1010

IDENTIFICATION OF STRUCTURAL DEGRADATION BY TIME-FREQUENCY SYSTEM ANALYSIS

Arata MASUDA[1] And Akira SONE[2]

SUMMARY

The purpose of this paper is to develop an identification method for time-varying transfer functions which represent instantaneous frequency response of linear time-varying systems, and to show its applicability to structural damage detection. We propose a three-step identification procedure including optimal time-frequency smoothing estimators which estimate nonstationary cross-spectra from observed data. A small-scale experiment and real strong motion records are investigated to show that the proposed identification procedure is applicable to the structural damage detection in the sense that it can track the structural changes in the time-frequency domain.

INTRODUCTION

Structures damaged by strong earthquakes have changes in their physical properties. Many researchers have studied on structural damage detection or health monitoring applying the system identification concepts. In particular, identification methods based on time-varying system representations have been receiving increasing attention because we can 'track' the structural degradation using these methods.

While most of the resent efforts in the structural engineering field are based on parametric time-varying models, many studies on the nonparametric representations of nonstationary signals or systems have been done over the past decades in the signal processing field. Recently, we have been working on the nonparametric time-frequency representation of time-varying systems and its application to the structural damage detection [Masuda et al. 1998, 1999]. In these literature, structures are modelled by linear time-varying (LTV) systems and their changing frequency response are characterized by Zadeh's time-varying transfer functions (TVTF) [Zadeh, 1950].

In this paper, we extend our work more practically. First, we reduce the TVTF identification problem into two nonstationary spectral estimation problems as described in [Masuda et al. 1998, 1999]. Then, an adaptive time-varying spectral estimator is newly developed which has an optimal time-frequency smoothing kernel. Finally, a small-scale experiment and real strong motion records are investigated to demonstrate the efficiency of the proposed method.

TIME-VARYING SYSTEM MODEL

Structures damaged by strong earthquakes have slow or abrupt changes in their physical properties such as stiffness. Linear time-varying (LTV) systems excited by nonstationary random processes provide adequate model of such degrading structures. The input-output relation of the model is given by

$$y(t) = (Hu)(t) + v(t) \tag{1}$$

[1] *Department of Mechanical and System Engineering, Kyoto Institute of Technology, Kyoto, Japan Email: masuda@ipc.kit.ac.jp*
[2] *Department of Mechanical and System Engineering, Kyoto Institute of Technology, Kyoto, Japan Email: sone@ipc.kit.ac.jp*

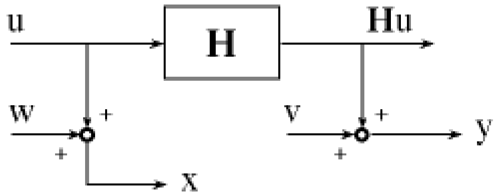


Figure 1: Unknown LTV system H excited by nonstationary random process u, and input and output observations x and y corrupted by white noise processes w and v

$$x(t) = u(t) + w(t) \tag{2}$$

where H is the unknown LTV system, $u(t)$ is the input, $x(t)$ and $y(t)$ denote the input and output observations corrupted by the zero-mean Gaussian stationary white noise processes $v(t)$ and $w(t)$ with their variance σ_v^2 and σ_w^2, respectively (see Figure 1). It is assumed that $v(t)$ and $w(t)$ are independent of each other and independent of $u(t)$.

The nonparametric representation of the LTV system H in the time-domain is given in the form of an integral operator as

$$(Hu)(t) = \int_s h(t,s)u(s)ds = \int_\tau h_0(t,\tau)u(t-\tau)d\tau \tag{3}$$

where $h(t,s)$ is the kernel function corresponds to the impulse response of the system and $h_0(t,\tau) := h(t,t-\tau)$. All integrals go from $-\infty$ to ∞ unless otherwise specified.

The nonparametric representation of the LTV system in the time-frequency domain is given by Zadeh's time-varying transfer function (TVTF):

$$Z_H(t,\omega) := \int_\tau h_0(t,\tau)e^{-i\omega\tau}d\tau \tag{4}$$

which represents the instantaneous frequency response of the system. Clearly, The TVTF reduces to the ordinary transfer function in the case of linear time-invariant (LTI) systems.

IDENTIFICATION

Three-Step Identification Procedure

We reduce the present identification problem into nonstationary spectral estimation problems using the three-step identification procedure [Masuda et al. 1998, 1999]. This subsection provides a brief summary of that.

First, we introduce the *decorrelation* of x defined by

$$z(t) = (R_u^{-1}x)(t) \tag{5}$$

where R_u^{-1} denotes the inverse of the LTV system R_u whose impulse response is given by the correlation function of input u. Then the following equation holds:

$$Z_H(t,\omega) = NS_{y,z}(t,\omega) \tag{6}$$

where $NS_{y,z}$ denotes the nonstationary cross-spectrum between y and z defined as

$$NS_{y,z}(t,\omega) := \int_\tau E\left\{y(t)\overline{z(t-\tau)}\right\}e^{-i\omega\tau}d\tau \tag{7}$$

where $E\{\cdot\}$ denotes the expectation operator.

Since it is difficult to make a direct evaluation of Eq. (5) except for some special cases, the following approximation can be used:

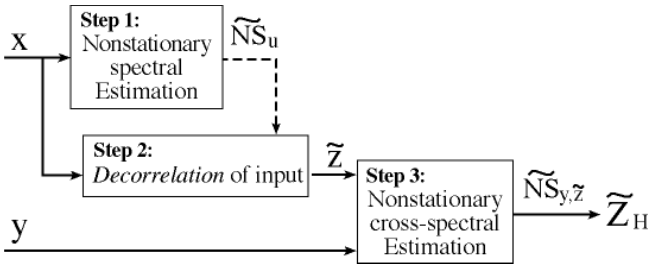


Figure 2: Three-step identification procedure

$$\tilde{z}(t) = \frac{1}{2\pi}\int_\tau\int_\omega \frac{1}{NS_x(t,\omega) - \sigma_w^2} x(t-\tau)e^{i\omega\tau}d\omega d\tau \tag{8}$$

where NS_x denotes the nonstationary spectrum of x defined as

$$NS_x(t,\omega) := \int_\tau E\left\{x(t)\overline{x(t-\tau)}\right\}e^{-i\omega\tau}d\tau \tag{9}$$

The results of Eqs (5) - (9) can be summarized in the following three-step identification procedure:

Step 1 Estimate $\widetilde{NS}_x$, the nonstationary spectrum of input.

Step 2 *Decorrelate* x by Eq. (8) to obtain $\tilde{z}$.

Step 3 Estimate $\widetilde{NS}_{y,\tilde{z}}$, the nonstationary cross-spectrum between y and $\tilde{z}$. Then $\tilde{Z}_H := \widetilde{NS}_{y,\tilde{z}}$ is the estimator of TVTF.

Figure 2 shows a schematic illustration of the procedure.

If the input is almost stationary, the following estimator can be used instead of the above procedure [Masuda et al. 1999]:

$$\tilde{Z}_H(t,\omega) := \frac{\widetilde{NS}_{y,x}(t,\omega)}{\tilde{S}_x(\omega) - \sigma_w^2} \tag{10}$$

where $\tilde{S}_x(\omega)$ is an estimation of the power spectrum of x.

Nonstationary Spectral Estimation by Time-frequency Smoother

As mentioned previously, two nonstationary spectral estimators of $\widetilde{NS}_x$ and $\widetilde{NS}_{y,\tilde{z}}$ are needed to complete the TVTF identification. In this and the next subsections, we develop an estimator for an arbitrary nonstationary cross-spectrum NS_{x_1,x_2}.

Since the nonstationary cross-spectrum NS_{x_1,x_2} is an expectation of the Rihaczek distribution RD_{x_1,x_2}, i.e.,

$$RD_{x_1,x_2}(t,\omega) := \int_\tau x_1(t)\overline{x_2(t-\tau)}e^{-i\omega\tau}d\tau \tag{11}$$

the following time-frequency smoothing estimator is reasonable:

$$\widetilde{NS}_{x_1,x_2}(t,\omega;\Phi^{(t)}) = \frac{1}{2\pi}\int_{t'}\int_{\omega'} RD_{x_1,x_2}(t',\omega')\Phi^{(t)}(t-t',\omega-\omega')d\omega' dt' \tag{12}$$

where $\Phi^{(t)}(t,\omega)$ is a time-varying smoothing kernel. The above estimator takes a local average over an appropriate region defined by $\Phi^{(t)}(t,\omega)$ instead of the ensemble average at the each point of (t,ω). The kernel function $\Phi^{(t)}(t,\omega)$ should be designed appropriately dependent on the local nonstationarity of processes.

Adaptive Smoothing Kernel

We introduce a window function $g(t)$ to determine the appropriate time-varying smoothing kernel $\Phi^{(t)}$. Rewriting the smoothing kernel as $\Phi^{(t)}(t,\omega) = g(-t)\Psi^{(t)}(t,\omega)$ and substituting it into Eq. (12) leads to

$$\widetilde{NS}_{x_1,x_2}(t,\omega;\psi^{(t)}) = \frac{1}{2\pi}\int_\nu\int_\tau A^g_{x_1,x_2}(t,\nu,\tau)\psi^{(t)}(\nu,\tau)e^{j(\nu t-\omega\tau)}d\tau d\nu \quad (13)$$

where $\psi^{(t)}(\nu,\tau) := F_{t\to\nu}F^{-1}_{\omega\to\tau}\left\{\Psi^{(t)}(t,\omega)\right\}$ and $A^g_{x_1,x_2}(t,\nu,\tau)$ is the short-time ambiguity function given by

$$A^g_{x_1,x_2}(t,\nu,\tau) := STFT^g_{t'\to(t,\nu)}\left\{x_1(t')\overline{x_2(t'-\tau)}\right\} \quad (14)$$

where $STFT^g$ denotes the short-time Fourier transform with the window function g. The following equation is also derived from the definition of the nonstationary spectrum:

$$NS_{x_1,x_2}(t,\omega;\psi^{(t)}) = \frac{1}{2\pi}\int_\nu\int_\tau EA^g_{x_1,x_2}(t,\nu,\tau)e^{j(\nu t-\omega\tau)}d\tau d\nu \quad (15)$$

where $EA^g_{x_1,x_2}(t,\nu,\tau)$ is the expected short-time ambiguity function given by

$$EA^g_{x_1,x_2}(t,\nu,\tau) := STFT^g_{t'\to(t,\nu)}\left\{E\left\{x_1(t')\overline{x_2(t'-\tau)}\right\}\right\}. \quad (16)$$

Comparing Eq. (13) with Eq. (15), we define the optimal smoothing kernel $\psi^{(t)}_{opt}$ as

$$\psi^{(t)}_{opt}(\nu,\tau) = \arg\inf_{\psi(\nu,\tau)} E\left\{\left|A^g_{x_1,x_2}(t,\nu,\tau)\psi(\nu,\tau) - EA^g_{x_1,x_2}(t,\nu,\tau)\right|^2\right\} \quad (17)$$

By the orthogonality principle, we have the solution as

$$\psi^{(t)}_{opt}(\nu,\tau) = \frac{\left|EA^g_{x_1,x_2}(t,\nu,\tau)\right|^2}{E\left\{\left|A^g_{x_1,x_2}(t,\nu,\tau)\right|^2\right\}} \quad (18)$$

and the optimal spectral estimator as $\widetilde{NS}_{x_1,x_2}(t,\omega;\psi^{(t)}_{opt})$.

In many practical problems, however, since it is almost impossible to know $EA^g_{x_1,x_2}$ and $E\left\{\left|A^g_{x_1,x_2}\right|^2\right\}$ *a priori*, we have to estimate these statistics from the observed realization. Further details can be seen in [Masuda, 1999].

EXAMPLES

Small-scale cantilever structure

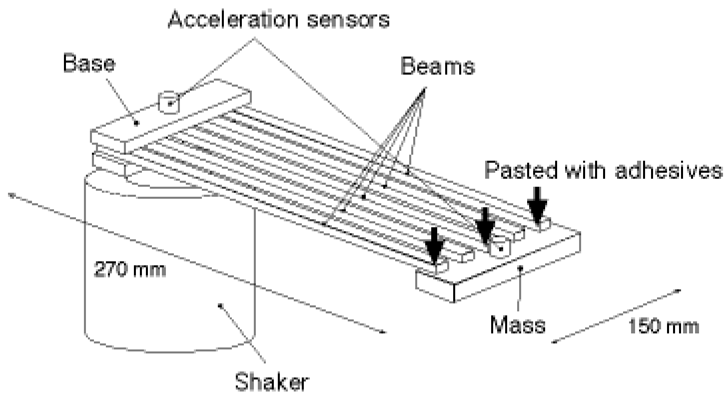


Figure 3: Small-scale cantilever structure

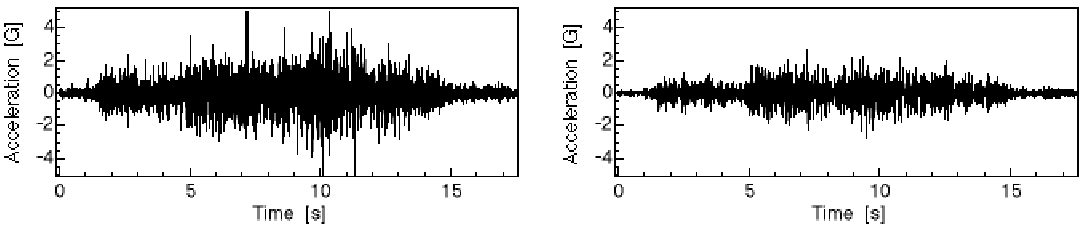
Figure 3 shows a small-scale cantilever structure consists of five parallel beams, a base and a mass. One end of each beam is fixed on the base mounted on a shaker. The other ends of two beams are fixed on the mass while those of remaining three beams are 'pasted' on the mass with adhesives. If the response of the structure becomes larger, the connecting joints between the beams and the mass will break one by one. Thus this is a small-scale model of a structure with welded joints which degrade under strong excitations.

First, we drived the shaker with small amplitude random noise to identify the frequency response of the structure under stationary conditions. The resulting fundamental frequencies are shown in Table 1.

Table 1: Identified natural frequencies under stationary conditions

Number of joints	First-mode [Hz]	Second-mode [Hz]
5	12.5	60.9
4	11.1	70.0
3	9.88	70.1
2	8.38	70.2

Then we drived the shaker with large amplitude nonstationary random noise and observed the acceleration responses at the base (input) and the mass (output). Before the excitation, there existed five joints between the beams and the mass. Then the number of 'healthy' joints reduced to two after the excitation.



(a) Input acceleration **(b) Output acceleration**

Figure 4: Observatioin data

The observed signals through the excitation are shown in Figure 4. and the waterfall plot of the identified TVTF is shown in Figure 5. From Figure 5, it seems that the system changes three times, i.e., at 5s, 10s and 12s. To confirm this impression, the time history of the identified fundamental frequency (which is obtained by the -π/2 phase contour lines of the identified TVTF) is plotted in Figure 6. The identified frequency agrees well with the fundamental frequency listed in Table 1.

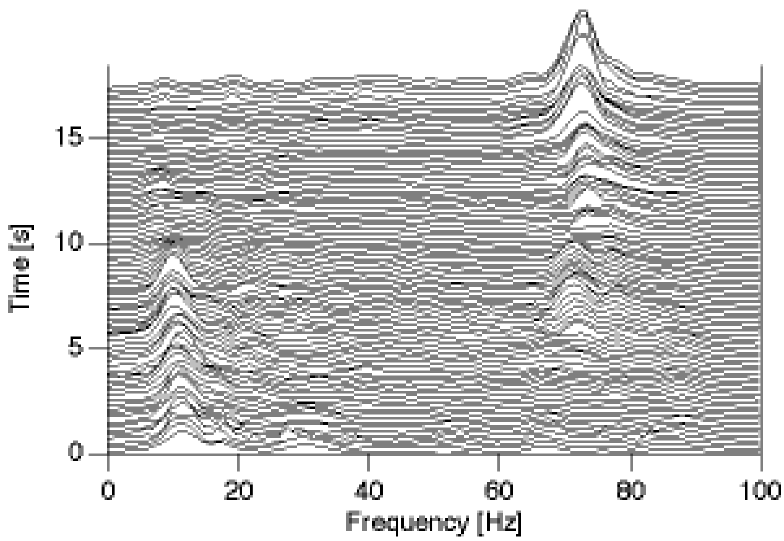


Figure 5: Identified TVTF for degrading cantilever structure

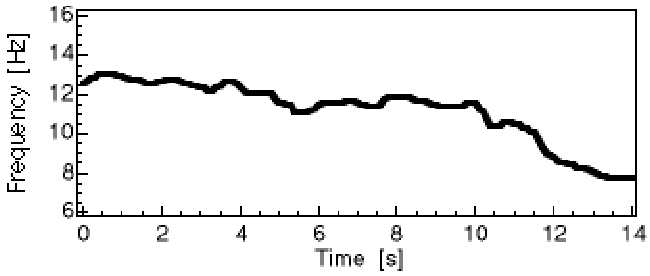


Figure 6: Identified TVTF for degrading cantilever structure

Strong motion records for a seven-storey building

Strong motion records collected by CDMG for a seven-storey building during the Northridge earthquake of January 17, 1994 are investigated. It has been reported that this building had severe damage to its concrete-frame columns at the base and mid-elevation [Aurelius, 1994]. Kunnath [1997] pointed out that the fundamental frequency of this building had changed to 0.42 Hz which were obtained through an ambient vibration test after the earthquake. Figure 7 shows the sensor location and observed responses.

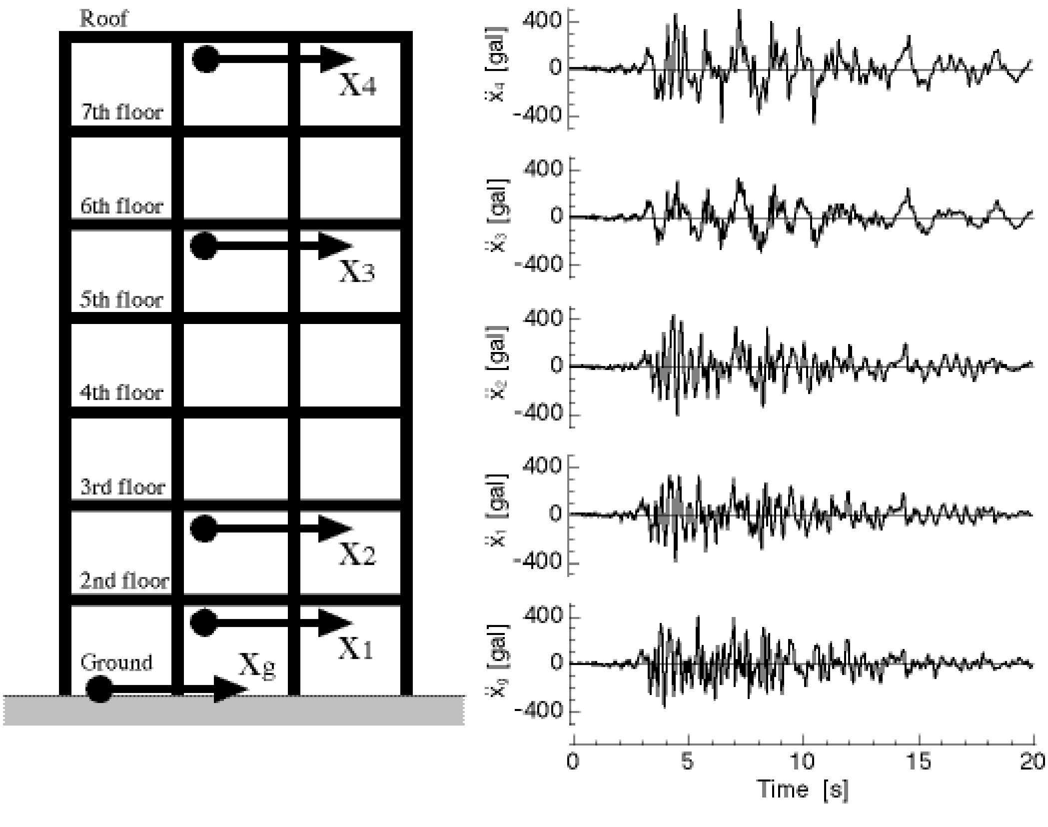
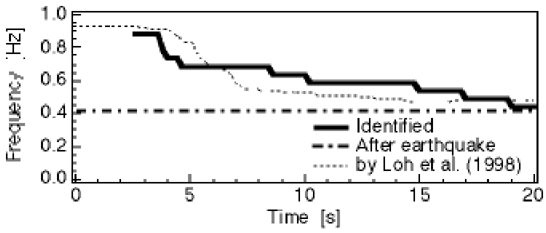
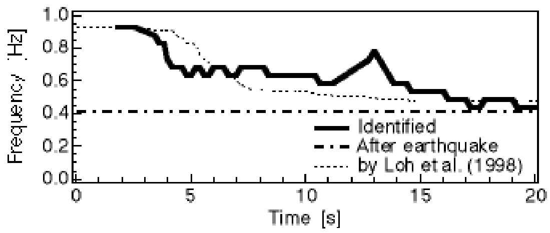


Figure 7: Identified TVTF for degrading cantilever structure

The time history of the identified fundamental frequency is shown in Figure 8 with solid line. The broken line shows the fundamental frequency measured after the earthquake and the dotted line shows the result by Loh et al. [1998] from the same data using the adaptive fading Kalman filter. The identified frequency asymptotically shifts toward the measured frequency and its time history looks reasonable comparing with the result by Loh et al.



(a) TVTF from $\ddot{x}_g$ to $\ddot{x}_3$



(b) TVTF from $\ddot{x}_g$ to $\ddot{x}_4$

Figure 8: Identified natural frequency and experimentally obtained natural frequency after the earthquake

CONCLUSIONS

This paper is concerned with the linear time-varying (LTV) system identification and its application to the structural damage detection. Nonparametric time-frequency representation of LTV systems, i.e., the time-varying transfer function (TVTF) is introduced and its identification problem is described.

We have developed the three-step identification procedure which contains optimal time-frequency smoothing estimators which estimate nonstationary cross-spectra from the observed data. A small-scale experiment and real strong motion records have been investigated to show that the proposed identification procedure is applicable to the structural damage detection in the sense that it can track the structural changes in the time-frequency domain.

REFERENCES

Aurelius, E. ed. (1994), An EQE Summary Report: The January 17, 1994 Northridge, CA Earthquake, EQE International.

Kunnath, S. (1997), "Predicting inelastic seismic response of structures: Modeling and calibration", *NCEER Bulletin*, 11-2.

Loh, C.-H., Wu, T.-S. and Lin, C.-Y. (1998), "Adaptive identification of nonlinear hysteretic systems", *Proc. of 2WCSC*, Kyoto, Japan, pp.2177-2186.

Masuda, A. (1999), *A study on mechanical and structural health monitoring*, (in Japanese), phD thesis, Kyoto Institute of Technology.

Masuda, A., Yamamoto, S. and Sone, A. (1998), "Adaptive identification of time-varying systems with application to the structural health monitoring", *Proc. of 2WCSC*, Kyoto, Japan, pp.2203-2211.

Masuda, A., Yamamoto, S. and Sone, A. (1999), "Nonparametric identification of linear time-varying systems (1st Report, Basic study on an identification method based on nonstationary spectral analysis)", (in Japanese), *Trans. of JSME*, 65, 634, C, pp.2243-2250.

Zadeh L. A. (1950), "Frequency analysis of variable networks", *Proc. of IRE*, 38, pp.291-299.