AND *MDOF*

Earthquake response analyses of both SDOF and MDOF systems for several buildings are executed to examine the validity of conversion from MDOF system into SDOF system shown above and its applicable scope. For buildings that the distribution of story strength and stiffness along the building height is extremely irregular, the predicting accuracy of the earthquake response of SDOF system is verified by comparing with that of MDOF system.

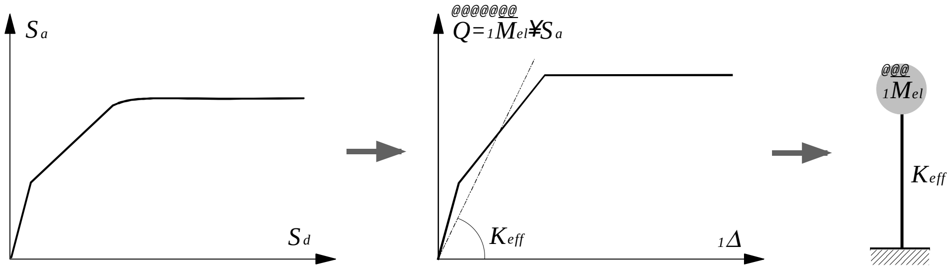

Fig. 3 Skeleton Curve for SDOF System

Modeling of Restoring Force Characteristic for Equivalent *SDOF* System

In order to execute earthquake response analyses of SDOF system, modeling of the restoring force characteristic is required. In this study, converting the capacity spectrum made by the method described in the above chapter into the tri-linear fitting curve, a skeleton curve for the $S_a - S_d$ relation is produced. As shown in Fig. 3, then, multiplying the S_a components of the skeleton curve by the equivalent mass corresponding to the first mode for elastic, ${}_1\overline{M}_{el}$, a skeleton curve for the representative shear versus representative displacement relation is obtained. The representative shear, ${}_1\overline{M}_{el} \times S_a$, is not agree with the base shear of the analyzed building when the equivalent mass, ${}_1\overline{M}$, changes after a portion of the building yields. In the analysis of SDOF system, however, since the lumped mass is assumed to be a constant value, ${}_1\overline{M}_{el}$, the maximum response (the representative displacement) obtained is not affected. Assuming that the restoring force characteristic of a building represents that of the stories and the characteristic of a story represents that of the members, hysteresis rules of the degrading tri-linear model (Takeda model) for RC buildings and the normal tri-linear model for steel buildings are used in SDOF system.

Method of Analysis

As shown in Fig.4, the method of analyses is as follows:

(a) Obtain the shear versus displacement relation of each story from a non-linear push-over analysis with the external force distribution corresponding to the first mode of vibration.

(b) Make the capacity spectrum ($S_a - S_d$ curve) using Eq.(14) and (15) and the analytical results obtained from

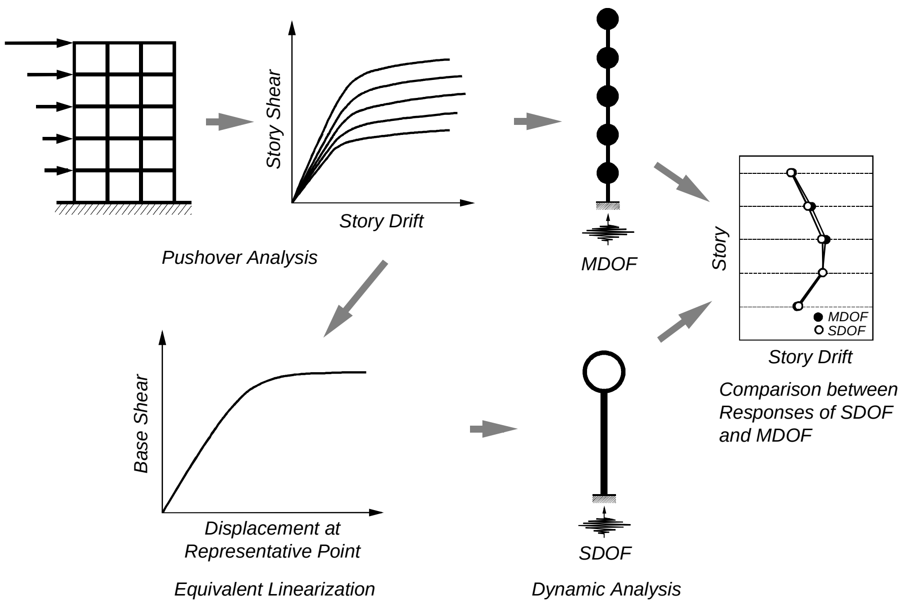

Fig. 4 Flow of Analysis

step (a), namely information on the external force and displacement of each story and base shear in each loading step.

(c) Make the skeleton curve for the base shear versus representative displacement relation by converting the $S_a - S_d$ curve into a tri-linear curve and multiplying S_a by the equivalent mass for the elastic first mode, ${}_1\overline{M}_{el}$. (see Fig.3)

(d) Execute earthquake response analyses for SDOF system using the equivalent mass for the elastic first mode, ${}_1\overline{M}_{el}$ and a restoring force characteristic with the skeleton curve made in step (c) and the assumed hysteresis rule for the structural type of analyzed buildings mentioned above.

(e) Seek a loading step on the $S_a - S_d$ curve made in step (b), which is corresponding or nearest to the maximum response of SDOF system obtained in step (d).

(f) Seek the displacement at each story on the shear versus displacement obtained from the push-over analysis in step (a), which is corresponding to the loading step obtained in step (e).

(g) Execute earthquake response analyses for MDOF system.

(h) Compare the results of SDOF system (step (f)) with those of MDOF system.

Analyzed Buildings and Analytical Assumption

The analyzed are three RC buildings that are 6, 10 and 19 stories and three steel buildings that are 5, 10 and 20 stories, respectively. For each building, four cases including one regular and three irregular shaped building models are then analyzed as shown in Fig.5. The irregular shaped buildings are the soft first story, the stiff first story and the soft middle story, respectively. The regular shaped building is modeled on the design example of a real building and the restoring characteristic of each story is obtained from nonlinear push-over analysis with the frame model. In the irregular shaped buildings, Models (b) to (d) in Fig.5, the stiffness and capacity of relatively strong stories are twice as large as those of the corresponding stories in the regular shaped building. Table 1 gives a brief outline of analyzed buildings.

Earthquake response analyses of both SDOF and MDOF are executed for the above 24 cases. The earthquake wave used is a simulated one with random phase referred to as the BCJ-L2 [Kitagawa, et al. 1994], of which the maximum acceleration and velocity are 355.7 cm/s^2 and 57.4 cm/s, respectively. In the response analyses, the viscous damping of buildings is assumed to be the transient stiffness proportioning type of 3% for RC buildings and the initial stiffness proportioning type of 2% for Steel buildings.

Results of Analysis

Figure 6 shows the story shear versus story drift relations obtained from push-over analysis for each regular shaped building. In the figure, circle and square marks are the maximum responses of each story obtained from

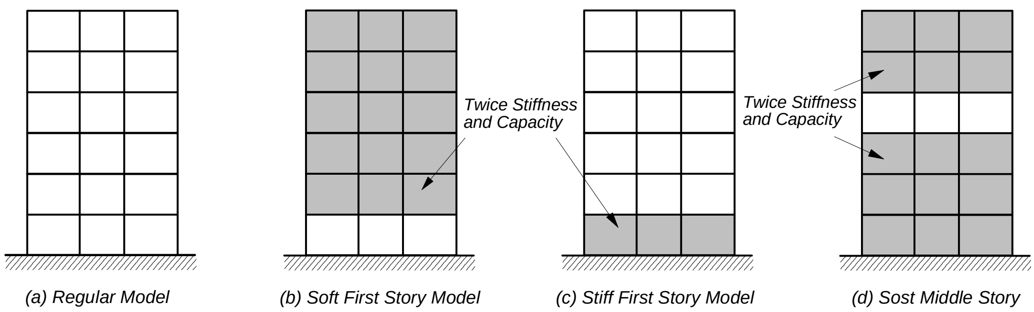

Fig. 5 Analyzed Buildings

Table 1 Relation of Analyzed Building and Skeleton Curve of Stories

Building Type	RC			Steel		
Number of Story	6	10	19	5	10	20
Total Height (m)	17.8	30.3	61.5	19.0	38.0	80.5
Total Weight (ton)	3,710	6,510	28,638	2,800	5,300	18,325
Natural Period (sec)	0.45	0.64	1.02	0.86	1.36	2.47

the earthquake response analysis of SDOF and MDOF systems, respectively.

In low-rise buildings regardless of structural type, RC or steel, the response of each story obtained from the analysis of SDOF system shows excellent agreement with that of MDOF system. In higher buildings, an increase in the displacement responses recognized as the higher mode effect is observed in the middle and/or upper stories for MDOF system. As a result, the displacement response of each story of SDOF system tend to be smaller than that of MDOF system in the middle and/or upper stories, because the higher mode effect can not be considered in SDOF system. The ratio of SDOF to MDOF on displacement response, ${}_s\delta/{}_m\delta$, of each story ranges from 0.8 to 1.1 in almost all models, although the ratio in the 19 story RC model ranges from 0.7 to 1.22. The ratio on shear response, ${}_sQ/{}_mQ$, of each story is between 0.9 and 1.1.

Comparison between the displacement responses of SDOF and MDOF systems for the soft first story, the stiff first story and the soft middle story models are shown in Figs. 7, 8 and 9, respectively.

For the soft first story models, as shown in Fig. 7, correspondence of the responses of SDOF system to those of MDOF system is almost good regardless of the structural type or the number of stories. In almost all models, the ratio, ${}_s\delta/{}_m\delta$, of the first story is between 0.8 and 1.2, and that of each story ranges from 0.7 to 1.2. The ratio, ${}_sQ/{}_mQ$, ranges from 0.8 to 1.15.

Correlation between the responses of SDOF and MDOF systems for the soft first story models is similar to that for the regular model. In all models except the 19 story RC model, the ratios on displacement response, ${}_s\delta/{}_m\delta$, of each story ranges from 0.75 to 1.15 and the ratio on shear response, ${}_sQ/{}_mQ$, is between 0.95 and 1.1. The ratios, ${}_s\delta/{}_m\delta$ and ${}_sQ/{}_mQ$, in the 19 story RC model range from 0.58 to 1.05 and from 0.58 to 1.05, respectively. Namely, the predicting accuracy of the responses of SDOF system for the 19 story RC model is worse than that for the other models due to the higher mode effect.

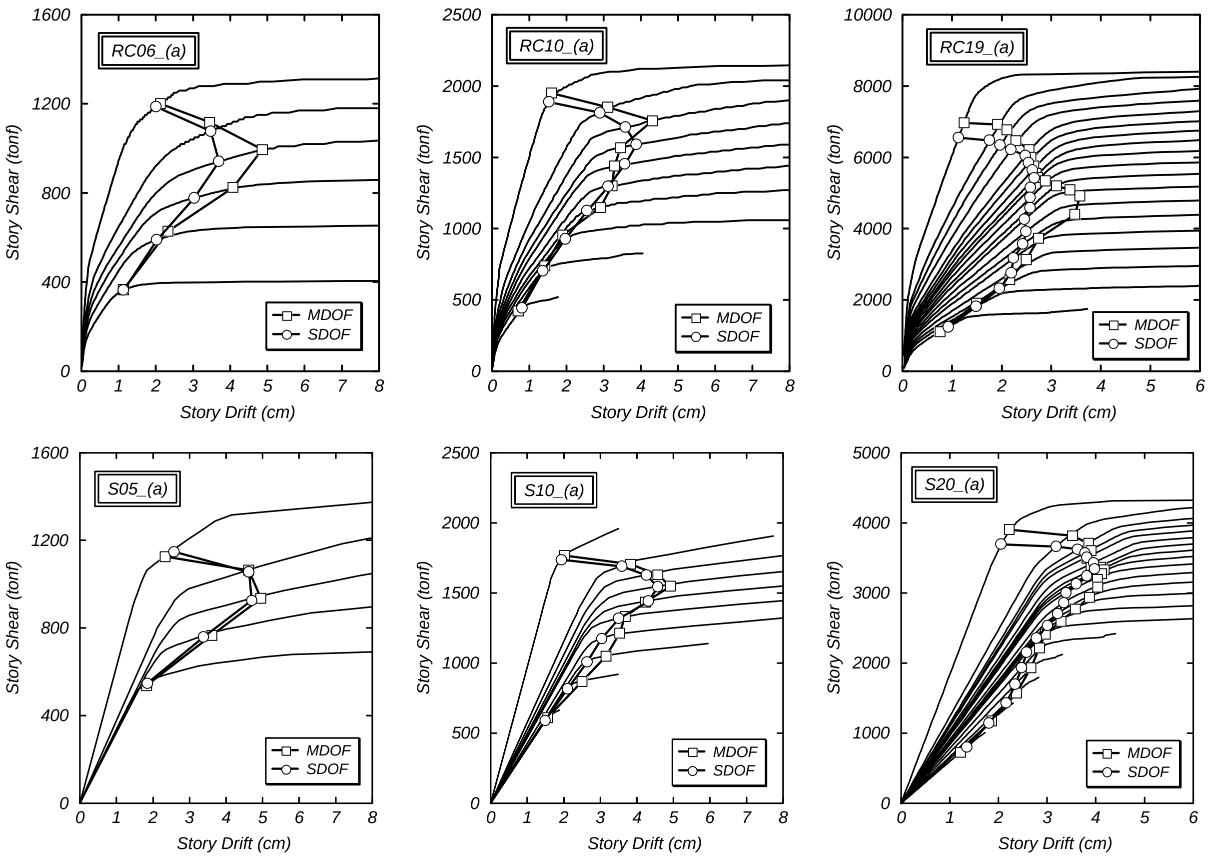

Fig. 6 Comparison between Response of SDOF and MDOF for Regular Shaped Buildings

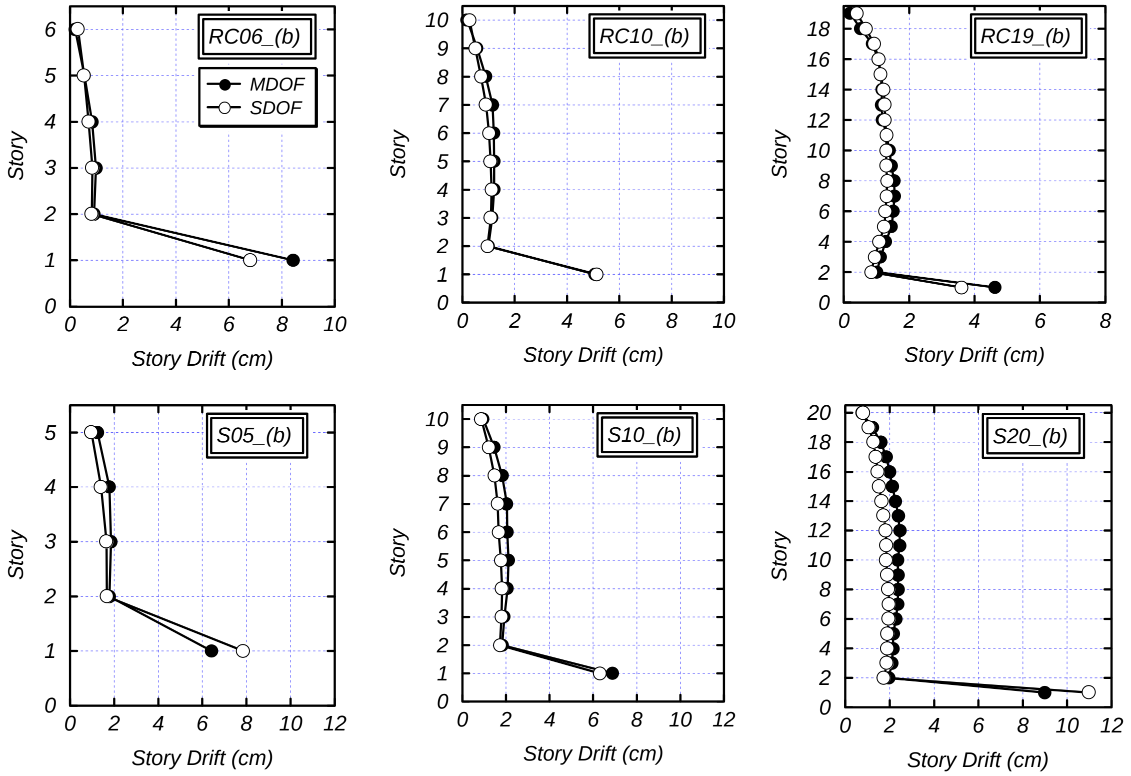

Fig. 7 Comparison between Response of SDOF and MDOF for Soft First Story Model

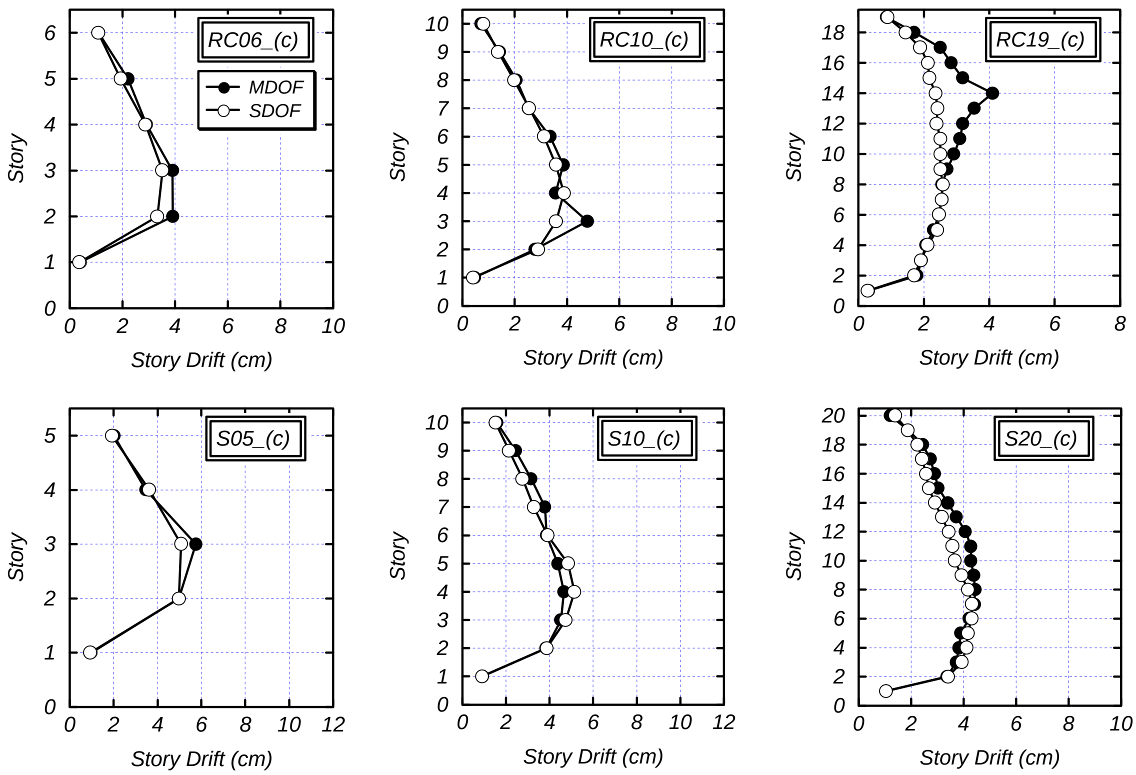

Fig. 8 Comparison between Response of SDOF and MDOF for Stiff First Story Model

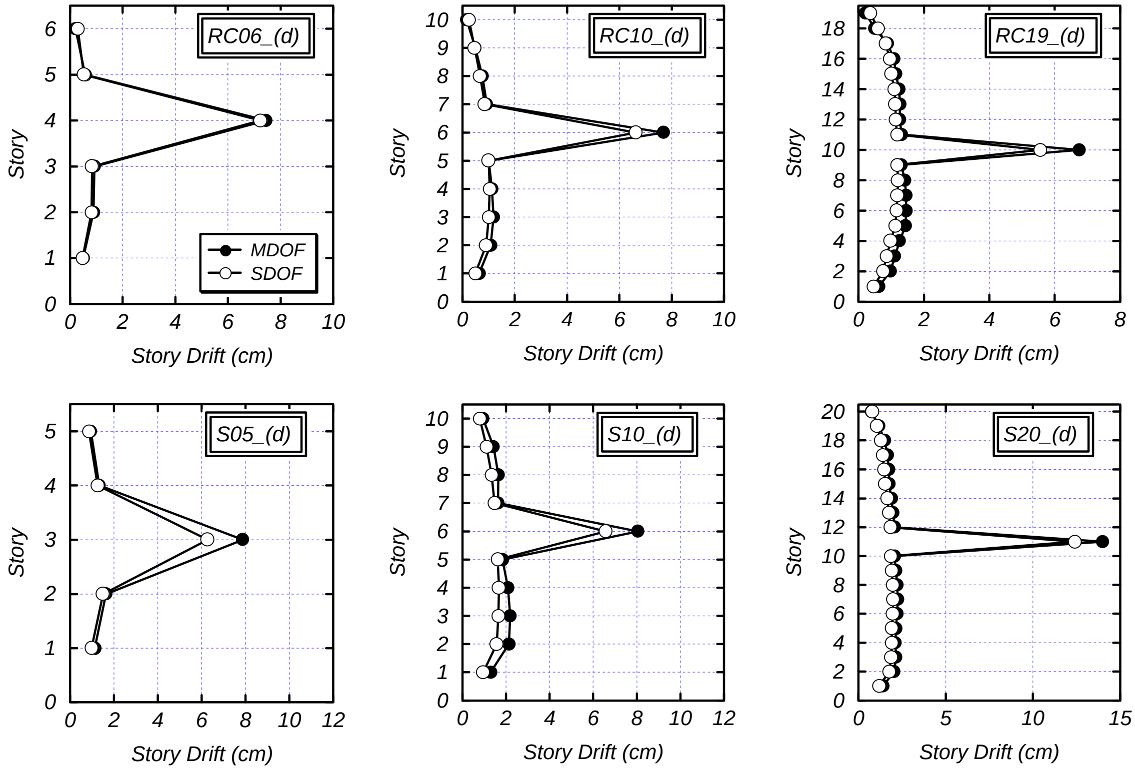

Fig. 9 Comparison between Response of SDOF and MDOF for Soft Middle Story Model

For the soft middle story models, correspondence of the responses of SDOF system to those of MDOF system is almost good, although there is a tendency that the responses of SDOF system are a little smaller than those of MDOF system over all stories. The ratio on displacement response, ${}_s\delta/{}_m\delta$, of the middle weak story ranges from 0.8 to 1.0 and that of the other stories are between 0.75 and 1.2. The ratio on shear response, ${}_sQ/{}_mQ$, of each story ranges from 0.8 to 1.2.

CONCLUSIONS

A method of converting the whole structural system of a building into the equivalent SDOF system was proposed in this paper. Using the method and executing earthquake response analyses of 24 cases, the predicting accuracy of the response of each story in a building obtained from the response of the equivalent SDOF system to that of MDOF system was investigated to grasp the applicable scopes of the capacity spectrum method. Conclusions obtained are:

1) In both RC and steel buildings of the regular shaped type, good agreement between the responses of SDOF and MDOF systems was observed regardless of the building height.
2) For the irregular shaped buildings, the responses of SDOF system could almost simulate those of MDOF system.
3) In high-rise buildings exceeding 10 stories, however, the displacement response of SDOF system tended to be smaller than that of MDOF system because of the higher mode effect in MDOF system. Accordingly, applying the capacity spectrum method for relatively high-rise buildings, the higher mode effect should appropriately be considered.

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