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STUDY ON INVERSE PROBLEM IN STRUCTURAL DAMAGE IDENTIFICATION

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SUMMARY

The objective of this paper is to present a system identification method to identify structural parameters when the input information is unknown. This method is named as Hybrid Inverse Method (HIM), which is essentially an iterative least-squares procedure in time domain. Since the information of input excitation is not available, the mechanical characteristic of the input force is used as an additional information to ensure the convergence of the iterative procedure. Steps of HIM are discussed in details in the paper. Practical input situation such as earthquake, ambient vibration is considered in numerical examples to verify the accuracy, reliability and robustness of the proposed algorithm. The results prove that using only the response measurements, HIM can estimate structural parameters and inverse input time history accurately for noise-free cases. For noise-included cases, HIM gives satisfied results as well even when the noise-signal ration is fairly high.

INTRODUCTION

Structural damage detection has recently received increasing attention in civil engineering. Among many nondestructive evaluation methods, the system identification (SI) techniques are very effective and appealing. The structural characteristics or mostly the parameters will change when damage occurs in the structure. On the basis of the fact the SI technique can be used for detecting potential damage in structures using the structural dynamic or static response measurements.

In the past decades many SI methods derived from automatics have been successfully transferred to civil engineering applications. Although most of these methods performance well in simulation examples, they still have several limitations that reduce their practical applications. A common feature of most available methods to-date is that the information of input excitation and output response must be known. However, common problems encountered in the ambient vibration survey are that the input excitation is generally unknown and the measurement of structural response is corrupted with ambient noise. For example, for the health monitoring of high buildings and the damage assessment of buildings suffering strong earthquake, And in many cases, it is extremely difficult to accurately measure the input information during actual circumstance. Initial efforts to solve this problem are often based on the assumption that the input excitation of the structure is impulsive or has zero mean and the output is free-decay type response. The random decrement technique proposed by Ibrahim (1977) is a representative of these methods. Considering that free-decay response is hardly to be measured for actual structures, methods that have no strict restriction for the statistical nature of the input or the type of output response are proposed recently. For example, Benedetii and Gentile (1994) presented a two-phase method in frequency domain, assuming that the responses at two locations of the system are known. Requirements for input information is elaborately avoided by using the ratio of the amplitudes of Fourier transforms of the two recorded signals. Hoshiya and Sutoh (1995) developed a procedure to evaluate the input excitation and system parameters of shear-type building under seismic load based on extended Kalman filter with a weighted global iteration (KF-WGI). Response measurements at all floors are needed in this procedure. The most promising method in time domain might be given by Wang and Haldar (1994), which solves this problem without any restriction on the type of excitation force. Discussion of the state of art in the time-domain SI techniques without input information

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and a comprehensive preferences are available in their excellent paper. More recently (Wang and Haldar 1997), they spread their method to the situation when response measurements recorded at only a few DDOFS by integrating KF-WGI technique.

A new time-domain method is proposed in this paper for structural parameters identification when the input excitation is unknown. The method is named as Hybrid Inverse Method (HIM), which can estimate structural parameters and inverse time history of input excitation simultaneously based on the structural response data. The HIM is essentially an iterative least-squares procedure in time domain and has no special requirement for the type of the input. Since the information of input excitation is not available, the mechanical characteristic of the input force is used as an additional information to ensure the convergence of the iterative procedure. Steps of HIM are discussed in details in the following sections..

PROCEDURE OF HIM

Consider a discrete MDOF linear system whose motion is assumed to be governed by the set of differential equations

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{f}(\boldsymbol{t}) \tag{1}$$

where M,C,K= mass ,damping , and stiffness matrices, each of order n by n, n= the number of DOF. X(t)= system displacement vector of order n, and f(t)=excitation vector of order n.

Using matrix transformation, (1) can be rewritten as (2). Similar approach has been used by Agababian et al. (1991), Wang et al. (1994), and more generally by Li (1996).

$$\boldsymbol{H\theta} = \boldsymbol{F} \tag{2}$$

where $\boldsymbol{H}$ = a matrix whose entries correspond to the system response at observation time t; $\boldsymbol{\theta}$ = a vector containing all the unknown system parameters, and $\boldsymbol{F}$= a vector containing the excitation measurements.

Suppose m is the number of total sample points, then (2) can be rearranged as

$$\boldsymbol{H}_{\boldsymbol{m}}\boldsymbol{\theta} = \boldsymbol{F}_{\boldsymbol{m}} \tag{3}$$

If the response quantities $\boldsymbol{X}$, $\dot{\boldsymbol{X}}$, $\ddot{\boldsymbol{X}}$ and input excitation $\boldsymbol{f}(\boldsymbol{t})$ are available at all DOFS, using least-squares approximation methods , the parameter vector $\boldsymbol{\theta}$ can be computed from

$$\boldsymbol{\theta} = [\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H}]^{-1}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{F} \tag{4}$$

However the information of the input excitation $\boldsymbol{f}(\boldsymbol{t})$ is unknown, so the above formulation can not be used directly to achieve unknown structural parameters.

Without losing any generality, a N-story shear building (as shown in Fig.1) subjected to base input is used here to describe the procedures of HIM. In that case, matrix in (1),(3) can be explained as

$$\boldsymbol{H}_{\boldsymbol{m}} = [\boldsymbol{H}(\boldsymbol{t}_1),\ \boldsymbol{H}(\boldsymbol{t}_2),\ \cdots,\ \boldsymbol{H}(\boldsymbol{t}_{\boldsymbol{m}})]^{\boldsymbol{T}} \tag{5}$$

$$\boldsymbol{\theta} = [\boldsymbol{c}_1,\ \boldsymbol{c}_2, \cdots, \boldsymbol{c}_{\boldsymbol{n}};\ \boldsymbol{k}_1,\ \boldsymbol{k}_2,\ \cdots,\ \boldsymbol{k}_{\boldsymbol{N}}]^{\boldsymbol{T}} \tag{6}$$

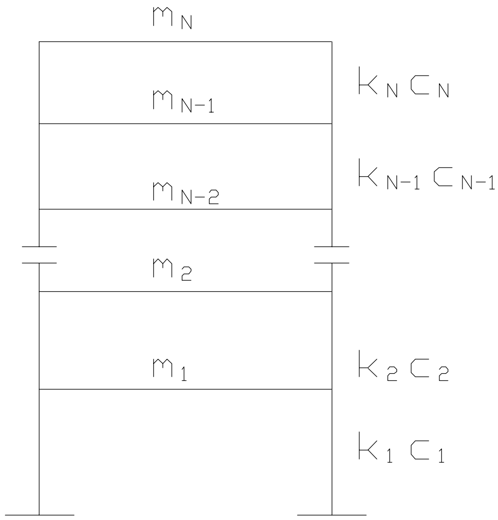


Fig. 1. N-Story Shear Building

$$\boldsymbol{F}_m = [\boldsymbol{F}(\boldsymbol{t}_1),\ \boldsymbol{F}(\boldsymbol{t}_2),\ \cdots,\ \boldsymbol{F}(\boldsymbol{t}_m)]^T \tag{7}$$

where $\boldsymbol{c}_i$, $\boldsymbol{k}_i$ (i=1,2,…,n)= the damping and stiffness respectively at the ith DDOF of the building, $\ddot{\boldsymbol{X}}_0$ =the ground acceleration, $\boldsymbol{H}(\boldsymbol{t}_1)$, $\boldsymbol{H}(\boldsymbol{t}_2)$, $\boldsymbol{H}(\boldsymbol{t}_m)$ are the response matrix at all DDOF at sample time $\boldsymbol{t}_1$, $\boldsymbol{t}_2$,…, $\boldsymbol{t}_m$, respectively and $\boldsymbol{H}(\boldsymbol{t}_i)$ (i=1,2,…,m) can further be expressed as

$$\boldsymbol{H}(\boldsymbol{t}_i) = \begin{bmatrix} \dot{x}_1 & \dot{x}_1 - \dot{x}_2 & 0 & 0 & x_1 & x_1 - x_2 & 0 & 0 \\ 0 & \dot{x}_2 - \dot{x}_1 & \dot{x}_2 - \dot{x}_3 & 0 & 0 & x_2 - x_1 & x_2 - x_3 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ 0 & 0 & 0 & \dot{x}_n - \dot{x}_{n-1} & 0 & 0 & 0 & x_n - x_{n-1} \end{bmatrix} \tag{8}$$

Similarly , the vector $\boldsymbol{F}(\boldsymbol{t}_i)$ (i=1,2,…,M) can be expressed as

$$\boldsymbol{F}(\boldsymbol{t}_i) = [\boldsymbol{p}_1(\boldsymbol{t}_i),\ \boldsymbol{p}_2(\boldsymbol{t}_i), \cdots,\ \boldsymbol{p}_n(\boldsymbol{t}_i)] \tag{9}$$

$$\boldsymbol{p}_l(\boldsymbol{t}_i) = -\boldsymbol{m}_l \ddot{x}_0(\boldsymbol{t}_i) - \boldsymbol{m}_l \ddot{x}_l(\boldsymbol{t}_i) \quad \boldsymbol{l} = 1, \cdots \boldsymbol{n} \tag{10}$$

where $\boldsymbol{m}_l$ (l=1,2,…,n) = the mass at ith DDOF of the building, $\ddot{\boldsymbol{x}}_0(\boldsymbol{t}_i)$ = the ground acceleration at sample point $\boldsymbol{t}_i$, $\ddot{\boldsymbol{x}}_l(\boldsymbol{t}_i)$ = the acceleration response at $\boldsymbol{lth}$ DDOF of the building at sample point $\boldsymbol{t}_i$.

Based on the above equations, a kind of hybrid inverse method has been developed to identify structural parameters and inverse the input process simultaneously. For a shear building under groundmotion input case, considering the fact that all the lumped massed of the building are subject to the same ground acceleration at any sample time, the algorithm of HIM can be constructed . For a shear building under seismic input case , the procedure of HIM is described as following

1. Assign initial values for the unknown structural parameters. For instance, let $\hat{\boldsymbol{\theta}}_0 = [1,\quad 1,\quad \cdots,\quad 1]$. It will be demonstrated later that the proposed algorithm is not sensitive to this initial assumption.

2. According to (3), the initial estimation of vector F can be calculated as

$$\boldsymbol{H}\hat{\boldsymbol{\theta}}_0 = \boldsymbol{F}_0 \tag{11}$$

3. Since the input excitation $\boldsymbol{F}_0$ is available after step 2, referring to (9) and (10), time history of the ground acceleration can be calculated at each DDOF of the building, which is

$$\begin{cases} \ddot{x}_0(t_i/1) = -\ddot{x}_1(t_i) - \dfrac{p_1(t_i)}{m_1} \\ \ddot{x}_0(t_i/2) = -\ddot{x}_2(t_i) - \dfrac{p_2(t_i)}{m_2} \\ \cdots \\ \ddot{x}_0(t_i/N) = -\ddot{x}_n(t_i) - \dfrac{p_n(t_i)}{m_n} \end{cases} \qquad i = 1,2,\cdots,m \tag{12}$$

where $\ddot{x}_0(t/l)$ (l =1,2,…n) = ground acceleration time history calculated from the lth DDOF of the building. According to the characteristic of seismic input mentioned above , all the $\ddot{x}_0(t/l)$ (l=1,2,…,n) should be the same. However they are not identical because of the difference between the initial guess of the structural parameters and the real ones. In order to make the estimated $\ddot{x}_0(t)$ coincide with the characteristic of seismic input, an average procedure can be executed to get a more reasonable estimation

$$\bar{\ddot{x}}_0(t_i) = \frac{1}{n}\sum_{i=1}^{n} \ddot{x}_0(t_i/i) \tag{13}$$

4. Referring to (9) and (10), new $p_l(t)$ (l=1,2, … , n) as well as $\boldsymbol{F}$ can be calculated using $\bar{\ddot{x}}_0(t)$ obtained in step 3.

$$\hat{p}_l(t_i) = -m_l \bar{\ddot{x}}_0(t_i) - m_l \ddot{x}_l(t_i) \qquad l = 1,\cdots n \tag{14}$$

$$\tilde{\boldsymbol{F}}(t_i) = \left[\hat{p}_1(t_i),\, \hat{p}_2(t_i), \cdots, \hat{p}_n(t_i)\right] \tag{15}$$

5. Further parameter estimation can be obtained using (4)

$$\hat{\boldsymbol{\theta}}_1 = [\boldsymbol{H}^T \boldsymbol{H}]^{-1} \boldsymbol{H}^T \tilde{\boldsymbol{F}} \tag{16}$$

6. Replace $\hat{\boldsymbol{\theta}}_0$ by $\hat{\boldsymbol{\theta}}_1$ in step 2 and then reiterated setup 2-6 until convergence of estimated parameters can be obtained with a predetermined accuracy.

$$\left\| \hat{\boldsymbol{\theta}}_i - \boldsymbol{\theta}_{i-1} \right\| \le \boldsymbol{\varepsilon} \tag{17}$$

where $\boldsymbol{\varepsilon}$ is the given accuracy. Once the algorithm converges, parameter vector got in step 5 gives the final identification of structural parameters, while the $\bar{\ddot{x}}_0(t)$ gives the time history of the ground motion.

NUMERICAL STUDIES

To verify the method proposed , a six-story shear building represented by a six DDOF system is considered first. The parameters of the building are listed in Table 1

Table 1 Structural parameters of the six-story building

No of DOF	1	2	3	4	5	6
m_i	6100	5800	5300	5300	5300	3600
k_i	271390	290360	282690	252480	233540	229940
c_i	2469.65	2874.56	2572.48	2297.57	2265.32	2092.45

Example 1. Stationary process excitation

The structure is presumed to be excited by a white noise input at the ground level, its time history is shown in Fig. 2.

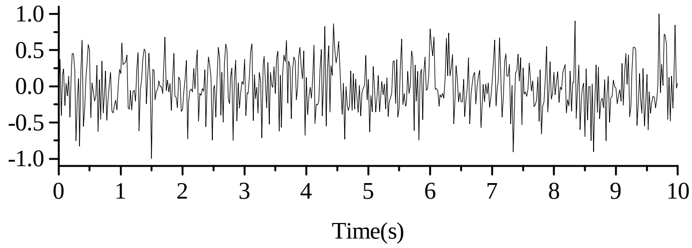


Fig 2 Case1: White Noise Input

Firstly, theoretical responses of all the DDOF of the structure are calculated using Wilson-θ method. All these response quantities are assumed to be measured at both locations of the structure. The objective is that using only these measured responses to estimate the structural parameters and inverse input time history through HIM procedure. To assess the influence of output measurement noise to HIM, numerical generated white noise with rms (root mean square) intensities of 5%, 10% and 20% are added to the responses data of all DDOF.

The sample time intervals is 0.02s, for noise free case, the response from 0.3s to 1.3s are considered in HIM, which means only 50 sample points are used, and 400 sample points from 0.3s~8.3s are used. The estimated results for noise-free case and noise included cases are shown in Table 2.

Table 2 Estimated results for example 1 through HIM

Structural parameters	Noise free		5% noise level		10% noise level		20% noise level	
	Results	Error%	Results	Error%	Results	Error%	Results	Error%
c1	2470.6	0.04	2500.0	1.23	2555.2	3.46	2729.1	10.5
c2	2873.1	0.05	2878.9	0.15	2903.6	1.01	3010.9	4.74
c3	2570.4	0.08	2560.7	0.46	2557.9	0.57	2578.8	0.24
c4	2295.7	0.08	2274.7	0.99	2253.2	1.93	2214.4	3.62
c5	2263.6	0.07	2218.2	2.08	2165.7	4.39	2045.2	9.72
c6	2091.3	0.05	2055.6	1.76	2010.2	3.93	1894.2	9.47
k1	271369	0.01	271233	0.01	271261	0.05	271863	0.17
k2	290345	0.01	290228	0.05	290245	0.04	290732	0.13
k3	282683	0.01	282641	0.02	282655	0.01	282881	0.07
k4	252479	0.01	252550	0.03	252566	0.03	252440	0.02
k5	233541	0.01	233743	0.09	233799	0.11	233464	0.03
k6	229941	0.01	230186	0.11	230271	0.14	229959	0.01

Several important observation can be made from Table 2. For noise free case, the estimated values of the parameters have little bias with the actual values. The maximum estimation is 0.08‰ and only 50 sample points are enough to get this accuracy. For noise-included cases, one need more sample points to reach a given accuracy. For 5% noise level, the maximum error is 2.08% in damping estimation and only 0.09% in stiffness estimation. Even for fairly high noise level, e.g. 20%, the correspond tow maximum errors about damping and stiffness are 10.5% and 0.17% respectively.

Example 2: Non-stationary process excitation

For case 2, the same structure is supposed to be excited by a seismic load at the ground level. The Elcentro earthquake is chosen as the seismic input.

Using the similar procedure as shown in example 1, theoretical responses of the structure under seismic input are calculated and then HIM is introduced to estimate structural parameters and inverse input time history. Estimated results of parameters for noise free cases are shown in Table 3.

Table 3 Estimated results for noise-free case through HIM

Structural parameters	Case 1			Case 2		
	Initial value	Results	Error(%)	Initial value	Results	Error(%)
c_1	1.0	2469.8	0.04	-10.0	2469.9	0.13
c_2	1.0	2874.4	0.06	8239.0	2874.5	0.03
c_3	1.0	2571.9	0.21	-213.0	2572.3	0.01
c_4	1.0	2296.8	0.32	311.0	2297.5	0.01
c_5	1.0	264.6	0.32	761.0	2265.2	0.04
c_6	1.0	2091.8	0.30	-82.0	2092.3	0.09
k_1	1.0	271388	0.01	0.763	271382	0.03
k_2	1.0	290358	0.01	872.0	290353	0.02
k_3	1.0	282689	0.01	1000.0	282684	0.02
k_4	1.0	252481	0.01	-1278.0	252476	0.01
k_5	1.0	233542	0.01	7812.0	233537	0.01
k_6	1.0	229942	0.01	-73.0	229938	0.01

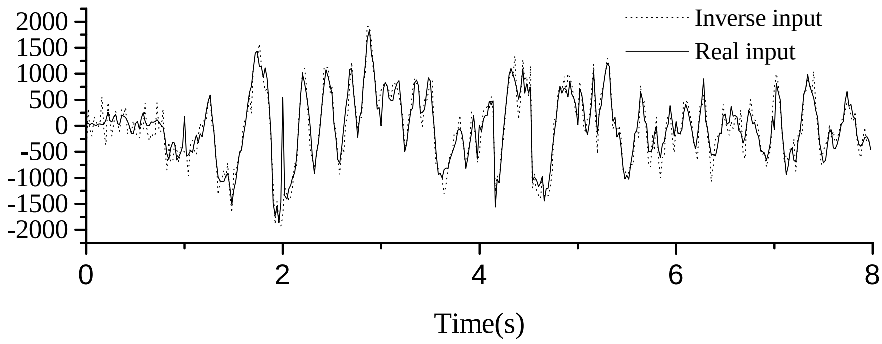
For both cases in Table 2, the sampling time interval is 0.02s. Only responses from 0.2s to 1.2s are used in case1 and 2.0s to 3.0s for case2. Although only 50 sample points are considered in both cases, the maximum error for stiffness estimation is only 0.003% and for damping parameter is 0.032%. The estimation errors are considerably smaller than those in other methods currently available in the literature. Different initial value of structural parameters are considered to verify its influence on the robustness and efficiency of the proposed method. It is clear from the Table 2 that even for a very poor initial guess: negative values for stiffness and damping parameters, the HIM procedure converge to exact values within little errors.

As a practical consideration, numerically generated white noise with rms intensities of 5%, 10% and 15% are added to the responses data of all DDOF. The estimated results through HIM for these three noise-included cases are given in Table 4. The sampling time interval is still 0.02s, but 400 sampling points are used in both cases.

Table 4 Estimated Results For Noise-Invluded Cases

Structural parameters	5% noise level		10% noise level		15% noise level	
	Estimated Result	Error (%)	Estimated result	Error (%)	Estimated Result	Error (%)
c_1	2520.7	2.07	2661.1	7.75	2885.2	16.8
c_2	2908.9	1.19	2997.5	4.28	3137.6	9.17
c_3	2594.8	0.87	2636.6	2.49	2697.0	4.84
c_4	2319.5	0.95	2355.7	2.53	2405.4	4.70
c_5	2274.3	0.39	2292.2	1.19	2318.9	2.37
c_6	2080.6	0.57	2052.9	1.89	2010.5	3.91
k_1	271418	0.01	271415	0.01	271367	0.01
k_2	290330	0.01	290220	0.04	290024	0.12
k_3	282578	0.01	282323	0.13	281929	0.27
k_4	252360	0.01	252090	0.15	251673	0.32
k_5	233202	0.01	232555	0.42	231608	0.83
k_6	229359	0.25	228267	0.73	226685	1.42

For 5% noise case, the maximum errors for stiffness estimation is 0.25% and for damping parameter is 2.07%. The corresponding maximum errors reported by Wang for a three-story shear building under seismic load are 1.75% and 5.43% when only 1% noise level is added to responses measurment of one story. The robustness of the proposed method can be seen from the results of 10% and 15% noise level cases. For 10% noise case, the maximum for damping coefficient is 7.75% and only 0.73% for stiffness estimation. The inverse time history of the input history for 10% case is shown in Fig.3



.Fig 3 Inverse Time History Of 10% Noise Level

It is evident from the numerical results that the proposed method is robustness and effective for structural parameter when input information is unknown.

CONCLUSION

A time domain sturctural parameters identification method with unknown input information , named as Hybrid Inverse Method (HIM) , is proposed in this paper. HIM has no striction for the type of the input excitation, and it can estmate structural parameters and inverse time history of input simultaneously using only response data. Integrated the physical/mechanical characteristic of the excitating force as a additional information into identification procedure is the base concept of HIM. Numerical example results show that for noise-free case HIM can give exact estimated results even when the data record is short. For noise-included case, HIM identified the structural parameter very well. It is evident that the proposed method is efficient in identifying actual existing structures when input excitation is hard to measured.

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