0977

POSTERIOR TIMESTEP ADJUSTMENT TECHNIQUE IN SUBSTRUCTURING PSEUDODYNAMIC TEST

Koichi KUSUNOKI[1], Yoshiaki NAKANO[2], Waon-Ho YI[3] And Li-Hyung LEE[4]

SUMMARY

The substructuring pseudodynamic test is a hybrid testing method consisting of a numerical simulation of the earthquake response of an analytical model and a loading test of a specimen. The substructuring pseudodynamic testing technique has been applied to various seismic experiments since it has advantages over the shaking table test to study dynamic behaviors of relatively large-scale structures. However, experimental errors are inevitable in substructuring pseudodynamic test. Some of these errors can be monitored during the test, but they cannot be fully eliminated due to limitations in control system. It is generally accepted that these control errors are significantly affecting dynamic responses of testing structures. This paper focuses on a technique to minimize the cumulative effect of such control errors. For this purpose, a posterior adjustment of the time increment from a target value Δt_n to an adjusted value is performed to minimize the effect of the control errors (Posterior Timestep Adjustment technique, PTA). Two integration methods using PTA techniques (PTA and Modified-PTA) are proposed in this paper, and linear and non-linear simulations of SDF and MDF shear systems considering undershooting control error at the first story which is assumed to be a loading test part were done to confirm the validity of these PTA techniques.

INTRODUCTION

The substructuring pseudodynamic test (referred to as SOL test subsequently) is the hybrid method in which restoring force vectors calculated in a computer with numerical hysteresis models and measured from a loading test are combined into a global restoring force vector of whole structure in each step of numerical integration. It has advantages over the shaking table test to study the dynamic response of relatively large-scale structure.

Computer-controlled actuators are applied to the loading system of SOL test. It is well accepted that random error appears at measuring and controlling instruments and control error does in actuator controlling system. These errors are significantly effective on the accuracy of SOL test [Kabayama, 1995]. New integration techniques in which time-step is adjusted to reduce the effects of control error, "Posterior Time-Step Adjustment Technique" and "Modified PTA" (referred to as PTA and MPTA respectively), are proposed in this paper. The purpose of this paper is to estimate the stability and accuracy of these techniques for SOL test.

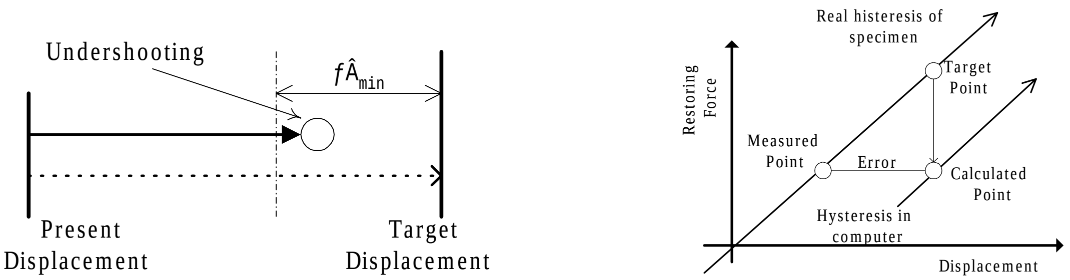
CONTROL ERROR

An actuator is usually controlled with analogue voltage and a computer is digitally controlled. Signals of loading system therefore must be controlled through the Analogue-Digital and Digital-Analogue transfer (referred to as A/D and D/A Transfer subsequently). A minimum controllable displacement $\delta_{\min}$ depends on the precision of A/D and D/A transfer, and it can be given as follows.

[1] *Institute of Industrial Science, University of Tokyo, Tokyo, Japan E-mail kusu@cc.iis.u-tokyo.ac.jp*
[2] *Institute of Industrial Science, University of Tokyo, Tokyo, Japan E-mail kusu@cc.iis.u-tokyo.ac.jp*
[3] *Department of Architectural Engineering, Kwang Woon University, Seoul, Korea E-mail whyi@daisy.kwangwoon.ac.kr*
[4] *Department of Architectural Engineering, Hanyang University, Seoul, Korea*

$$\delta_{\min} = \frac{2\ell}{2^n} \tag{1}$$

In this Equation, $\pm\ell$ represents actuator stroke and 2^n represents the precision of A/D and D/A transfer board equipped in a computer. It is impossible to let an actuator impose to a target displacement at a step when the distance from a target displacement is less than $\delta_{\min}$ (**Fig. 1** (a)). Usually undershooting is applied to define the target displacement in order to avoid the damage to specimen caused by overshooting displacement. If undershooting is applied, the measured displacement does not reach the target displacement almost every step, and a hysteresis characteristic used in a numerical integration becomes difference from that of real specimen. Especially when the response displacement is relatively small in linear domain, the hysteresis curve shows an anti-clockwise loop (**Fig. 1** (b)). It is well accepted that nonetheless the value of control error is very small, the calculated response is much amplified compared with the real response because of this additional energy input due to anti-clockwise loop [Kabayama, 1995].



(a) Undershooting control error **(b) The effect of undershooting control error**

Fig. 1 Undershooting control error and the effect of undershooting control error

OS INTEGRATION TECHNIQUE

Nakashima et al. proposed the operator-splitting integration technique (referred to as OS technique subsequently) for SOL test of multi-degree-of-freedom system [Nakashima et. al., 1990]. The procedures of OS technique are shown as follows.

At n-th step, predictor displacement $\{y^*_{n+1}\}$ is calculated at first with responses of n-th step with the assumption that response acceleration at (n+1)-th step is zero (Eq. (2)).

$$\{y^*_{n+1}\} = \{y_n\} + \{\dot{y}_n\}\Delta t + \frac{1}{4}\{\ddot{y}_n\}\Delta t^2 \tag{2}$$

In this equation, $\{y_n\}$, $\{\dot{y}_n\}$, $\{\ddot{y}_n\}$ and Δt represent displacement, velocity and acceleration vector at n-th step and fixed time-step for integration which is defined before analysis respectively.

As a second step, the restoring force vector $\{Q^*_{n+1}\}$ at predictor displacement $\{y^*_{n+1}\}$ is measured from the specimen or calculated in a computer. Response displacement vector at (n+1)-th step $\{y_{n+1}\}$ is calculated from the motion equation with $\{y^*_{n+1}\}$ and $\{Q^*_{n+1}\}$ as follows.

$$[M]\{\ddot{y}_{n+1}\} + [C]\{\dot{y}_{n+1}\} + \{Q^*_{n+1}\} + [K^I](\{y_{n+1}\} - \{y^*_{n+1}\}) = -[M]\{\ddot{y}^0_{n+1}\} \tag{3}$$

In this equation, [M] and [C] represent mass and damping matrix, and $\{\ddot{y}^0_{n+1}\}$ represents a ground acceleration vector. Initial stiffness matrix is usually applied to $[K^I]$. Then acceleration and velocity vector at (n+1)-th step, $\{\ddot{y}_n\}$ and $\{\dot{y}_n\}$, are calculated with Eq. (4).

$$\{\ddot{y}_{n+1}\} = \frac{1}{\beta \cdot \Delta t^2} \cdot \{y_{n+1}\} - \frac{1}{\beta \cdot \Delta t^2} \cdot \{y_n\} - \frac{1}{\beta \cdot \Delta t} \cdot \{\dot{y}_n\} - \left(\frac{1}{2 \cdot \beta} - 1\right) \cdot \{\ddot{y}_n\}$$
$$\{\dot{y}_{n+1}\} = \{\dot{y}_n\} + \frac{1}{2} \cdot (\{\ddot{y}_n\} + \{\ddot{y}_{n+1}\}) \cdot \Delta t \tag{4}$$

POSTERIOR TIME-STEP ADJUSTMENT TECHNIQUE (PTA)

It can be said that appropriate response can not be calculated with SOL test using OS technique, which does not consider the effect of control error. Procedures of Posterior Time-step Adjustment technique based on OS technique shown in 3rd chapter are introduced to reduce the effect of control error as follows [Yi and Peek, 1993]. The value of control error displacement measured in SOL test is calculated from the difference between the predictor displacement $\{y^*_{n+1}\}$ (Eq. (2)) and measured displacement $\{\tilde{y}^*_{n+1}\}$ of experiment part in SOL test. The new predictor displacement vector $\{\tilde{y}^{**}_{n+1}\}$ with variable time-step is then defined as Eq. (5).

$$\{\tilde{y}^{**}_{n+1}\} = \{y_n\} + \{\dot{y}_n\}\Delta t_n + \frac{1}{4}\{\ddot{y}_n\}\Delta t_n{}^2 \tag{5}$$

And the value of variable time-step Δt_n, which minimizes the norm $\| \{\tilde{y}^*_{n+1}\} - \{\tilde{y}^{**}_{n+1}\} \|$, can be found through Newton-Raphson convergent technique. The effect of control error on response displacement at n-th step obtained from Eq. (3) can be reduced using Δt_n in stead of Δt and $\{\tilde{y}^{**}_{n+1}\}$ instead of $\{y^*_{n+1}\}$. In other words, Δt_n is the appropriate time-step, which minimizes the difference between the predictor and measured displacement of a specimen.

In order to reduce control error of both experiment and numerical part, whole displacement data are used to calculate the norm (**Fig. 2 (a)**). The norm of whole displacement vector is generally formulated as Eq. (6). If weight matrix [G] can be determined properly, an appropriate Δt_n can be obtained. However, it can be accepted that it is difficult to define [G] properly. Time range parameter θ needs to be determined to avoid a negative or relatively large Δt_n value as shown in Eq. (7).

$$\| \{\tilde{y}^*_{n+1}\} - \{\tilde{y}^{**}_{n+1}\} \| = \sqrt{(\{\tilde{y}^*_{n+1}\} - \{\tilde{y}^{**}_{n+1}\})^t [G] (\{\tilde{y}^*_{n+1}\} - \{\tilde{y}^{**}_{n+1}\})} \tag{6}$$
$$(\Delta t_{n\,\min}, \Delta t_{n\,\max}) = [(1-\theta)\Delta t, (1+\theta)\Delta t] \tag{7}$$

MODIFIED POSTERIOR TIME-STEP ADJUSTMENT TECHNIQUE (MPTA)

As mentioned in previous chapter, it is difficult to determine the appropriate weight matrix [G] in PTA technique, appropriate Δt_n can not be found by PTA technique. To overcome this disadvantage of PTA technique, whole structure is divided into numerical part and experiment part in MPTA technique. The value of Δt_n is calculated from only the displacement data of experiment part, which contains control error. The procedures of MPTA technique are introduced as follows. In these procedures, force and displacement vectors of whole structure $\{\ \}$ need to be divided into experiment part $\{\ \}_E$ and analytical part $\{\ \}_A$ through reassembling nodal numbers appropriately so that $\{\ \}^t$ can form $\{\{\ \}^t_E, \{\ \}^t_A\}$.

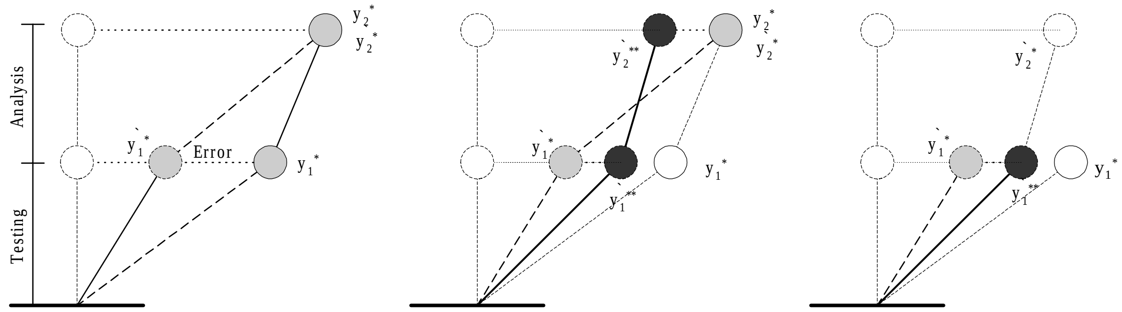
The predictor displacement value of testing part $\{y^*_{n+1}\}_E$ is calculated with fixed time-step Δt as a first step. Then actuator is imposed toward the target displacement $\{y^*_{n+1}\}_E$. Restoring force vector $\{Q^*_{n+1}\}_E$ and displacement vector $\{\tilde{y}^*_{n+1}\}_E$ of experiment part are measured when the distance between target displacement and measured displacement of specimen is less than $\delta_{\min}$ (**Fig. 2 (c)**). The variable time-step Δt_n, which minimizes the norm $\| \{\tilde{y}^*_{n+1}\}_E - \{\tilde{y}^{**}_{n+1}\}_E \|$, can be found with Newton-Raphson convergent technique. The equation of this norm is shown in Eqs. (8) and (9), where [I] represents unit matrix.

$$\left\{ \tilde{y}^{**}_{n+1} \right\}_E = \{y_n\}_E + \{\dot{y}_n\}_E \Delta t_n + \frac{1}{4}\{\ddot{y}_n\}_E \Delta t_n^{\ 2} \tag{8}$$

$$\left\| \left\{ \tilde{y}^{*}_{n+1} \right\}_E - \left\{ \tilde{y}^{**}_{n+1} \right\}_E \right\| = \sqrt{\left(\left\{ \tilde{y}^{*}_{n+1} \right\}_E - \left\{ \tilde{y}^{**}_{n+1} \right\}_E \right)^t [I] \left(\left\{ \tilde{y}^{*}_{n+1} \right\}_E - \left\{ \tilde{y}^{**}_{n+1} \right\}_E \right)} \tag{9}$$

The predictor displacement vector of numerical part $\left\{ y^{*}_{n+1} \right\}_A$ can be calculated with Δt_n as Eq. (10) (**Fig. 2 (c)**). The restoring force vector of numerical part $\left\{ Q^{*}_{n+1} \right\}_A$ at $\left\{ y^{*}_{n+1} \right\}_A$ can be calculated with numerical model.

$$\left\{ y^{*}_{n+1} \right\}_A = \{y_n\}_A + \{\dot{y}_n\}_A \Delta t_n + \frac{1}{4}\{\ddot{y}_n\}_A \Delta t_n^{\ 2} \tag{10}$$

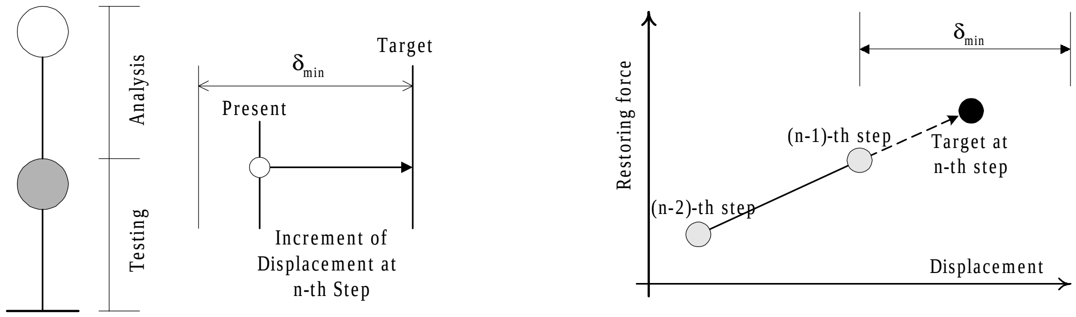


(a) SOL test **(b) PTA Technique** **(C) MPTA Technique**

Fig. 2 SOL test with PTA and MPTA techniques for two-degree-of-freedom system

The restoring force vector of whole structure $\left\{ Q^{*}_{n+1} \right\}$ can be calculated to combine $\left\{ Q^{*}_{n+1} \right\}_E$ and $\left\{ Q^{*}_{n+1} \right\}_A$. Finally response displacement vector at (n+1)-th step $\left\{ y_{n+1} \right\}$ can be obtained from Eq. (3) and Δt_n. The range of Δt_n is defined as shown in Eq. (7) to avoid a negative or relatively large Δt_n value as PTA technique.

If the increment of predictor displacement of experiment part at n-th step $\left\{ \Delta y^{*}_n \right\}_E$ is less than $\delta_{\min}$, the actuator keeps present position (**Fig. 3 (a)**). If the increment of measured displacement is zero, Δt_n always becomes negative or zero. When the increment of measured displacement is zero, Δt is used in Eq. (3) instead of Δt_n and restoring force of experiment part is calculated numerically with linear extrapolation technique (**Fig. 3 (b)**).



(a) Increment displacement becomes zero **(b) Linear extrapolation method**

Fig. 3 Linear extrapolation method

ANALYTICAL EXAMINATION WITH PTA AND MPTA

Linear and non-linear dynamic response analyses were carried out with single-degree-of-freedom and six-degree-of-freedom shear mode vibration system (referred to as SDF and MDF subsequently) considering the effect of control error numerically to estimate basic characteristics of PTA and MPTA. First story of each system is assumed as a experiment part. Unit matrix [I] is used as weight matrix of PTA in this paper.

Weight of each floor is 172.80 tonf and height of each story is 4.0 m. Damping coefficient is assumed 0.0%, since main purpose of this paper is to investigate the effect of control error. Takeda Tri-Linear Model is used as a hysteresis model of each story [Takeda et. al., 1970]. Restoring forces and displacements at each stiffness degradation point are calculated as follows (**Fig. 4**). Each parameter of SDF and MDF are shown in **Table. 1**.

1. Fy Ultimate lateral resistance in each story (base shear coefficient is assumed 0.3).
2. Dy = Story height/150.
3. Fc = Fy / 3.0.
4. Dc Calculated with Fc, elastic stiffness and stiffness degrading ratio (0.4).
5. Fu, Du Calculated so that post yielding stiffness $Fu - Fy / Du - Dy$ is Fc / Dc times 1/1000.

Table. 1 Parameters list for SDF and MDF (tonf, cm) [AIJ, 1992]

System	Story	Fc	Dc	Fy	Dy	Fu	Du
SDF	1	18.00	0.36	54.00	2.67	54.54	13.33
MDF	1	108.00	0.36	324.00	2.67	327.24	13.33
	2	98.74	0.36	296.23	2.67	325.85	13.33
	3	88.87	0.36	266.61	2.67	293.27	13.33
	4	80.23	0.36	240.69	2.67	264.75	13.33
	5	63.36	0.36	190.08	2.67	209.09	13.33
	6	37.03	0.36	111.09	2.67	122.19	13.33

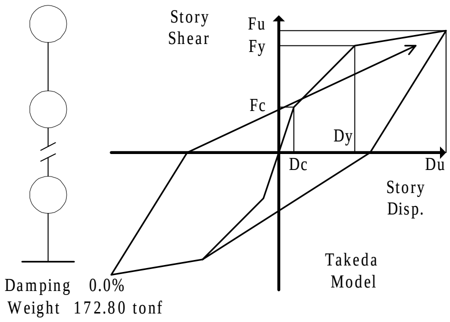


Fig. 4 Parameters for analysis

The input acceleration records to the pseudodynamic test conducted by Kabayama et al. is used, scaling its peak acceleration to 500 gal (**Fig. 5**).

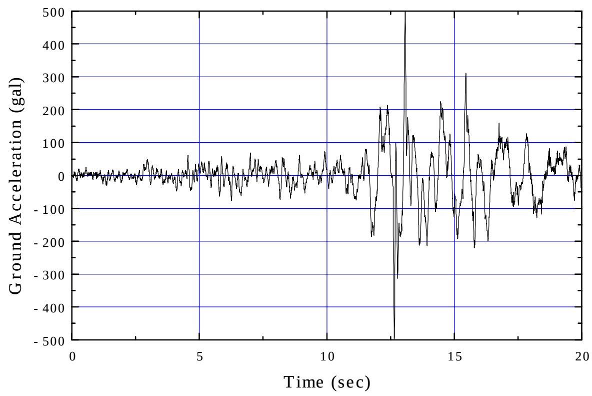


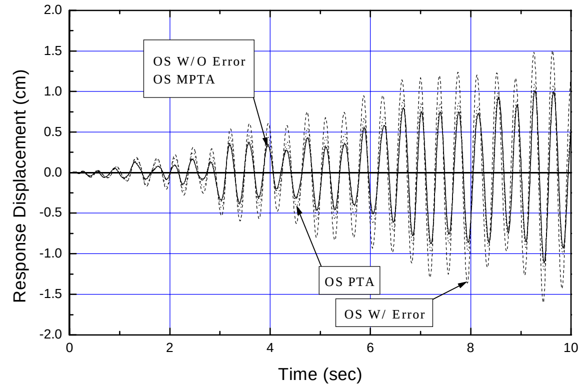
Fig. 5 Input wave

Max/Min stroke of actuator is assumed $\pm 15cm$ and precision of A/D and D/A transfer board are adjusted 11 bits. As the result, $\delta_{\min}$ is 0.0146 cm. Undershooting control error is applied and the value of undershooting control error is calculated to simulate the SOL test with $\delta_{\min}$ in each step. θ and weight matrix are assumed 0.5 and unit matrix [I] as mentioned above. It should be noted that PTA and MPTA techniques proposed in this paper could be applied to both undershooting and overshooting control errors.

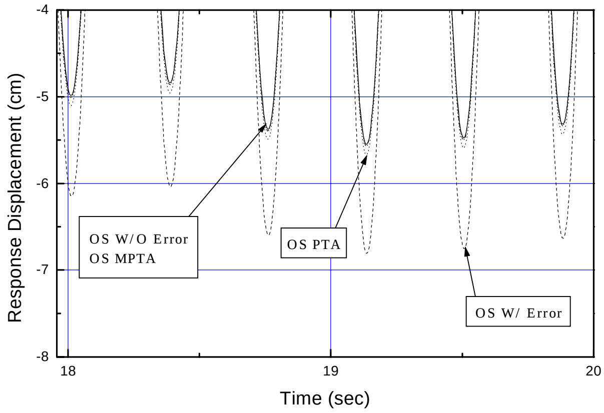
SDF SYSTEM

Fig. 6 (a) and **Fig. 6 (b)** show the time histories of response displacement. It can be recognized that the response displacement with error (referred to as OS W/ Error subsequently) are amplified compared with that without error (referred to as OS W/O Error subsequently), but response displacements of PTA and MPTA are almost same as that of OS W/O Error. As shown in **Fig. 6 (c)**, structural responses calculated using OS W/ Error, PTA and MPTA techniques show anti-clockwise hysteresis loops. However, the areas for PTA and MPTA are smaller than OS W/ Error and their responses are successfully improved.

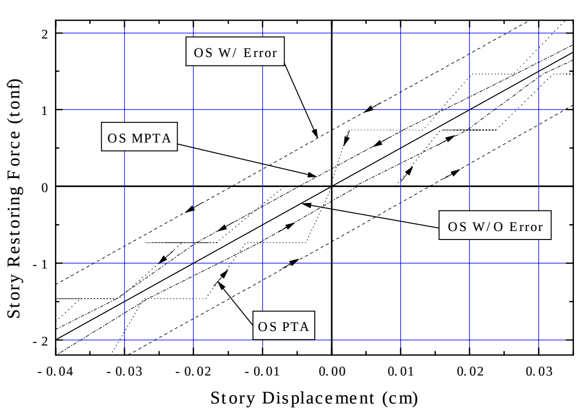
Fig. 6 (d) and **Fig. 6 (e)** shows time histories of non-linear response displacement and **Fig. 6 (f)** shows the envelope curves of restoring force – response displacement relationship. The PTA technique shows a little difference from OS W/O Error during the first 10 seconds, but it compares well with OS W/O Error during the next 10 seconds, while the MPTA compares well with OS W/O Error over the all 20 seconds. Considering the



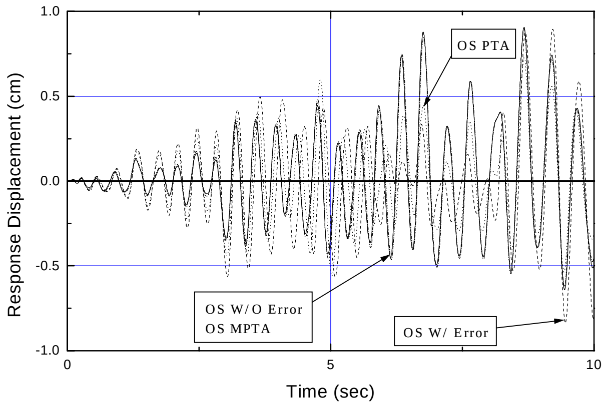
(a) Response displacements(Linear: SDF : 0~10 sec)



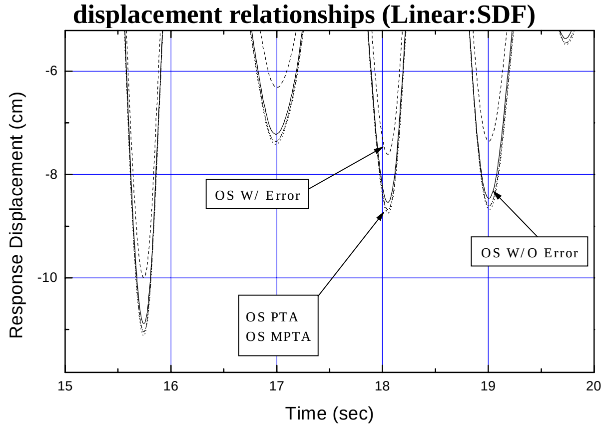
(b) Response displacements (Linear:SDF:18~20 sec)



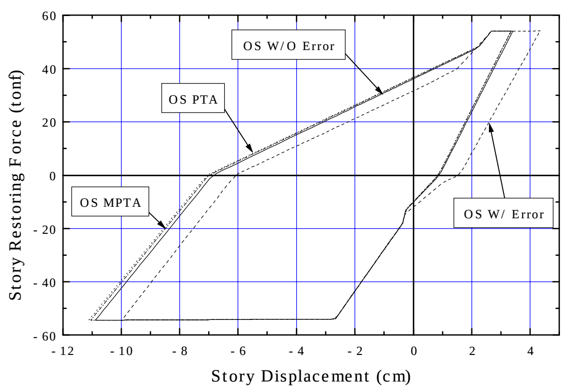
(c) Enveloped curves of story restoring force- story displacement relationships (Linear:SDF)



(d) Response displacements (Non-linear:SDF:0~10 sec)



(e) Response displacements (Non-linear:SDF:15~20 sec)



(f) Enveloped curves of story restoring force- story displacement relationship(Non-linear:SDF)

Fig. 6 Linear and non-linear responses of SDF system

simple formulation for PTA, however, PTA can be a usable technique to simulate relatively large responses of a SDF system.

MDF SYSTEM

Fig. 7 (a) shows the time histories of linear response displacement at first story. It can be said that the response displacement of OS /W Error is amplified compared with that of OS W/O Error. **Fig. 7 (b)** and **Fig. 7 (c)** also show the time histories of response displacement at first story. Response of OS W/ Error is amplified by the effect of undershooting control error, and spurious high mode effect of OS W/ Error is predominant because of undershooting control error [Nakashima et. al., 1982 and 1983]. Furthermore, PTA can not reduce the effect of control error, and spurious high mode effect of PTA responses is also predominant as the result of OS W/ Error. One of the reasons why PTA can not reduce the effect of undershooting control error is the assumption [G]=[I]. One can easily imagine that it is difficult to define weight matrix [G] properly. On the other hand, MPTA reduces the effect of undershooting control error sufficiently and the response of MPTA is almost same as OS W/O Error. As shown in **Fig. 7 (d)**, structural responses calculated using OS W/ Error, PTA and MPTA techniques show anti-clockwise hysteresis loops. However, the area for MPTA is smaller than OS W/ Error and PTA, and its response is successfully improved.

Fig. 7 (e) and **Fig. 7 (f)** show the time histories of non-linear response displacement. Responses of OS W/ Error and PTA are amplified by the effect of undershooting control error, and spurious high mode effect of these responses are predominant as is found in the linear analysis results. MPTA reduces the effect of control error sufficiently and the response of MPTA is almost same as OS W/O Error.

CONCLUDING REMARKS

Posterior Time-step Adjustment technique and Modified PTA technique were proposed to reduce the effect of control error. Linear and non-linear dynamic response analyses of SDF and MDF shear mode systems considering undershooting control error at first story were carried out to confirm the validity of these techniques. Results obtained from the investigations can be summarized as follows.

1. Dynamic responses were amplified by the effect of undershooting control error, and spurious high mode effect of MDF responses were predominant.
2. Both PTA and Modified PTA techniques can reduce the effect of undershooting control error on dynamic responses successfully in both linear and non-linear responses of SDF system.
3. Modified PTA technique can reduce the effect of control error successfully in both linear and non-linear responses of MDF systems, while PTA technique cannot reduce it if weight matrix of PTA cannot be provided properly.

REFERENCES

AIJ/Architectural Institute of Japan (1992), “Design Guidelines for Earthquake Resistant Reinforced Concrete Buildings Based on Ultimate Strength Concept”

KABAYAMA, K. (1995), “*Study on High Accurate On-line Test*”, Doctoral Thesis of Univ. of Tokyo

NAKASHIMA, M., KATO, H. and KAMINOSONO, T. (1982), “Simulation of Earthquake Response by Pseudo Dynamic Test Technique Part 1 and 2”, *Summaries of technical papers of annual meeting*, AIJ, pp.951-954, Oct.

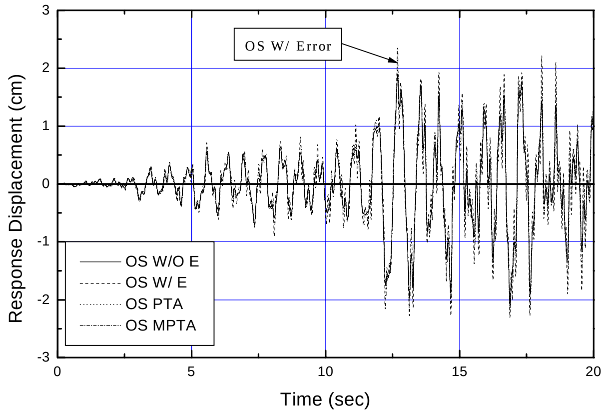
NAKASHIMA, M., KATO, H. and KAMINOSONO, T. (1983), “Simulation of Earthquake Response by Pseudo Dynamic Test Technique Part 3 and 4”, *Summaries of technical papers of annual meeting*, AIJ, pp.445-448, Oct.

NAKASHIMA, M., ISHIDA, M. and ANDO, K. (1990), “Integration Techniques for Substructure Pseudo Dynamic Test (Pseudo dynamic test using substructuring techniques)”, *Journal of Struct. Constr. Engng*, AIJ, No417, Nov.

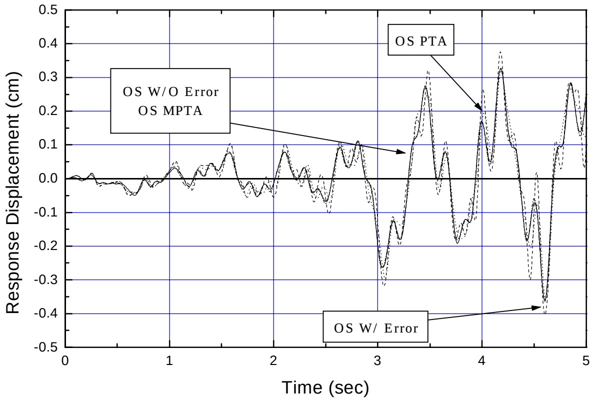
Plesha, M. E. and Belytschko (1985), T., ”A Constitutive Operator Splitting Method for Nonlinear Transient Analysis”, *Computers & Structures*, Vol.20, No.4

Takeda, T., Mete A. Sozen, N. Norby Nielsen (1970), ”Reinforced Concrete Response to Simulated Earthquake”, *Journal of Structural Division*, Vol. 96, ST 12

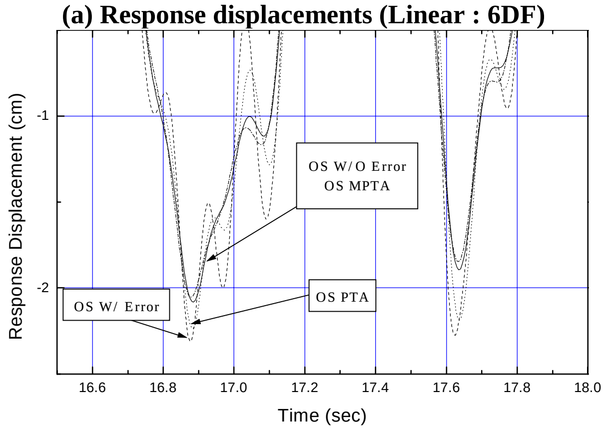
Yi, W. H. and Peek, R. (1993), ”Posterior Time-Step Adjustment in Pseudodynamic Testing”, *Journal of Engineering Mechanics*, Vol.119, No. 7, July



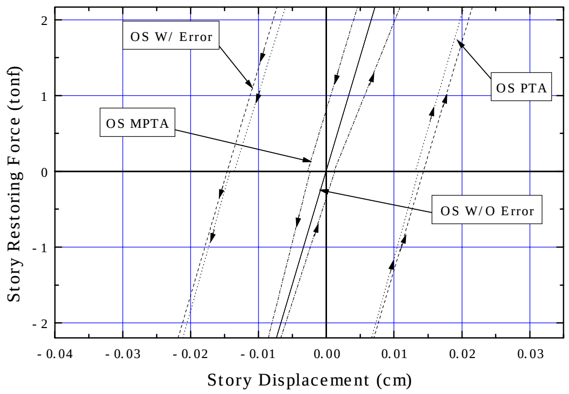
(a) Response displacements (Linear : 6DF)



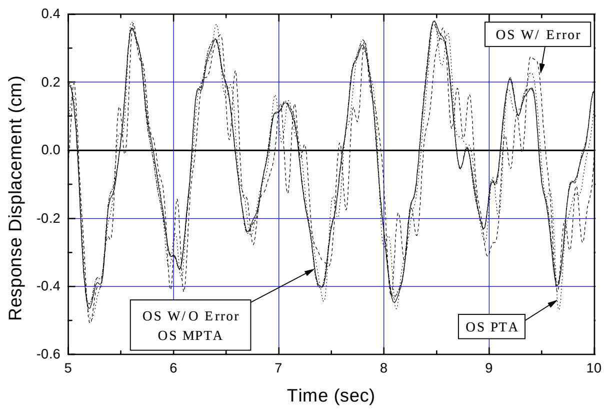
(b) Response displacements (Linear:6DF:0~10 sec)



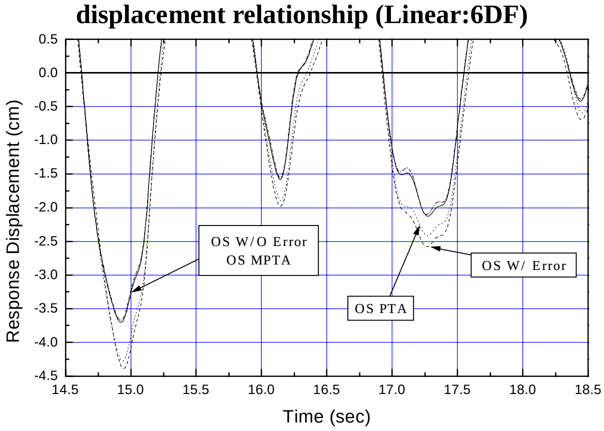
(c) Response displacements (Linear:6DF:16.5~18 sec)



(d) Enveloped curves of story restoring force- story displacement relationship (Linear:6DF)



(e) Response displacements (Non-linear:6DF:5~10 sec)



(f) Response displacements (Non-linear:6DF:14.5~18.5 sec)

Fig. 7 Linear and non-linear responses of MDF system