

EFFECT OF THE AXIAL FORCE AND OF THE VERTICAL GROUND MOTION COMPONENT ON THE SEISMIC RESPONSE OF R/C FRAMES

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SUMMARY

In this paper the effect of the vertical motion on the response of R/C frames is investigated. An analytical model, which was developed with the possibility to consider the effects of uncoupled axial force variations, such as caused by the vertical motion, was used. The analysis concern a five-story frame subjected to different strong motion records. A comparison of the response with and without the vertical component was performed. The results confirmed that the vertical excitation could have a detrimental effects on the damage of R/C frames, and on the various aspects of its response, particularly as a consequence of a deterioration of the column behaviour.

INTRODUCTION

In the analysis and design of earthquake resistant structures, and particularly of R/C frames, the vertical ground motion component is frequently not considered, since it is retained that most of the damage is due to the horizontal component, and that the vertical acceleration is small. The current seismic codes recommend a vertical spectrum with values which vary from half to three quarters of that of the horizontal spectra. This approach seems to be unconservative in light of measurements of ground motion during recent earthquakes, which indicate that the vertical acceleration could reach values even higher than that of the horizontal acceleration. Also field observations proved that many buildings and bridges experienced significant damage attributable to high vertical forces. A wide compendium of field evidence supported by analytical considerations was presented by Papazoglou and Elnashai, [1996], which confirmed that certain failure modes are correlated with the high axial forces due to the vertical motion.

A great obstacle in the study of the seismic behaviour of structures under strong horizontal and vertical motion was the analytical difficulty which arises in the representation of all the effects of the varying axial forces induced by the vertical component. More refined model are required than with the horizontal motion alone, since with the vertical motion the interaction between the axial force and the flexural and shear behaviour of the structural elements becomes a fundamental aspect. This problem, although the progress made in the developing analytical models, and in the reducing the computing time, is still actual. Moreover the analysis of the behaviour of structures subjected to horizontal and vertical motion was considered only recently by many researchers. Among these it is possible to mention Saadehvaziri and Foutch, [1991]. They analysed, with inelastic plane stress elements, the response of bridge columns and piers subjected to vertical and horizontal excitation, and these columns resulted to show unstable hysteresis loops and little energy dissipation. In the case of R/C frames the study conducted by Ghobarah and Elnashai [1998] can be mentioned. They found a loss in the strength of the structures considered and a modification of their behaviour factor. The purpose of the present study is to give a further contribution to the analytical investigations on the behaviour of R/C frames in presence of vertical motion. It was performed in order to comprehend the influence of the vertical motion on the various aspect of the structural response, and also to distinguish the effects of strong motion records with different characteristics.

OBSERVATIONS ABOUT THE VERTICAL GROUND MOTION COMPONENT

Generally the vertical ground motion attenuates more rapidly than the horizontal motion, and the effects of the vertical component are more evident in the near field of an earthquake. In fact it was observed that the vertical to horizontal peak acceleration ratio $PVGA/PHGA$ tends to assume greater values in the near field and to decrease

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as the epicentral distance increases. Unfortunately most of the strong motion data are poor of near field records, especially the earlier data, and in the past years the vertical motion was difficult to estimate. Recently it was possible to have many records for several earthquakes, with more detailed information also in the near field, and more refined evaluation methods allowed to have better estimations of the vertical motion. Now it is evident that the earlier data conducted to an underestimation of the *PVGA/PHGA* ratio [Abramson and Litehiser, 1989], and that this ratio in the vicinity of moderate to strong earthquakes can also exceed unity.

Some records of the Imperial Valley earthquake of Oct. 15, 1979, showed peak vertical acceleration of 0.72g and values of the vertical to horizontal ratio equal to 1.54 at epicentral distance of 29 km. The maximum vertical values registered was 1.66g. The Northridge earthquake of Jan. 17, 1994, was the first earthquake where high vertical acceleration was recorded in a large number of stations, and where many buildings exhibited damage correlated the with vertical motion. Values of 1.18g of the vertical acceleration was registered, while the *PVGA/PHGA* reached values, in the near field, of 1.79. This ratio was equal to unity in stations located at epicentral distance of 20 km, where the vertical acceleration was 0.62g. The Kobe earthquake, of Jan. 17, 1995, was similar to the Northridge event for the vertical accelerations registered, and for the damage observed.

Another aspect that in the past years was not taken in consideration is the frequency content of the vertical motion, which was often noticed to be significantly higher than that of the horizontal motion. Therefore the vertical component may be more dangerous as it was retained, since it may be close to the vertical frequencies of free vibration of many structures. There are a lot of records of instrumented buildings, especially from the Northridge and the Kobe earthquakes, which show a great amplification of the vertical motion along the building high. For example in the Wilshire Building, the *PHGA* and the *PVGA* values recorded during the Northridge earthquake were 0.21g and 0.07g, while at the roof the peak accelerations became respectively 0.37g and 0.33g, showing an amplification factor of 4.7 in the vertical direction [Ghobarah and Elnashai, 1998].

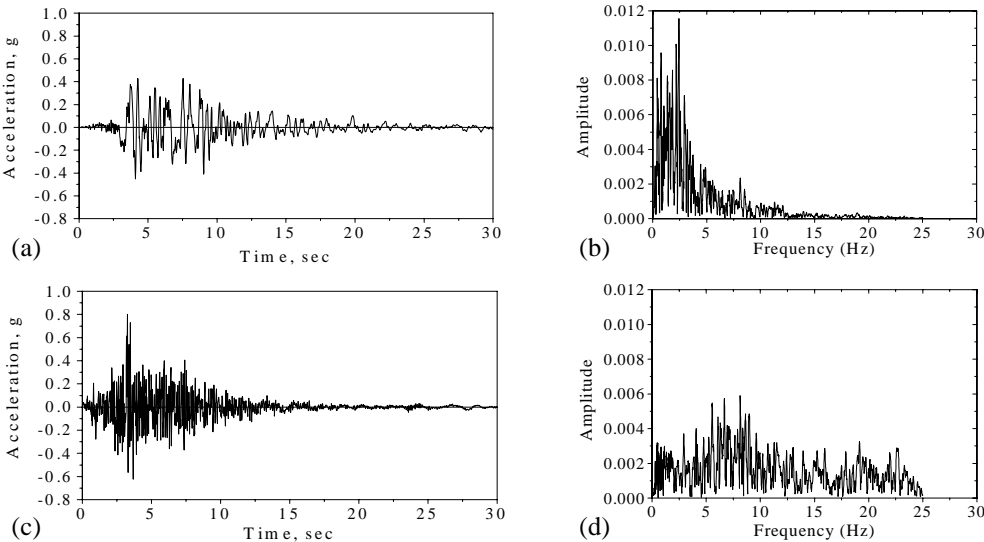


Figure 1: Records from the Northridge earthquake, Saticoy St. station. a) horizontal acceleration, b) FFT of the horizontal acceleration, c) vertical acceleration, d) FFT of the vertical acceleration.

With eigenvalue analysis it is possible to identify the first vertical periods of R/C frames, and to compare these periods with the horizontal ones. Considering moment resisting frames with a storeys number varying from 1 to 8, the horizontal period varies from about 0.1 to 0.8, the vertical period varies from about 0.04 to 0.12, and the ratio of vertical to horizontal period varies from 2.5 to 7 [Papazoglou and Elnashai, 1996]. This means that the vertical periods are not significantly influenced by building height and, as a consequence, that the vertical amplification is similar for buildings with different height. As pointed earlier, the frequency content of the vertical motion is often higher than that of the horizontal motion, and could include the frequency range, indicated above, of vertical vibration of buildings, thus causing great vertical amplification. As an example of the characteristics of the vertical motion in the near field, the horizontal and the vertical acceleration, recorded at the station of Saticoy Street during the Northridge event, are reported in Figure 1 with the corresponding frequency contents obtained with the fast Fourier transform.

The principal effect of the vertical motion is the generation in columns of fluctuating axial forces uncoupled from the lateral forces. These axial forces are added to the axial forces correlated with the over-turning moments and greater level of compression and tension could be reached in columns than with the horizontal motion alone. The vertical motion could have a detrimental effect on the structural behaviour of columns if it produces significant axial forces simultaneously with the horizontal components peak effects. In fact, in the non linear range, the flexural and shear behaviour of the structural elements is strongly affected by the axial force variations, and great importance assumes the axial force-moment interaction. Some effects of this interaction combined with the vertical motion may be the subsequent: significant fluctuation of stiffness and strength, increase in lateral shear and moment due to increase of compressive loads, reduction of shear capacity due to tensile loads, increase of ductility demand and possibility of pullout and buckling of reinforcing bars [Saadehvaziri and Foutch, 1991].

ANALYTICAL MODEL FOR THE SEISMIC BEHAVIOUR OF R/C FRAMES

One of the main problem associated with the analysis of R/C frames subjected to a vertical and horizontal seismic action is the representation of the effects of nonproportionally varying axial loads. The numerical modelling of the non linear behaviour of RC frames with many degree of freedom is commonly carried out by means of global models, based on the direct introduction of constitutive laws at the section level, rather than with fiber or more complex models. The global models, in fact, allows to obtain enough realistic results with a relative computation simplicity, but they present the disadvantage of neglecting the axial force influence. In this study a particular global model developed by the authors is used [Diotallevi and Landi, 1999], which has the capability to include the axial force-moment interaction also in the case of uncoupled variation of axial force. The accuracy of the model was verified with the simulation of available experimental data regarding a scale model of a frame tested on a shaking table.

In the numerical model a structural member, like a beam or a column, could be schematised with just a single element. Since the inelastic deformations, under seismic actions, are typically located in element end zones, each element is subdivided into three zones of variable length: an elastic central zone, and two inelastic zones at the ends. The stiffness properties of the non linear zones are obtained from the study of the cyclic behaviour of the extreme sections, which is carried out through an hysteretic model for the moment-curvature relationship. The length of the plastic zones are updated, at each loading increment applied to the structure, on the basis of the current value of the yielding moment, M_y , which is a function of the axial load. The element model could include at the element ends also two rigid links, if the joints are assumed to be rigid, and two inelastic springs between the links and the plastic zones, if the fixed-end rotations due to bond-slip of the anchored bars in joints are considered. The second order effects, which can be non negligible in the presence of high vertical and horizontal forces, are taken into account through the usual $P-\Delta$ method. The moment-curvature law adopted for the sections is based on a bilinear monotonic diagram with strain hardening. The cyclic behaviour of the sections, for a constant axial force, is studied with an hysteretic model similar to that proposed by Chung et al. [Chung, Meyer and Shinozuka, 1989]. This model is able to account for typical aspects of the seismic response of R/C elements such as stiffness degradation, pinching of the loops, and strength deterioration.

The analysis method is extended to the case of a changing axial force starting from the assumption that axial force variations produce shifts between moment-curvature diagrams corresponding to different levels of axial force. In the case of cyclic loading the individuation of a series of moment-curvature diagrams for different values of axial force N is made through some assumptions (Fig. 2a). The unloading branches for different values of N are determined assuming that they start from points with the same curvature, and that they are all directed to the same point, (S_I), on the curvature axis. The subsequent reloading branches start from the same point where the preceding unloading branches end, and they are directed to points with a curvature equal to the maximum curvature previously reached. The determination of the shifts between different branches is performed through an iterative procedure (Fig. 2b), since the flexural stiffness, with a varying axial force, depends upon the ratio $\Delta N/\Delta\phi$, between the axial force increment and the curvature increment. At each loading increment initial values of the stiffness of sections are fixed, then the values of ΔM , ΔN and $\Delta\phi$ are calculated. Hence it is possible to individuate the corresponding point at the end of the iteration, P_1 , and to evaluate the unbalance moment ΔM_{u1} , which arises if P_1 is not located on the branch relative to $N+\Delta N$. If the unbalance moments of all sections are not enough small, new stiffness values are calculated considering in each section the point P_1' on the branch corresponding to $N+\Delta N$, and another iteration is performed.

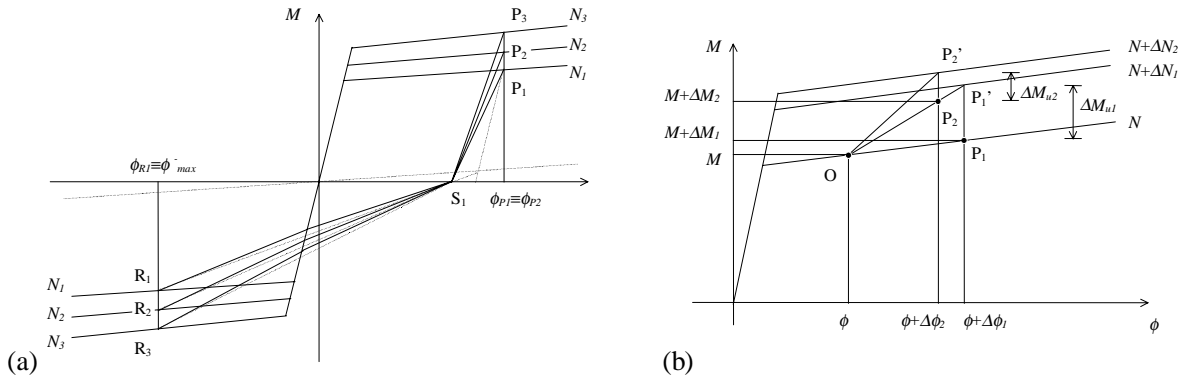


Figure 2: Axial force-moment interaction model. a) unloading and reloading branches with a varying axial force, b) iteration procedure for the determination of the shifts.

ANALYTICAL INVESTIGATION ON THE EFFECTS OF THE VERTICAL MOTION

The effects of the vertical motion were studied, with the model above described, considering a five storey, three bay moment resisting R/C frame, which represents a recurrent building structure. The dimensions of the frame, the cross sections and the material properties are reported in Figure 3. The structure was designed with aseismic criteria, and it is characterised by relatively strong columns. The assumed total mass per floor is equal to $53 \cdot 10^3$ kg, and this mass is associated to a gravity load of 35.3 kN/m applied on beams. The first horizontal and vertical periods were computed, before the seismic analysis, considering the stiffness of the elements in the cracked flexural condition, and their values are respectively 0.9 second and 0.074 second, corresponding to frequencies of 1.1 Hz and of 13.5 Hz. The effects of the bond-slip of the bars in joints are assumed to be not prominent, and they are not considered in the present analysis. The behaviour of the frame was studied considering various ground motion records, and performing for each record two analysis, the first under the horizontal component alone, the second under the horizontal and the vertical component.

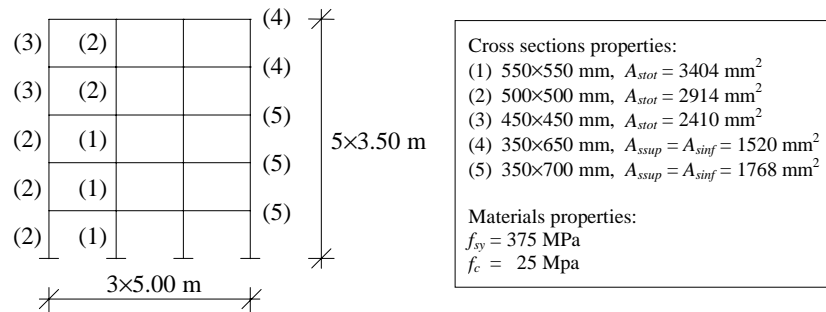


Figure 3: Characteristics of the R/C frame.

The selected strong motion data consists of five records from the Northridge, the San Fernando and the Imperial Valley events. The parameters of the records are shown in Table 1, where also the obtained amplification factors are reported. These records present different characteristics in relation to the damage capability of the vertical component. The record no. 1, whose components are also shown in Figure 1, has high horizontal and vertical peak values which occur at the same time, and its vertical component has a high frequency content. The record no. 5 has properties similar to that of the no. 1, except for the non contemporaneity of the peak values. The other records show lower peak vertical values, and among them the no. 3, and the no. 4 present a vertical motion with a lower frequency content then the other records, even if it is higher then that of the horizontal motion. The values of the vertical amplification factors for the records no. 1 and no. 5 are very high, and they are greater then in the horizontal direction. Therefore with these records the effects of the vertical motion are expected to be greater.

As it was previously noticed, the main effect of the vertical motion is the generation of axial forces, which are uncoupled to that due to lateral forces, and which have a lower vibration period, corresponding to the vertical period of the structure. In the exterior columns, the only where the axial forces due to the overturning moments are significant, the contribution of the vertical motion to the total axial force can be comparable, in the ground columns, to that of the horizontal motion. This can be observed for the record no. 1 in Table 2, where the

extreme values of axial forces in the left exterior columns are reported. In the upper floors the contribution of the vertical motion increases and it reaches, at the roof, the 76 % of the total axial force. From Table 2 it results that not only greater values of compression are obtained, but also significant tensile axial forces, indicated with the negative sign. In the interior columns the axial force variations, whose amplitude can be very high, are induced practically by the only vertical component. The great values of compressive and tensile axial forces can lead also to the possibility of direct failure in compression or tension. In the Table 2 the effects of the different records are evident, and the contribution of the vertical motion with the records no. 3, and no. 4 is not considerable as with the other records.

Table 1: Parameters of the selected records and amplification factors.

No.	Record	Epic. (km)	PGA		V/H	Peak Roof A		Amplification	
			H (G)	V (G)		H (G)	V (G)	H	V
1	Northridge: Saticoy St.	2.2	0.45	0.80	1.77	1.14	3.02	2.53	3.77
2	Northridge: ColdWater Can. Av.	12.1	0.30	0.26	0.86	0.66	0.63	2.20	2.42
3	Northridge: W. Lost Can. Rd.	25.7	0.45	0.28	0.62	1.22	0.41	2.71	1.46
4	San Fernando: Castaic OldRidge	28.8	0.30	0.15	0.50	0.77	0.30	2.56	2.00
5	Imperial Valley: Diff. Array	28.7	0.48	0.46	0.95	0.87	1.57	1.81	3.41

Table 2: Minimum and maximum axial forces in ground and roof left external columns.

Record No.	Ground external column : axial forces (kN), $N_0=500$				Roof external column : axial forces (kN), $N_0=87$				Contribution of vertical motion to total axial force (%)	
	H		H+V		H		H+V		Ground	Roof
	Nmin	NMAX	Nmin	Nmax	Nmin	NMAX	Nmin	Nmax		
1	-73	1173	-758	1896	31	131	-127	291	53	76
2	-19	1049	-124	1097	40	129	19	164	13	39
3	-178	1191	-192	1208	21	145	6	148	2	13
4	-63	1922	-112	942	15	136	9	145	7	11
5	-61	1143	-304	1367	28	148	-9	238	28	64

In the Figures 4 to 7 a series of diagrams are reported, which regard important aspects of the structural response, as the displacement, the plastic zones, the plastic deformations, the energy dissipation, and the maximum moment and shear values. They show the results obtained for each record in both cases, with and without the vertical motion. Also from these diagrams it appears that the detrimental influence of the vertical motion is significant with the records no. 1, and no. 5. Among them, the latter causes slightly lower effects due to a lower peak vertical acceleration and to the non contemporaneity with the peak horizontal one.

The maximum roof displacement (Fig. 4a) is resulted to increase of a non negligible amount with the vertical motion, due to the more damage induced in the structure, and probably also to the $P-\Delta$ effects in presence of high compression forces. The maximum interstory drift (Fig. 4b), which tends also to increase, exhibits for the records no. 1 a different trend then the other aspects, but it has to be considered that the vertical action causes greater drift in all upper storeys. In the analytical model description it was pointed that the model takes account for the gradual diffusion of plasticity from the end sections of the elements. The Figure 4c reports the average values of the extension of the plastic zones. The average, as in the following diagrams, is evaluated on all the end sections, and not only on the plasticized sections. This is made in order to give a more realistic measure of the damage sustained by the structure. The remarkable increment of the plastic zones length, which can arise with the vertical motion, is due to a greater extension of the plastic zones both in columns and beams, and to the generation of new plastic zones in columns, caused by axial force fluctuation especially in interior and upper columns.

The estimation of the plastic deformations of the sections is performed through the index $PD=\max(\phi_p/\phi_{pf})$, equal to the maximum value, attained during the section loading history, by the ratio between the current plastic curvature ϕ_p and the failure plastic curvature ϕ_{pf} , where the latter is calculated as a function of the axial load. The mean values of this index are calculated for the column sections (Fig. 5a), for the beams sections (Fig. 5b), and for all the end sections of the frame (Fig. 5c).

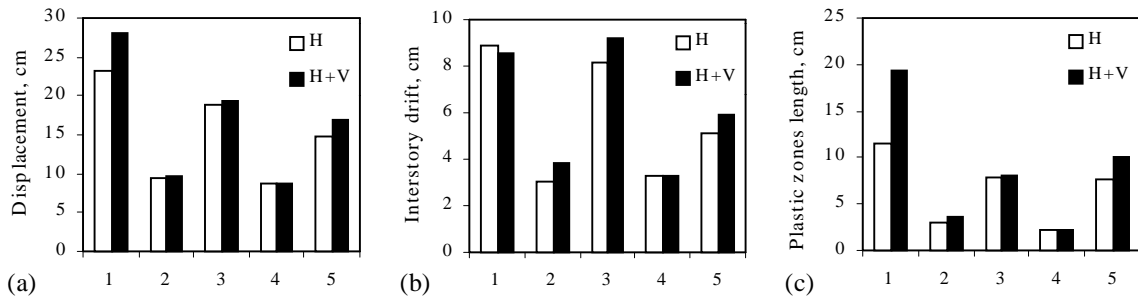


Figure 4: a) maximum displacement, b) maximum interstory drift, c) mean length of the plastic zones.

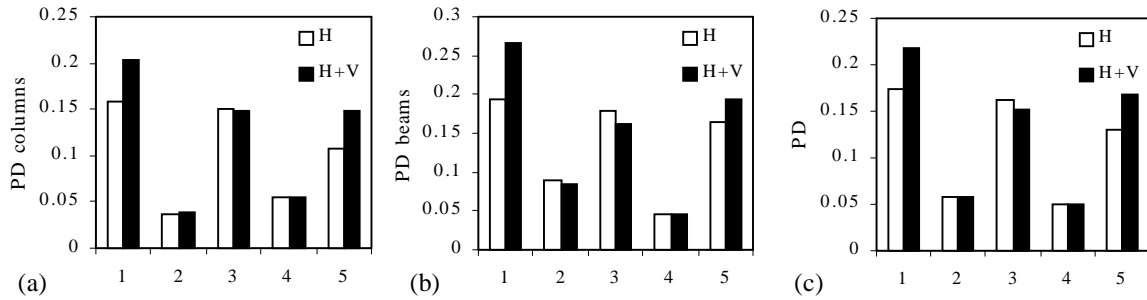


Figure 5: Plastic deformations. a) mean value of PD in columns, b) mean value of PD in beams, c) mean value of PD in all the structure.

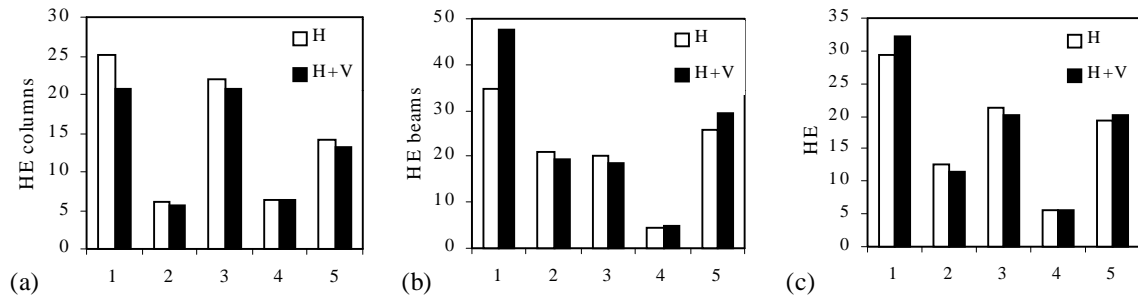


Figure 6: Hysteretic energy. a) mean value of HE in columns, b) mean value of HE in beams, c) mean value of HE in all the structure.

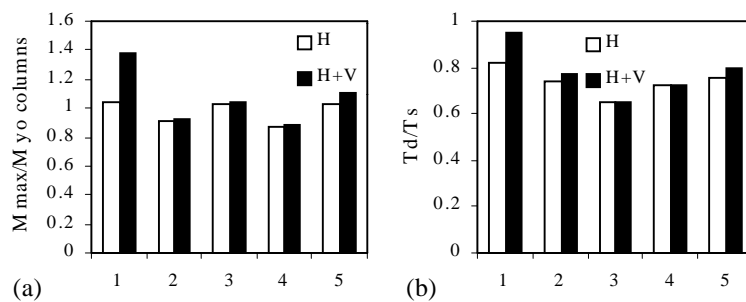


Figure 7: a) mean value of M_{max}/M_{y0} in columns, b) maximum value of T_d/T_s .

Considerable increase are obtained, with the vertical motion, both for columns and beams, and as a consequence, for the structure. But between beams and columns there are differences in the causes of the increase of the mean values. In the columns there are various causes, such as the greater number of plasticized sections, and the increment of the maximum PD values of all columns. This increment is correlated with the drop in the ductility supply induced by the high axial compression, and it can be considerable. For example with the record no. 1 the PD index reaches, in the left exterior ground columns, a value near to unity, indicating an imminent collapse of the section. In the beams the maximum PD values do not increase, and the only cause of the greater mean value is the increase of the curvatures reached in sections where the plasticization with the horizontal motion are small. This increase is particularly evident in upper beams.

The energy dissipated by the hysteresis loops of the plasticized sections are reported in a normalized form through the ratio $HE=E/(M_{y0}\phi_{y0})$, between the energy E and the product $M_{y0}\phi_{y0}$ of the yielding moment with the yielding curvature corresponding to the initial axial force N_0 . Also for the energy index, mean values for the columns (Fig. 6a), for the beams (Fig. 6b), and for the total structure (fig. 6c) are reported. It is possible to notice that the mean value in columns decreases with the vertical motion, while the mean value in beams increases. The reason of the reduction of the energy dissipation in columns could be searched in the strange and irregular shape of their moment-curvature hysteresis loops. In the beams the increase of the dissipated energy can be explained with the greater amplitude, in terms of curvature, of many moment-curvature loops in the sections of upper beams. The mean value of HE in all the structure shows little increase, and the reduction of column energy is compensated by the increase of the beam energy. The damage evaluation of the structure was not performed with a functional which combines the methods based on energy and ductility, since in this way it was possible to distinguish the behaviour of the single index. Moreover these functionals was developed for elements with constant axial force, and their application to elements subjected to uncoupled lateral and axial force, which have irregular hysteresis loops, should still be studied.

Since the axial force variations affect the strength of the elements, the columns, with the greater axial compression caused by the vertical motion, tends to reach greater values of the moment in the plastic range. This increment of the maximum moment can be very high in the interior and in the upper columns. In the Figure 7a the mean values in columns of the ratio M_{max}/M_{y0} between the maximum moment reached and the initial yielding moment are reported. The greater values of the maximum moment are also associated with a greater shear demand. Although the shear strength of the columns increases with the axial force, the ratio T_d/T_s between the shear demand and the shear supply can significantly increase. Also the greater tensile axial forces can have a negative effect on the T_d/T_s ratio, since they cause a loss of the shear strength. The maximum values of the T_d/T_s ratio attained in columns are reported in Figure 7b. The shear strength was evaluated with a method reported by Priestley et al. [Priestley, Verma and Xiao, 1995], which is able to consider, among various effects, also the axial force influence.

In order to observe directly some of the mentioned effects of the vertical motion, the axial force time histories of the right ground columns and the moment-curvature diagrams of the right exterior ground column, obtained with the record no. 1, are shown, respectively, in the Figures 8 and 9. In the Figure 8a, relative to the exterior ground column, it is evident the superposition of the axial force variations induced by the vertical motion with that due to the horizontal motion, which has a lower vibration period. In the Figure 8b, relative to the interior ground column, it appears how minimal is the axial force variations correlated with the overturning moments in the interior columns, and how great could be there the extreme values reached by the axial force with the vertical motion. The maximum compression axial force becomes, in fact, 3.4 times the initial axial force due to gravity loads, while the maximum tension axial force becomes 1.25 times the initial axial force.

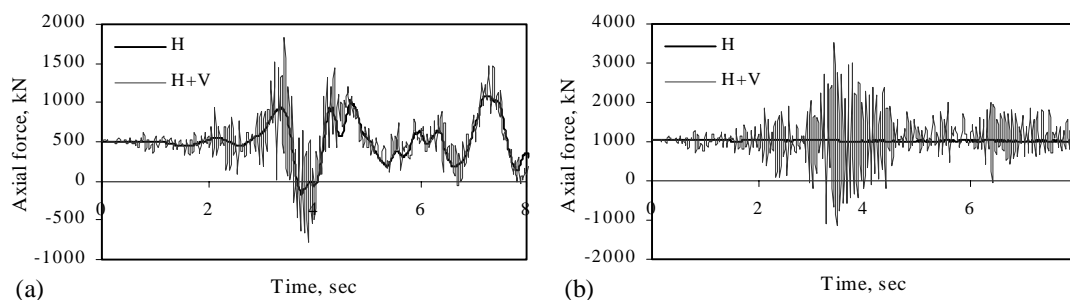


Figure 8: Axial forces obtained with the record no. 1 for the right ground columns. a) exterior column, b) interior column.

The moment-curvature diagram of the exterior ground column, obtained without the vertical motion (Fig. 9a), shows the characteristics of the column behaviour under coupled axial and lateral force variation, as, for example, the strength asymmetry. The diagram obtained with the vertical motion reveals (Fig. 9b), as it was previously pointed, an unusual and irregular shape with significant fluctuations in strength and stiffness, which reflect, through the axial force-moment interaction, the effects of the vertical vibration. The maximum curvatures reached by the loops in figure 9b are about half of those reached in Figure 9a, and this indicates the lower dissipation capacity of the column considered with the vertical motion. However this is not the only reason of the lower dissipated energy in columns, since also in columns which experience greater curvatures the hysteresis energy resulted lower, and probably another important cause is the irregular shape of the loops. The decrease of the maximum curvature does not lead to a decrease of the PD index, but, on the contrary, to an increase from

about 0.55 to 0.6, because of the loss of ductility with high axial forces. It is also evident in the Figure 9 the increase of the maximum moment, which is equal to about the 30% of the initial yielding moment.

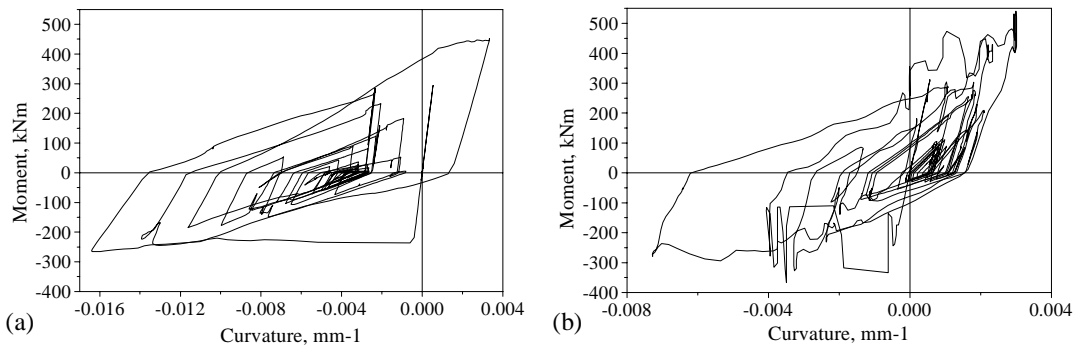


Figure 9: Moment-curvature diagrams obtained with the record no. 1 for the right exterior ground column. a) with horizontal motion, b) with horizontal and vertical motion

CONCLUSIONS

The effects of the vertical motion on the seismic response of a R/C frame were studied with an analytical model, which was developed in order to take account for the axial force-moment interaction also in the case of nonproportionally varying axial loads. Among the several records used in the investigation, the most detrimental was resulted to present a great peak vertical acceleration, a high frequency content of the vertical motion, and a simultaneous occurrence of the peak vertical and horizontal effects. The significant axial force fluctuations due to the vertical motion affected the columns behaviour and as a consequence the global structural response. With the vertical motion a greater roof displacement and a greater diffusion of damage in the frame were observed. The number of the plasticized regions increased in the columns, and their extension increased in all elements. A reduction in the ductility caused by the high axial compression, and a reduction in the energy dissipation due to the irregular shape of the moment-curvature loops was noticed in the columns. On the contrary the beams experienced a greater energy dissipation. The high axial forces induced in columns determined a noticeable increase of the maximum moment and shear values. Both the greater shear values and the greater tensile axial loads caused an approach to the shear failure in columns. The results of this investigation showed the importance in the seismic analysis of the vertical motion, and suggest to improve the study of the vertical motion characteristics and of the structural behaviour under this type of action.

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