



## AN ENHANCED HYSTERETIC MODEL FOR SITE RESPONSE ANALYSIS

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### SUMMARY

A new nonlinear model for site response analysis was developed to more realistically simulate the observed soil nonlinearity in cyclic shear. The formulation is based on the Perfectly Hysteretic model with the tangent stiffness related to the most recent stress reversal only. The current (tangent) stiffness is expressed as a harmonic mean of the small strain and large strain stiffnesses with an additional component describing the maximum shear stiffness,  $G_{max}$ , as a function of maximum shear stress ratio. The model has three material parameters which allows the nearly independently adjustment of the shear modulus and damping curves. As a result, the model is able to closely reproduce realistic shear modulus degradation and damping curves obtained experimentally for clays and sands, in contrast to other existing nonlinear models with the same degree of complexity. The proposed model has been implemented in a general dynamic finite element code and used to perform 1-D site response analyses. Evaluation of the model performance showed that the new nonlinear model provides better predictions than the conventional equivalent linear method and similar to those obtained with other, more complex, nonlinear models.

### INTRODUCTION

Site response analyses is an important component of all analyses in the field of earthquake engineering. It is typically the first step in the evaluation of structural performance in any soil structure interaction problem. It was perhaps after the 1906 San Francisco earthquake, that local site conditions were first recognized to play an important role on the observed level of structural damage. Similar observations were obtained from the Niigata and Great Alaska Earthquakes of 1964 among others. As a result, a significant effort has been devoted to quantify the elements contributing to particular aspects of observed performance such as ground amplification and liquefaction over the last three decades.

The assessment of ground response under seismic excitation is generally performed by solving the simplified equations of motion resulting from the vertically propagating shear waves in the soil profile with appropriate boundary conditions. Given a geological profile and the input ground motion at prescribed locations, the objective of the analysis is to determine the soil response at selected locations (e.g., ground surface). The input ground motion for seismic site response analyses is generally specified as the acceleration time history at the lowest layer of the geological profile which typically corresponds to rigid bedrock. Characterization of the geological profile must include the vertical (and sometimes horizontal) stratigraphy as well as material properties for each identified layer. Since soil is a highly nonlinear material, the properties used in the analyses must also reflect these important characteristics. Once the input parameters for the site response analysis are defined, the problem becomes the determination of how the soil responds to the prescribed loading. The primary objective is to accurately model the soil non-linearity and yet accomplish this with a computationally efficient and robust numerical formulation. Methods used to describe nonlinear soil response for site response analyses fall into one of two categories: a) time or frequency domain equivalent linear and b) time domain nonlinear methods. Although the current State-of-the-Practice focuses heavily on the use of the equivalent linear method, recent developments in testing and modeling, a constant increase in computer speed and the need to analyze soil response at increasingly higher levels of ground acceleration has motivated a renewed interest in nonlinear methods of analyses (e.g., Lee and Finn, 1978; Li et al. 1992; Matasovic, 1993).

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One of the major shortcomings of existing nonlinear models is the inability to simultaneously generate realistic modulus reduction and damping curves such as the ones backcalculated from field behavior or measured in carefully performed laboratory experiments. In general, the following key elements should be considered in the development of nonlinear constitutive model to be used for site response analyses: a) accurate modeling of non-linearity and damping characteristics at relevant strain levels, b) the ability to independently match the observed shear modulus degradation and damping for a given soil and c) correctly track stress and strain reversals. The following sections describe a new simplified model developed at UC Berkeley [Lok, 1999] developed to describe the observed soil behavior, while at the same time providing for ease of numerical implementation and robustness of the computational algorithm. The following paragraphs briefly describe the key elements of the proposed model with particular emphasis in the simulation of clay response. Model performance is illustrated by comparison with predictions using SHAKE/SHAKE 91 [Schnabel et al., 1972, Idriss and Sun, 1992] and DESRA-2 [Lee and Finn, 1978] computer codes which are widely available and extensively used in geotechnical engineering practice. Further comparisons of model predictions with observed response at Treasure Island during the Loma Prieta Earthquake show that the formulation accurately captures the increased soil nonlinearity on higher levels of shaking.

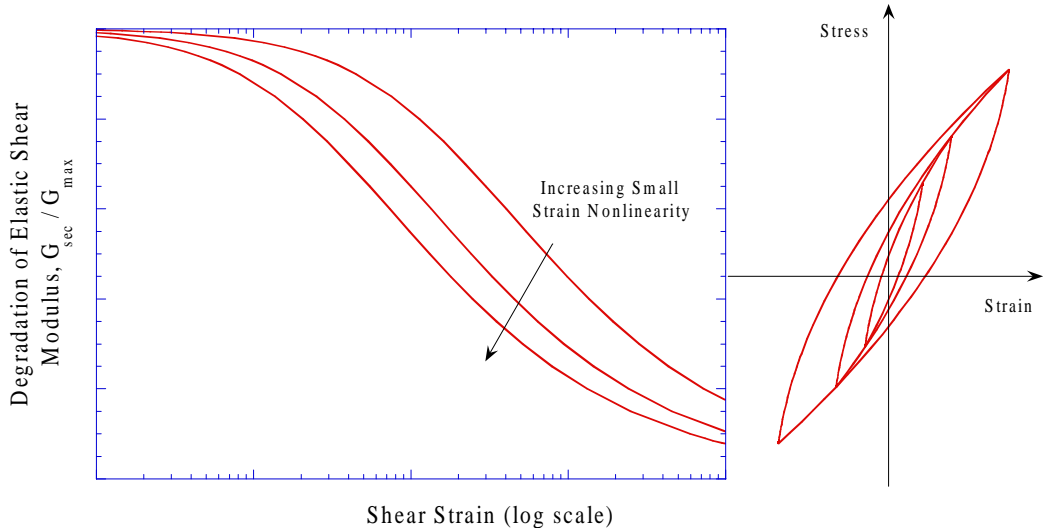
**MODEL FORMULATION**

The proposed model uses the concept of “Perfect Hysteretic” response originally proposed by Hueckel and Nova [1979] as shown in Figure 1. The model relates the tangential stiffness to the most recent stress reversal state as suggested by several researchers [e.g., Pyke, 1979; Hight et al., 1983]. It is possible to demonstrate that for monotonic loading, the model is equivalent to 'damage-based' formulations and to Rasberg-Osgood and Davidendok models which have been used with the help of the Masing rules to describe cyclic response. Pestana [1994] proposed a simple equation to describe the small strain non-linearity in shear and has been used for monotonic loading of sands, clays, and silts [Pestana, 1994; Pestana and Whittle, 1999]. The current shear modulus,  $G$ , is described as:

$$\frac{G}{G_{max}} = \frac{1}{(1 + \omega_1 \xi_s)} \tag{1}$$

where  $G_{max}$  is the shear modulus immediately at load reversal (i.e., strains  $\sim 10^{-4}\%$ ),  $\omega_1$  is a material constant, and  $\xi_s$  is a dimensionless measure in stress space given by:

$$\xi_s = \|\eta - \eta_{rev}\| \tag{2}$$



a) Typical Shear Modulus Degradation Curve for Soils      b) Hysteretic Loops of Increasing Amplitude

Figure 1: Perfect Hysteretic Conceptual Model

where  $\eta$  is the current shear stress ratio ( $= s/p$ , where  $s$  is the deviatoric stress tensor and  $p$  is the mean effective stress) and  $\eta_{rev}$  is the stress tensor at stress reversal. The modulus degradation and damping curves predicted by

this model are shown in Figure 2. For a given value of  $\omega$ , the secant shear modulus decreases and the damping increases with increasing shear strain level which is qualitatively similar to observed laboratory response. Similarly, as the parameter  $\omega$  increases, the secant shear modulus decreases and the damping level increases at any given strain level. Nevertheless, changes in the secant shear modulus are uniquely related to changes in damping through the hysteretic formulation.

### Stress Reversal Determination

The stress reversal point is defined by the direction of strain rates, which is based on the observation that the non-linearity of soil is most appropriately described in strain history [Hight et al., 1983]. This also becomes a more efficient approach than stress based algorithms when trying to implement in a displacement driven finite element code such as FEAP [Taylor, 1998]. The loading/unloading condition is based on the sign of the vector product of the accumulated deviatoric strain since last reversal and the current increment of strain:

$$\chi \cdot \delta\chi = \Delta\epsilon_s : \delta\epsilon_s = (> 0 \text{ loading}, \quad \leq 0 \text{ unloading}) \quad (3)$$

where  $\Delta\epsilon_s$  is the accumulated deviatoric strain from last reversal point, and  $\delta\epsilon_s$  are the current deviatoric strains.

### Enhanced Hysteretic Model Formulation

The perfect hysteretic formulation described previously has the ability to model the small strain non-linearity for soil and to keep track of strain reversal. However, the model described by only one parameter does not have enough flexibility to independently model the shear modulus degradation and damping behavior of most soils.

The following improvements are proposed to provide the modeling capabilities to adjust the non-linearity and damping for larger strain as well as to adjust non-linearity and damping independently, while maintaining the need of additional input parameters to a minimum. The fundamental assumption is that the tangential stiffness can be described as the harmonic mean of the small strain and larger strain shear-strain dependent stiffness:

$$\frac{1}{G} = \frac{1}{G_h} + \frac{1}{G_p} \quad (4)$$

The small strain shear stiffness  $G_h$  is described by the perfectly hysteretic formulation described previously:

$$\frac{G}{G_{\max}} = \frac{(1 + c \|\eta_{\max}\|)}{(1 + \omega_1 \xi_s)} \quad (5)$$

where  $G_{\max}$  is the small strain shear modulus at the beginning of loading,  $\omega_1$  is a material parameter describing the small strain non-linearity ( $< 10^{-2}\%$ ),  $\|\eta_{\max}\|$  is the norm of the maximum shear stress ratio in the loading history, and  $c$  is a material parameter describing the change of  $G_{rev}$ . Increasing values of  $c$  decreases the reversal stiffness at a given load reversal decreases with increasing the maximum stress ratio,  $\eta_{\max}$ , achieved during loading. The large strain stiffness,  $G_p$ , is given by:

$$G_p = \frac{G_h}{\omega_2 \xi_s^2} \quad (6)$$

As can be seen in figure 3, the effect of parameter  $\omega_2$  is similar to that of parameter  $\omega_1$  but only affects the response at larger strains (i.e.,  $> 10^{-2}\%$ ). In contrast, the effect of parameter  $c$  is such that as it increases, the secant shear modulus as well as the damping values decrease at any given strain level. This allows the model to nearly independently target a) the small strain nonlinearity ( $< 10^{-2}\%$ ) through parameter  $\omega_1$ , b) nonlinearity in the intermediate range ( $10^{-2}\% < \gamma < 1\%$ ) through primarily parameter  $\omega_2$ ; and c) damping levels with the additional parameter  $c$ .

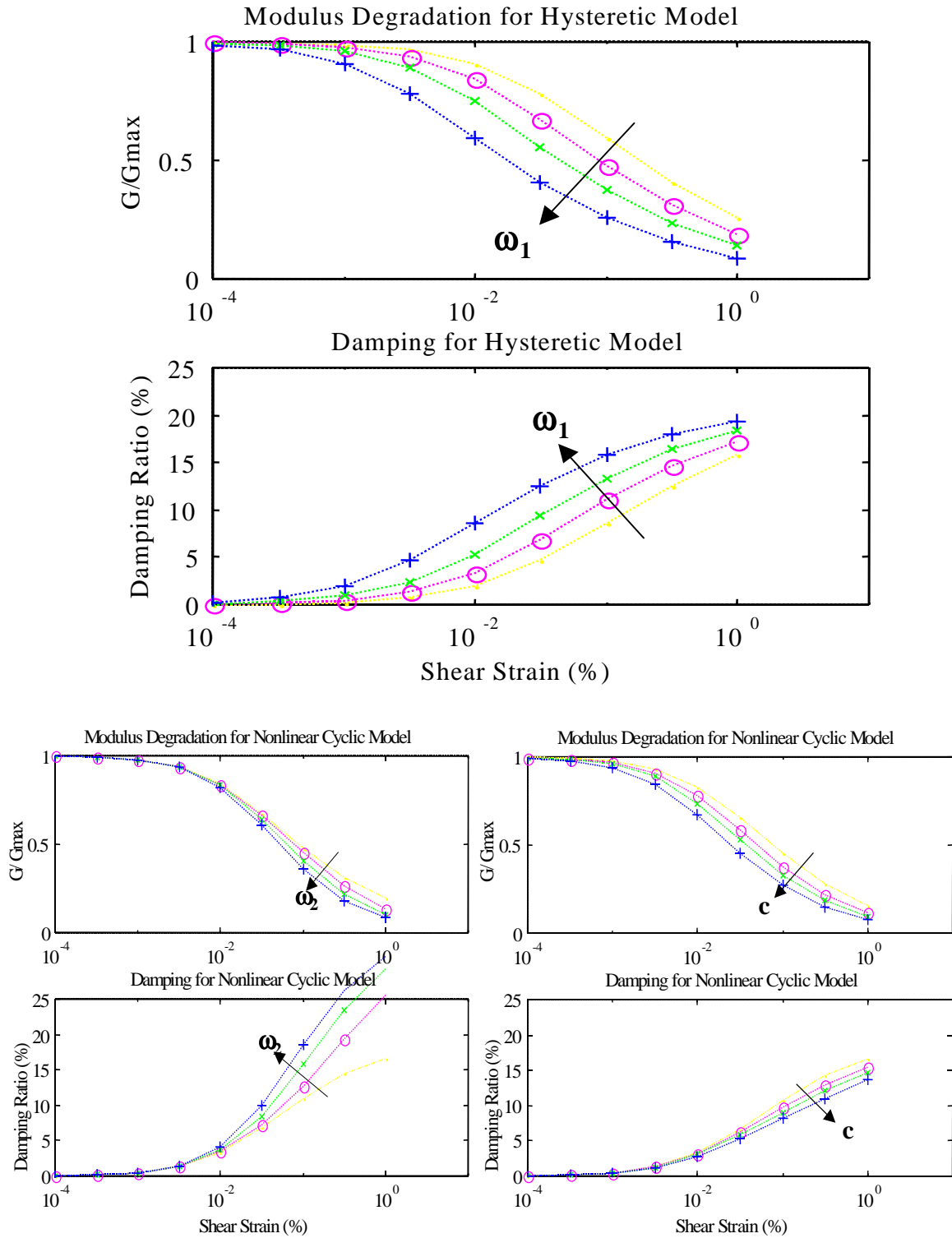


Figure 2: Effect of Material Parameters on the Predicted Shear Modulus Degradation and Damping.

### Description of Shear Modulus Degradation and Damping for Clays

The proposed model was used to simulate the cyclic behavior of clays. Figure 3 compares the modulus degradation and damping curves for clays obtained experimentally by Vucetic and Dobry [1991] with model simulations. An optimization procedure was employed to determine input parameters to match the observed behavior automatically. Other modulus degradation and damping curves can also be approximated with this model by carefully choosing the model parameters as described by Lok [1999]. In general, the model parameters can be chosen easily for a given set modulus degradation and damping curves. As can be seen from the graph,

the model simulates secant shear modulus which are in excellent agreement with measured behavior. The model predicts damping values which are in excellent agreement with measured values for strain levels larger than  $10^{-2}$  %. It is worth noticing that the “perfectly” hysteretic model describes near zero damping at small strains ( $10^{-4}$  %~ $10^{-3}$  %), where as measured response for clays seems to suggest a finite damping of 0.5-1% for this strain range and can not be captured with current formulation.

The new model was implemented in the computer code FEAP, a general-purpose finite element program developed at University of California, Berkeley (Taylor, 1998). The first version of this model was implemented as a 2-dimensional plane strain 4-node element. When performing one-dimensional site response analysis, static condensation is used to reduce the total number of degrees-of-freedom.

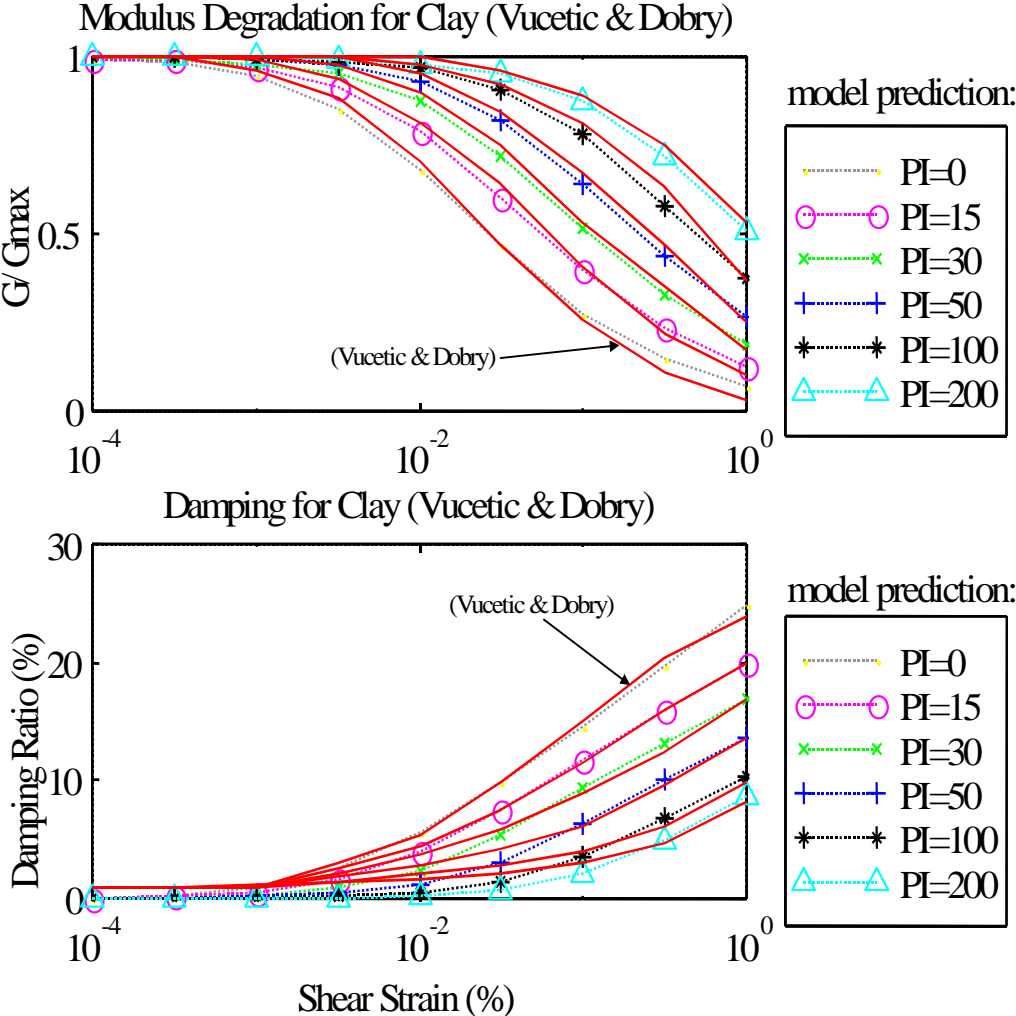


Figure 3: Model Prediction of Modulus Degradation and Damping for Clays

**MODEL EVALUATION**

A study was performed to analyze the Treasure Island records of the Loma Prieta Earthquake. The Loma Prieta earthquake occurred on October 17, 1989 along the San Andreas fault in the Santa Cruz area has a moment magnitude of MS 7.1. The records used in this study were obtained at the rock outcrop of Yerba Buena Island and the soil surface of Treasure Island, which are located about 80 km northwest of the epicenter. The peak acceleration ranged from 0.067 g at the rock outcrop to 0.16 g at the soil surface.

The Treasure Island profile consists of about 13 m sandy fill, which is underlain by about 16 m thick of Young Bay Mud. Underlying the Young Bay Mud are alternating layers of dense sand and Old Bay Mud to a depth of about 91 m. Weathered shale extends from this depth to about 98 m, where the more competent sandstone is encountered. Detailed information of the soil profile of the Treasure Island site and the modulus degradation and

damping curves for clayey and sandy soils can be found in the literature [e.g., Idriss, 1993] and will not be discussed here.

This event was analysed using the newly implemented model to examine the modelling ability for such magnitude. The results of the analyses are shown in Figure 4 for the two horizontal components together with the results obtained by Idriss [1993] using the equivalent linear procedure implemented in SHAKE91 [Idriss and Sun, 1992]. Overall, the new model provides a better match of the frequency content for both horizontal components. For the stronger component, the equivalent linear method provided a little better prediction in the range of period from 0.2 to 0.4 sec. However, the new model better predicts the peak ground acceleration. Figure 4 also includes the results obtained by Idriss [1993] using the non-linear code DESRA-2. In general, both methods provide reasonably match with the observation but the spectral accelerations are underestimated around the predominant period of the earthquake (i.e., 0.6-0.7 sec), which is also observed by the Idriss study. This may be partially attributed to local site conditions and surface wave effects at the Treasure Island, which is not accounted for by one-dimensional analyses. However, comparing to DESRA-2, the newly implemented matches more favourably with the observed spectral accelerations around the site period (i.e.,  $\approx$  1.3-1.5 sec) as well as peak ground acceleration.

## CONCLUSIONS

A new nonlinear model for site response analysis was formulated based on the perfectly hysteretic approach. The model has only three parameters and is able to reproduce the modulus degradation and damping curves experimentally obtained for clays with different degrees of plasticity. The model was implemented in a general purpose finite element program and evaluated against the Treasure Island site with the Loma Prieta Earthquake. Evaluation of the model performance showed that the new nonlinear model provides better predictions than the conventional equivalent linear method and similar to those obtained with other, more complex, nonlinear models.

The model was also used for the numerical analyses and soil-pile-superstructure shaking table experiments performed at U.C. Berkeley [Lok, 1999] and they are described by Lok et al. [1999]. In general, the analysis of the free field response with the newly-developed enhanced hysteretic model was very successful. The enhanced hysteretic model was able to provide a better description of the free field response with the same amount of the information used in the equivalent linear procedure.

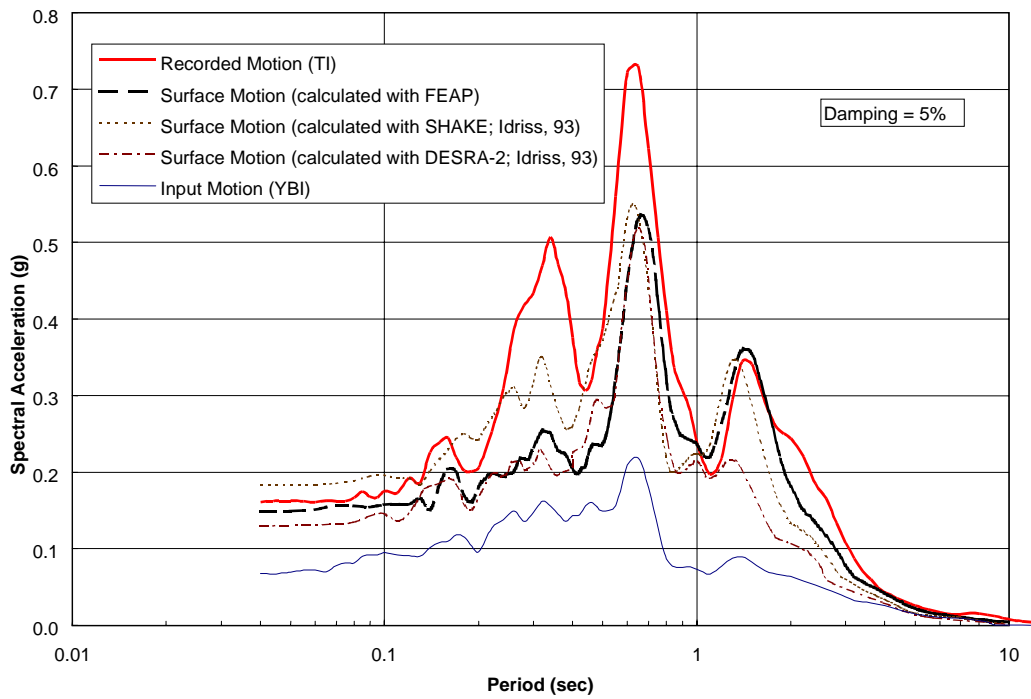
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Acceleration Spectra for Surface Motion at Treasure Island (90 degree)



Acceleration Spectra for Surface Motion at Treasure Island (0 degree)

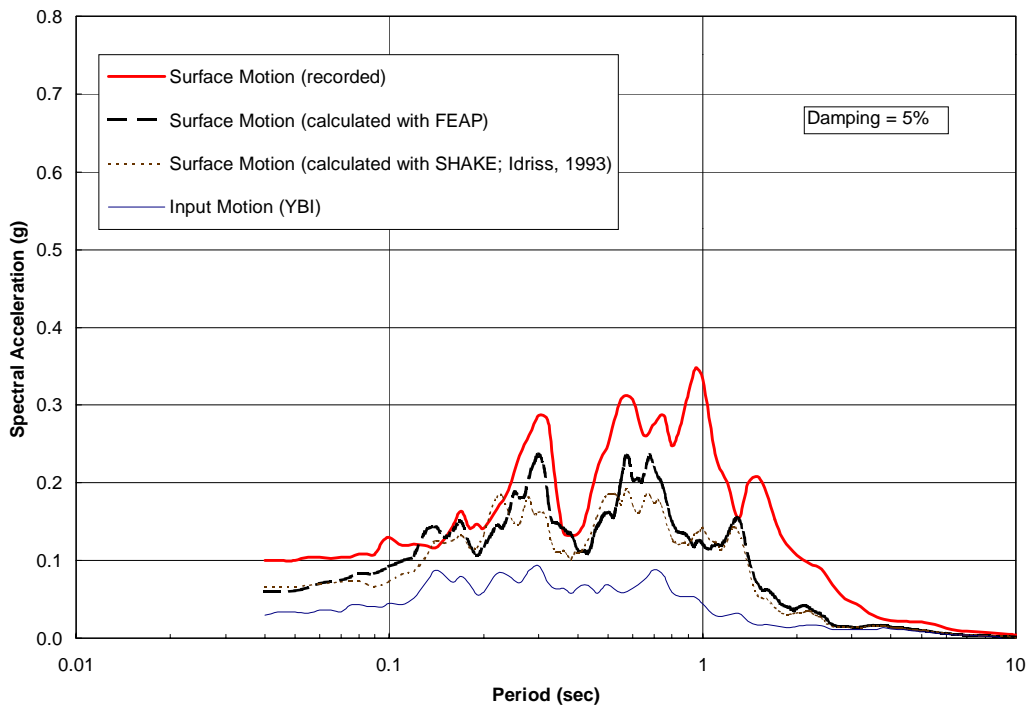


Figure 4 Comparison of Observed and Computed Response Spectra of Loma Prieta earthquake recorded at Treasure Island

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