RELIABILITY OF STRUCTURAL SAFETY OF SCHOOL GYMNASIUMS IN HEAVY SNOW REGIONS AGAINST COMBINED LOAD OF SNOW AND EARTHQUAKE

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SUMMARY

Reliability Index of structural safety of school gymnasiums is investigated against a combined load of dead load, snow load, and earthquake load. First, a theoretical study is conducted assuming that snow and earthquake loads can be modeled as stochastic processes with fluctuation in occurrence, duration, and intensity, while dead load and resistant load capacity are independent stochastic variables with no fluctuation in time domain. Contour lines between two design factors, one for snow load and the other for earthquake load, which give the same reliability index, are obtained. This theoretical result shows that the design factor for snow load is more sensitive to the reliability index of school gymnasiums than that for earthquake load in the vicinity of the combination of usually adopted design factors for both loads. This means that the important factor assigned to the school gymnasium as a shelter for local inhabitants should be considered as snow load rather than earthquake load to achieve a certain amount of extra reliability for its retrofit. Secondly, a Monte Carlo simulation is followed to confirm this theoretical result. An interaction curve between two ultimate strengths for vertical and horizontal loads is applied to represent the resistant capacity of a school gymnasium. Contour lines giving the same reliability index between these two design factors are obtained again. These new contour lines confirm the conclusion obtained by the former theoretical study.

INTRODUCTION

School gymnasiums are considered very important structures since many of the school gymnasiums in Hanshin District, Hyogo Pref., were used as shelters for the emergency evacuation of local inhabitants after the Hyogo-ken Nanbu Earthquake [5]. In the case of earthquake diagnosis or retrofit of existing school gymnasiums, it becomes usual that a new design factor, a so-called important factor, is assigned for earthquake design load [8]. In the snow region of Japan, however, the important factor of snow load should also be considered in addition to that of earthquake load because snow load has direct effects upon the design of long span structures like school gymnasiums. In Japan, the current design factor of snow load is defined as 0.35 in combination with that of earthquake load of 1.0. There is a problem concerning this combination of load factors [2],[3], but in this paper, we do not discuss these load factors themselves but inspect the effect of snow load on structural reliability of the school gymnasiums designed by this current combination of these load factors. We aim to clarify the relation between structural reliability of school gymnasiums and the combination of load factors for snow and earthquake loads, and to demonstrate the reasonable extra design factors, that is to say, important factors for snow and earthquake loads.

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Snow and earthquake loads are assumed as stationary stochastic processes with fluctuation in occurrence, duration, and intensity. The former is represented as a discontinuous rectangular pulse process and the latter as a Poisson impulse process as shown in Fig.1 [1],[4]. Dead load and resistant capacity of school gymnasiums are assumed as independent stochastic variables with no fluctuation in time. The stochastic parameters of these random variables are chosen according to AIJ Standard [1] as shown in Table 1.

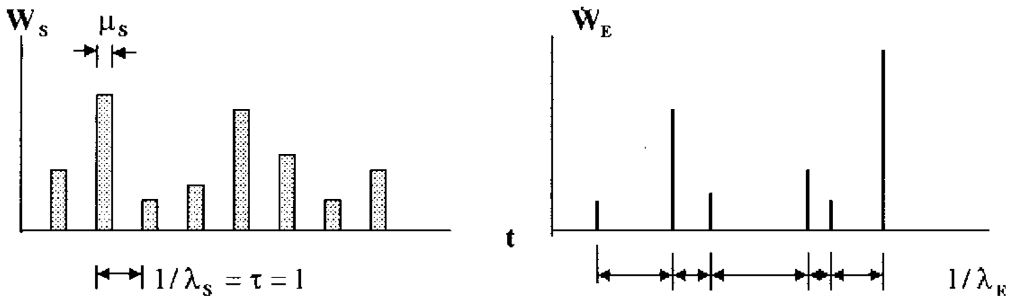


Figure 1: Stochastic Processes of Snow and Earthquake loads

Table 1: Stochastic parameters

	Dead Load (D)	Snow Load (S)	Earthquake Load (E)	Resistant Capacity (R)
average/nominal : $\overline{W}_X / W_{Xn}$	1.0	0.45	0.42	1.10
covariance : V_X	0.10	0.48	0.80	0.15
occurrence ratio : $\overline{\lambda}_X$	-	1.0 (fixed)	1/3	-
duration : $\overline{\mu}_X$	-	1/3 (fixed)	-	-

Note: W_X denotes X load.

3. Exceeding Probability and Structural Reliability

Exceeding probability of revel r during T years under a combination of snow and earthquake loads is represented by Equation (1) [1].

$$P_f(r:T) \approx 1 - \exp\left\{-\lambda_S T\left(\lambda_E(1-\mu_S/\tau)\tau F_E^c(r) + 1 - F_{S+E.\mu_S}(r)\right)\right\} \qquad (1)$$

where $$F_{S+E.\mu_S}(r) = \int_0^r \exp\left\{-\lambda_E \mu_S F_E^c(r-w)\right\} f_S(w)dw \qquad (2)$$

$F_X(r)$: Cumulative distribution function of X load, $F_X^c(r) = 1 - F_X(r)$

$f_X(r)$: Probability density function of X load

On the other hand, a design criterion under a combination of snow and earthquake loads is shown as follows,

$$\text{Prob}[Z_1 - Z_2 \geq 0] \geq \Phi(\beta) \qquad (3)$$

where $Z_1 = R - W_D$, $Z_2 = \max(W_S + W_E)$

R : Resistant capacity of school gymnasiums

W_D : Dead load

β : Reliability index

Let F_{Z1} and F_{Z2} denote cumulative distribution functions of Z_1 and Z_2 respectively. Translation to the equivalent standard normal variates (u_1, u_2) is achieved via the following equations.

$$u_1 = \Phi^{-1}(F_{Z1}(z_1)) , \quad u_2 = \Phi^{-1}(F_{Z2}(z_2)) \qquad (4)$$

Reliability index β is represented by the length of the perpendicular from the origin to the tangent of a failure surface. The following expressions, therefore, are obtained.

$$u_1^* = \alpha_{Z1}\beta \ , \quad u_2^* = \alpha_{Z2}\beta \tag{5}$$

where u_{Z1}^*, u_{Z2}^* : Coordinates of the foot of perpendicular on a failure surface

α_{Z1}, α_{Z2} : Direction cosines and $\alpha_{Z1}^2 + \alpha_{Z2}^2 = 1$

Using the result of Eq. (4) and Eq. (5) in Eq. (3), the design criterion is expressed as follows,

$$\mathrm{Prob}\left[\max\{W_S + W_E\} \le Z_2^*\right] = \Phi(\alpha_{Z2}\beta) \tag{6}$$

Considering that snow load will happen in T years without fail, Eq. (1) is applied to Eq. (6), then Eq. (7) is obtained.

$$-\lambda_E(1-\mu_S)F_E^c(r) - 1 + F_{S+E,\mu_S}(r) = \frac{1}{T}\ln\left\{(1-e^{-T})\Phi(\alpha_{Z2}\beta) + e^{-T}\right\} \tag{7}$$

The first term of the left side of Eq. (7) means the exceeding probability under earthquake load only, thus we can neglect this term when long span structures in a heavy snow region are considered. So we obtain the next equation,

$$F_{S+E,\mu_S}^c(w) = -\frac{1}{T}\ln\left\{(1-e^{-T})\Phi(\alpha_{Z2}\beta) + e^{-T}\right\} \tag{8}$$

Let $\hat{\beta}$ denotes $-\frac{1}{T}\ln\left\{(1-e^{-T})\Phi(\alpha_{Z2}\beta) + e^{-T}\right\}$, then we obtain

$$\mathrm{Prob}[W_S^* + W_E^* \le Z_2^*] = \Phi(\hat{\beta}) \tag{9}$$

Finally the load factors γ_X and resistant factor ϕ under a combined load of dead, snow and earthquake are represented as follows,

$$\left.\begin{aligned}
\gamma_D &= \frac{1}{\sqrt{1+V_D^2}}\exp\left(\alpha_D\alpha_{Z1}\beta\sigma_{\ln D}\right)\\
\gamma_S &= \frac{1}{\sqrt{1+V_S^2}}\exp\left(\alpha_S\hat{\beta}\sigma_{\ln S}\right)\\
\gamma_E &= \frac{1}{\sqrt{1+V_E^2}}\exp\left(\alpha_E\hat{\beta}\sigma_{\ln E}\right)\\
\phi &= \frac{1}{\sqrt{1+V_R^2}}\exp\left(-\alpha_R\alpha_{Z1}\beta\sigma_{\ln R}\right)
\end{aligned}\right\} \tag{10}$$

where $\sigma_{\ln X}$: Standard deviation of ln X

$\alpha_S, \alpha_E, \alpha_R, \alpha_D$: Direction cosines , and $\alpha_S^2 + \alpha_E^2 = 1, \alpha_R^2 + \alpha_D^2 = 1$

Using design factors of z_S and z_E for snow and earthquake load respectively, nominal resistant capacity R_n is expressed as follows,

$$R_n = \theta(W_{Dn} + W_{Sn} z_S)(1 + c a_n z_E) \quad (11)$$

where θ : Safety factor
W_{Xn} : Nominal design value of X load
c : Effective factor of earthquake lateral load when evaluated as a vertical load
a_n : Nominal base shear coefficient for design

In order to realize a given reliability, the following relation should be satisfied.

$$\phi R_n \geq \gamma_D W_{Dn} + \gamma_S W_{Sn} + \gamma_E W_{En} \quad (12)$$

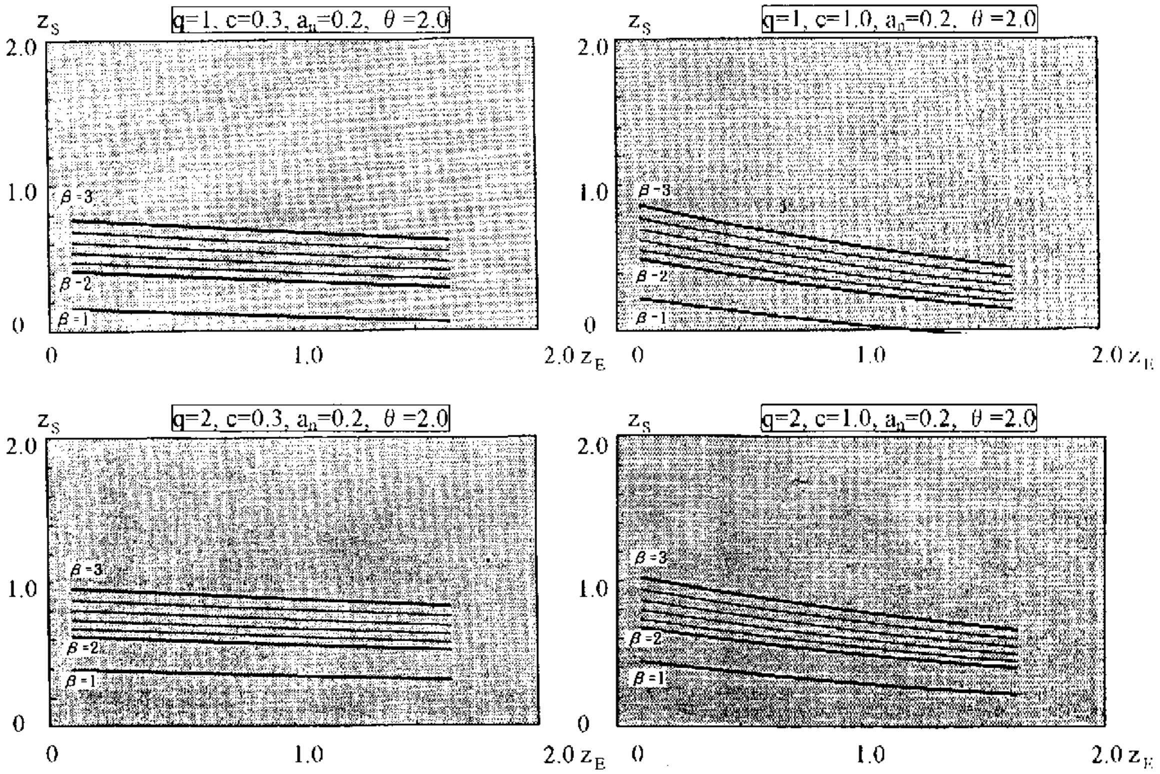
From Eq. (12), we can obtain the necessary relation between z_S and z_E to keep the given reliability as follows,

$$\theta(1 + q z_S) - \frac{1}{\phi}(\gamma_D + q\gamma_S) + c\left\{\theta(1 + q z_S) z_E - \frac{\gamma_E}{\phi}(1 + q)\right\} a_n \geq 0 \quad (13)$$

where $q = W_{Sn} / W_{Dn}$

4. ANALYTICAL RESULTS

Pairs of z_S and z_E which give the same reliability index are obtained for a combination of parameters of c, q, and θ. These results are represented by contour lines in a z_S - z_E plane as shown in Fig. (1). Configurations of these contour lines vary from case to case. The relation between z_S and z_E for any reliability indexes seems to be inversely proportional. And it should be noted that the density of contour lines in the direction of z_S is far greater than that in the direction of z_E for any combination of parameters. In other words, the reliability index of the school gymnasiums in a heavy snow region can be improved by raising the value of z_S more easily than z_E.



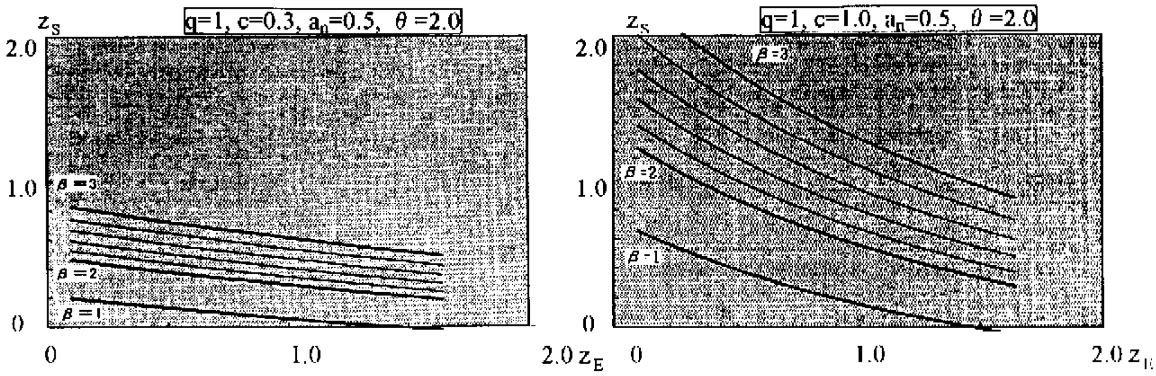


Figure 1: Contour lines of β in the plane of z_S and z_E obtained by theoretical study

5. MONTE CARLO SIMULATION

5.1 Estimation of Resistant Capacity by R_V R_H Interaction Curves:

Resistant capacity of a school gymnasium for a combination load of vertical and horizontal directions is assumed to be represented by an elliptic interaction curve. This interaction curve is given by Equation (14) where parameters of R_V^0, R_H^0, m, n are decided via elasto-plastic analyses of actual school gymnasiums in the heavy snow regions.

$$\left(\frac{\overline{R}_V}{\overline{R}_V^0}\right)^m + \left(\frac{\overline{R}_H}{\overline{R}_H^0}\right)^n = 1.0 \qquad (14)$$

where

$\overline{R}_V$: Vertical component of average ultimate strength of school gymnasiums against a combined load.

$\overline{R}_H$: Horizontal component of average ultimate strength of school gymnasiums against a combined load.

$\overline{R}_V^0$: Average ultimate strength of school gymnasiums against a vertical load only

$\overline{R}_H^0$: Average ultimate strength of school gymnasiums against a horizontal load only

snow level	a	b	m	n
1 m	1.80	1.90	1.40	3.30
2 m	1.87	2.04	1.60	3.40
3 m	1.94	2.18	3.90	3.60

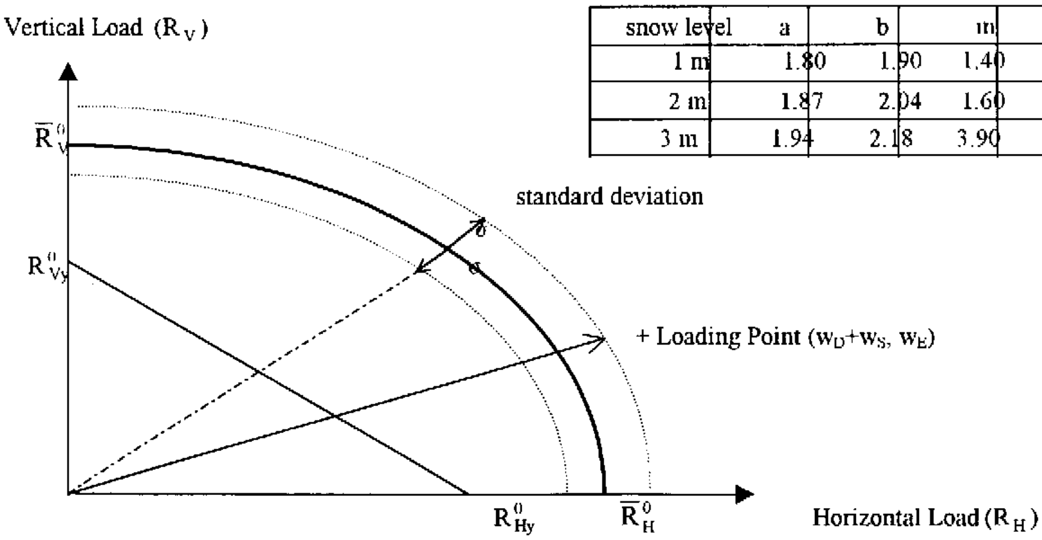


Figure 2: Interaction Curve of Resistant Capacity (R_V, R_H)

5.2 Stochastic Description of Resistant Capacity:
An interaction curve of the average resistant capacity of school gymnasiums is assumed to have a log-normal probability density distribution with a standard deviation, σ, along the line drawn through the origin as shown in Figure 2. This interaction curve is obtained by analyzing the ultimate strength of actual school gymnasiums in the heavy snow regions which have been designed using the standard design factors of 0.35 for snow and 1.0 for earthquake load. We can apply this original interaction curve in order to obtain an interaction curve for any other pair of design factors as shown in follows. Introducing $\bar{r}_V$, $\bar{r}_H$, a and b, we obtain a non-dimensional expression of Equation (14).

$$\left(\frac{\bar{r}_V}{a}\right)^m + \left(\frac{\bar{r}_H}{b}\right)^n = 1.0 \tag{15}$$

where

$\bar{r}_V = \bar{R}_V / R^0_{Vy}$, $\bar{r}_H = \bar{R}_H / R^0_{Hy}$, $a = \bar{R}^0_V / R^0_{Vy}$, $b = \bar{R}^0_H / R^0_{Hy}$

R^0_{Vy} : Yield strength of school gymnasiums against a vertical load only

R^0_{Hy} : Yield strength of school gymnasiums against a horizontal load only

By assuming that Equation (15) is true for any combinations of both vertical and horizontal design factors which differ from the original ones, we can obtain any corresponding interaction curves for newly assigned pairs of design factors.
Let z_S and z_E denote a new combination of design factors, then the corresponding resistant capacity is obtained as follows,

$$\left.\begin{aligned} \bar{R}_V &= \bar{r}_V R^0_{Vy} = \bar{r}_V \frac{z_V (R^0_{Vy})_{z_V=0.35}}{0.35} \\ \bar{R}_H &= \bar{r}_H R^0_{Hy} = \bar{r}_H \frac{z_H (R^0_{Hy})_{z_H=1.00}}{1.00} \end{aligned}\right\} \tag{16}$$

5.3 Calculation of Reliability Index β:
Ten thousand interaction curves, all of which have the same average ultimate strength given by Equation (16), are created for a new combination of design factors of Z_S and Z_E applying an inverse transform method to the log-normal distribution of resistant capacity. On the other hand, a duration time of 50 years is assigned to simulate a series of various combinations of loading for each structure of 10,000, which have a prescribed distribution of resistant capacity. Then a reliability index is obtained from the following judgement using Figure 2.

$$\beta = \Phi^{-1}\left[\mathrm{Prob}\left\{\vec{R}_V{}^2 + \vec{R}_H{}^2 \geq (w_D + w_S)^2 + w_H{}^2\right\}\right] \tag{17}$$

where

$\vec{R}_V$: Vertical component of the resistant capacity along with the line drawn from the origin to a loading point in R_S and R_H plane. (see Figure 2)

$\vec{R}_H$: Horizontal component of the resistant capacity along with the line drawn from origin to a loading point.

w_D, w_S, w_E : Combination of Dead, Snow, and Earthquake respectively when either of w_S, w_E and ($w_S + w_E$) attains its peaks during 50 years.

6. MONTE CARLO SIMULATION RESULTS

Stochastic parameters shown in Table 1 are adopted in Monte Carlo simulation analysis. Earthquake load, w_E, is represented by base shear coefficient of α, which is defined as $w_E/(w_D + z_S w_S)$, and its nominal value, which is denoted as α_n, is decided to be 0.2 according to Japanese standards [1]. Monte Carlo simulation study is

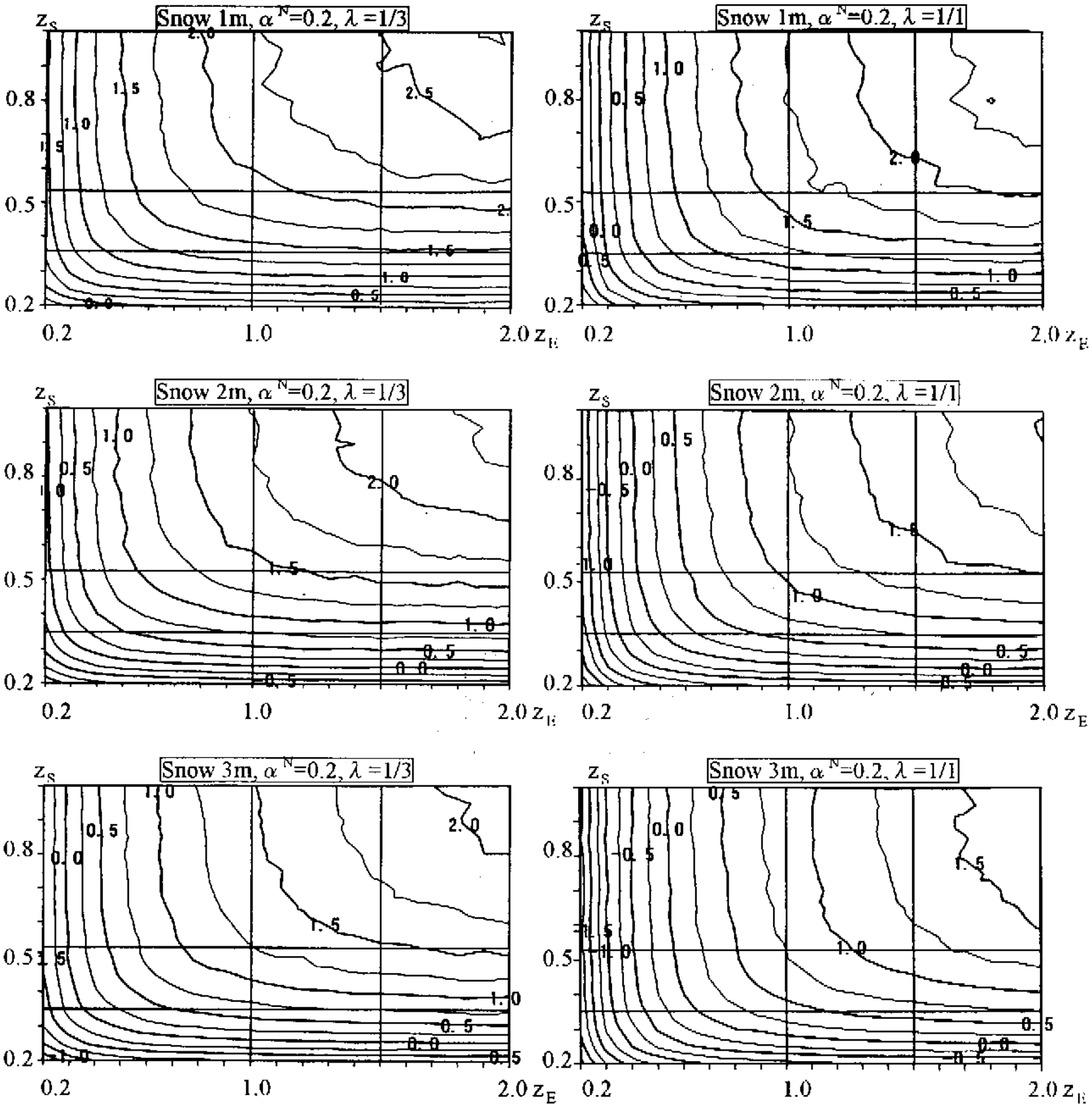


Figure 3: Contour lines of β in the plane of z_S and z_E obtained by simulation study

conducted for the structures in the three regions of which maximum snow heights are assigned as 1m, 2m, and 3m. Simulation results are shown by contour lines each of which gives the same reliability index of β in a z_S - z_E plane. Some of the results are shown in Figure 3. Configurations of contour lines in Figure 3 appear rather different from those of theoretical study in Figure 2 at first sight. The tendency of the contour lines, however, is similar. That is to say, these configurations indicate more clearly that the relation between z_S and z_E is inversely proportional, so it becomes evident that the contour line has its turning point from the region almost parallel to z_S axis to the region parallel to z_E. In this simulation study, considering the region of higher seismicity, a higher earthquake occurrence ratio, $\overline{\lambda}_E$ = 1/1, is added to the originally adopted value of 1/3 and the results for these tow values are shown in Figure 3. What is obvious on comparing these two groups, the group for $\overline{\lambda}_E$ =1/3 and the other for $\overline{\lambda}_E$ =1/1, is that counter line curves move laterally a little bit to the right as $\overline{\lambda}_E$ changes to the greater one. This indicates that the turning point moves to the right and reliability indexes for a certain z_E become smaller because of this movement. This drop in reliability is more severe when the smaller z_E is taken. As long as we take the value of z_E around 1.0, the turning point is still on the left of z_E and the tangents of counter lines are still gentle. The density of contour lines in the direction of z_S is, therefore, greater than that in the direction of z_E even though in the case of $\overline{\lambda}_E = 1/1$. We can examine easily how much the reliability index is improved by 1.5 times of current standard value of either z_S or z_E, 0.35 or 1.0 respectively, using the parallel lines drawn in Figure 3. From this examination, it becomes evident again that the reliability index of the structures in a heavy snow region can be improved by raising the value of the current z_S more easily than z_E.

6. CONCLUSIONS

1. The structural reliability of school gymnasiums under a load combination of Dead. Snow, and Earthquake is improved more easily by raising the design factor for Snow load than that for Earthquake load as far as the vicinity of the current design factors in Japan is concerned.
2. High seismicity region like earthquake occurrence ratio of 1/year make the sensitivity of Snow load to the reliability a little bit low, but the effect of Snow load on the reliability is still larger than Earthquake load.
3. Although the difference between results by theoretical studies and that by Monte Carlo simulations in the configuration of contour lines is rather great, the effect of z_S and z_E on the reliability is similar in both cases.

7. REFERENCES

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