THE ENGINEERING THEORY AND COMPUTATION METHOD FOR DESIGNING ISOLATION SYSTEM OF RUBBER BEARING AND ITS COMPOSITE DEVICES

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SUMMARY

An exquisite non-dimensional horizontal stiffness formula for rubber bearing is proposed. However the extensive discussions focus on stability and lateral stiffness characteristics for bearing-column serial system and composite isolator consisting of two rubber bearings with different cross sections connected by rigid plate in between. All the computation formulas of critical force and lateral stiffness are deduced on the base of engineering theory of Harings & Gent.

INTRODUCTION

Steel plate laminated rubber bearing as an elastomeric element, is such kind of seismic isolation device with very high anti-compression capability and low shear resistance. Usually, its vertical stiffness is hundred even over thousand times of its horizontal stiffness. In other word, its lateral displacement is dominated by shear deformation of rubber layers, and its critical buckling load is quite low although it likes a short column with small slender proportion. Hence the stability of the bearing against buckling and corresponding reduction in lateral stiffness usually should be put into account in design of base isolated structures. To estimate the buckling load and horizontal stiffness of rubber bearing, an elastic analysis model is adopted in which the multilayer bearing is regarded as a continuous column with bending and shear flexibility. And the analysis results of this model are basically consistent with experimental data. Extensive studies have been conducted by Kelly [1996] and Imbimbo & Kelly [1997] for post-buckling behavior or stability at large displacement and the influences of end plate rotation on buckling load and horizontal stiffness of elastomeric isolator. A new non-dimensional general expression and more delicate approximate formula for horizontal stiffness of rubber bearing was put forward by Zhou et al [1998]. Recently we have developed relatively rigorous expression of horizontal stiffness [Zhou et al 1999] for composite isolator proposed by Trics [1994] and a serial system of rubber and R/C column. In this paper, a more general solution for stability and lateral stiffness of composite isolator consisting of two rubber bearings connected each other by rigid plate, is given. The methodology of analysis is based on Haringx [1948~49] & Gent's [1964] engineering theory .

A PRACTICABLE EXPRESSION FOR HORIZONTAL STIFFNESS OF RUBBER BEARINGS

The horizontal stiffness $K_H(P)$ of rubber bearing, with boundary condition of one end to be fixed and the other to be free to move in horizontal direction but restricted against rotation, can be expressed by the following form [Zhou 1998].

$$\frac{K_H(P)}{K_H(0)} = \left(1 + \frac{1}{12\lambda^2}\right)\frac{\lambda p^2}{2\lambda p_\lambda \tan\frac{p_\lambda}{2} - p} \qquad (1)$$

where

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$$K_H(0)=\left(\frac{h}{GA}+\frac{h^3}{12EI}\right)^{-1} \qquad p=\frac{Ph}{\sqrt{EIGA}} \qquad \lambda=\sqrt{\frac{EI}{GAh^2}} \qquad p_\lambda = p\sqrt{1+\frac{1}{\lambda p}}$$

Eq. (1) can be replaced by the following approximate form proposed by Zhou [1998].

$$\frac{K_H(P)}{K_H(0)}=\left(1-\frac{p}{p_{cr}}\right)\left(1+\frac{p^*}{p_{cr}}\right) \tag{2}$$

where

$$p_{cr}=\sqrt{\frac{1}{4\lambda^2}+\pi^2}-\frac{1}{2\lambda} \qquad \frac{p^*}{p_{cr}}=0.79\lambda^{-1.57}+0.88$$

It can be pointed out that following widely used simplified formula is adoptable only if $\lambda \geq 4$

$$\frac{K_H(P)}{K_H(0)}=1-\left(\frac{p}{p_{cr}}\right)^2 \tag{3}$$

The relations between $K_H(p)/K_H(0)$ and p for different value of λ given by Eq.(1) ~ Eq.(3) are shown in Fig.1. It can be seen from Fig.1 that Eq. (1) gives good results for various values of λ. For the non-dimensional horizontal stiffness of rubber bearing Eq.(1) is an ideal form to reflect the influence of axial compression force and easy to prepare design diagram. In addition, it is very effective for sensitivity analysis of various parameters. Eq.(2) is a good approximation of Eq.(1) for wide range of λ.

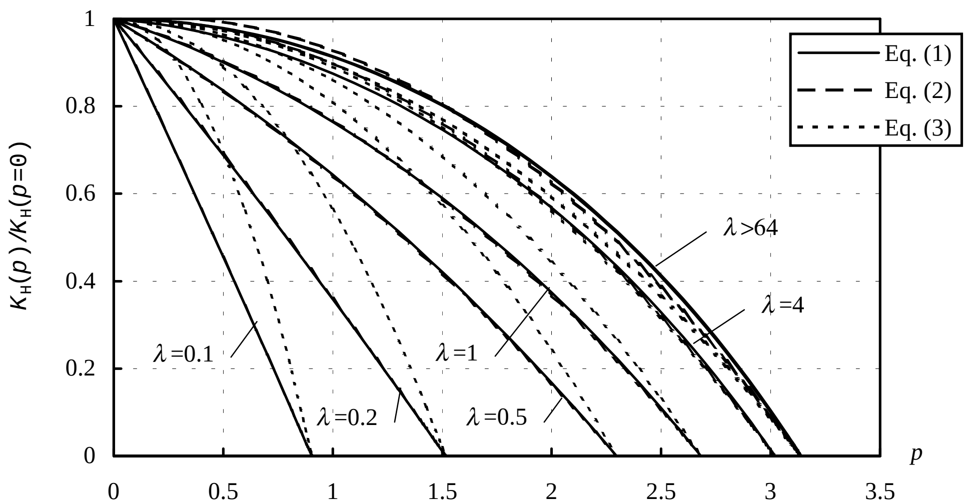

Fig.1 Comparison of the results from accurate formula and approximate ones (non-dimension results)

THE HORIZONTAL STIFFNESS COEFFICIENT OF THE LAMINATED RUBBER BEARING ISOLATOR IN SERIES WITH R/C COLUMN

In order to increase available space and save the amounts of building materials and construction work, the building owner and designer prefer install laminated rubber bearings at the top of R/C columns of the lowest story rather than at the bottom of it. In such a case the rubber bearing and R/C column composite of a serial system of seismic isolation. In bridge structures, the rubber bearing usually is installed at top of pier and also forms a serial system. It is assumed as followings so as to deduce computation formula of the horizontal stiffness coefficient of the rubber bearing isolator:

1) The top surface of the rubber bearing is free to move horizontally but controlled against rotation along the two orthogonal horizontal axes by the R/C column in series with it;
2) The slender proportion of the R/C column is moderate, so that the influence of the axial force on translation displacement and shear deformation can be neglected;
3) The lateral deformation of rubber bearing subjected to axial and horizontal loading can be analyzed according to the approximate theory developed by Haringx [1948~1949] & Gent [1964].

On the base of above assumptions the following expression of horizontal stiffness of the rubber bearing in the bearing-column serial system has been obtained by Zhou et al [1999].

$$\frac{K_H(p)}{K_H(0)}=\frac{p\left[1+\frac{1}{12\lambda^2(1+\gamma)}\left(1+4\gamma+3\gamma\frac{h_c}{h_b}\right)\right]\left(\frac{p\sin p_\lambda}{p_\lambda}+p\gamma\cos p_\lambda\right)}{2\left(1+\frac{p\gamma}{4\lambda}\frac{h_c}{h_b}\right)-\left[2+\frac{p\gamma}{\lambda}\left(1+\frac{h_c}{2h_b}\right)\right]\cos p_\lambda+\left(\gamma p_\lambda-\frac{p}{\lambda p_\lambda}\right)\sin p_\lambda} \tag{4}$$

and

$$K_H(0)=\frac{G_sA_s}{h}\frac{1+\gamma}{1+\gamma+\frac{1}{12\lambda^2}\left[1+\gamma\left(4+\frac{3h_c}{h_b}\right)\right]} \tag{5}$$

where

$$p=\frac{Ph_b}{\sqrt{E_bI_bG_bA_b}} \qquad \lambda=\sqrt{\frac{E_bI_b}{G_bA_bh_b^2}} \quad \gamma=\frac{h_c}{h_b}\frac{E_bI_b}{E_cI_c} \qquad p_\lambda=p\sqrt{1+\frac{1}{\lambda p}} \tag{6}$$

and P is the applied axial compression force. The subscript b and c is representative of rubber bearing and R/C column respectively and the meanings of other notations are obvious.

It is interesting to point out that if $\lambda\to\infty$, $K_H(0)=G_sA_s/h$, $K_H(p)/K_H(0)$ is independent of h_b/h_c directly, and Eq. (4) can be simplified as follows

$$\frac{K_H(p)}{K_H(0)}=\frac{p(\sin p+p\gamma\cos p)}{2(1-\cos p)+p\gamma\sin p} \tag{7}$$

For various values of γ, the $K_H(p)/K_H(0)$~ p relation can be calculated by Eq. (4) and the results are shown in Fig.2 by solid lines. However the R/C column in the serial system will constrain the rubber behaving like a moment spring if $\lambda\to\infty$, and now the serial system is just as the case of a rubber bearing with flexible end condition at top plane as it is studied in Ref. (Imbimbo & Kelly [1997]). For the sake of comparing the accuracy of the following approximate formula

$$\frac{K_H(p)}{K_H(0)}=1-\frac{1+4\gamma}{1+\gamma}\frac{p^2}{12} \tag{8}$$

given by Imbimbo & Kelly [1997], the corresponding results calculated from Eq. (8) are also shown in Fig.2. As Imbimbo & Kelley have pointed out that the numerical coefficient $\sqrt{12}$ in p of Eq. (8) can be changed into π. Hence the calculation results from Eq. (8) are shown in Fig.2 by dotted and dashed lines respectively corresponding to the numerical coefficient 12 or π^2. It can be seen from the curves shown in Fig.2 that the results by the approximate Eq. (8) contain somewhat difference whenever the numurical coeffient in Eq. (8) is 12 or π^2.

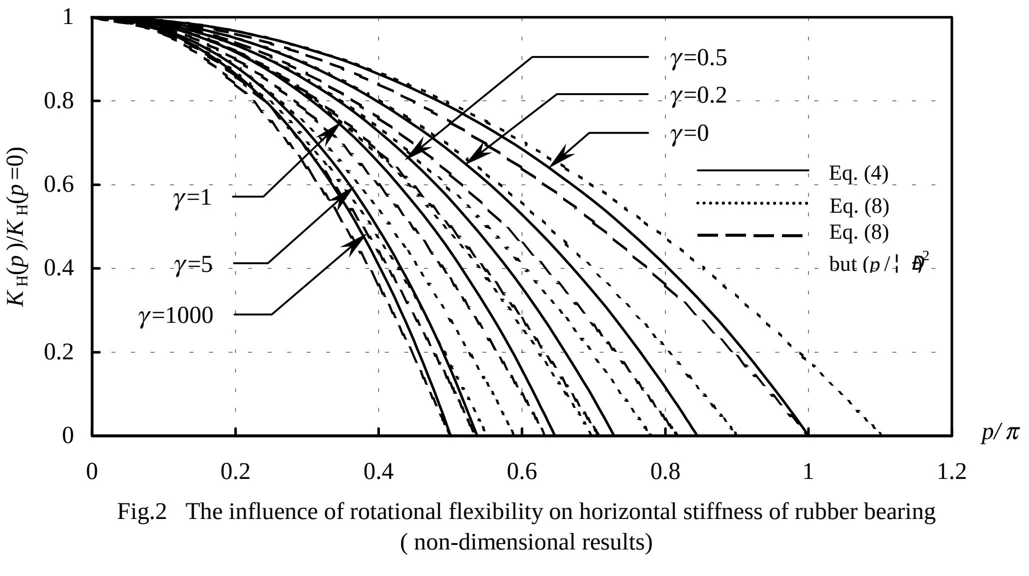

Fig.2 The influence of rotational flexibility on horizontal stiffness of rubber bearing (non-dimensional results)

THE DEFORMATION PATTERN OF THE COMPOSITE ISOLATOR SUBJECTE TO LATERAL FORCE

The composite isolator considered in this paper is made from two single isolators that are linked together by using rigid plate with infinitive stiffness in between (Fig.3). The low end of the composite isolator is fixed and the upper one is free to move in horizontal direction but restrained against rotation. Suppose a lateral force F and vertical load P are applied on top of the isolator simultaneously, the deformation patter of the isolator has been shown in Fig.4. Thus the horizontal stiffness of the composite isolator K_H becomes

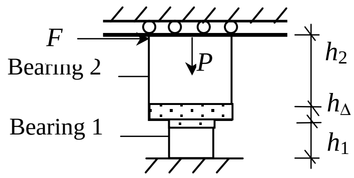

Fig.3 Composite isolator

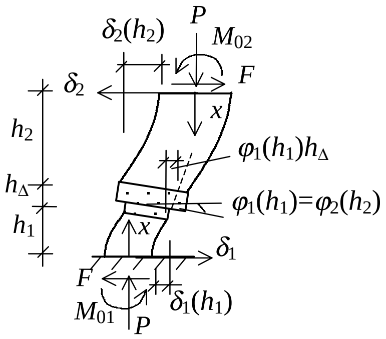

Fig.4 The deformation and reaction force of composite isolator

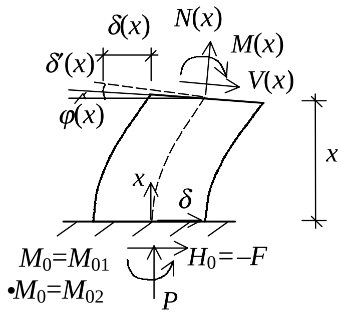

Fig.5 Isolator

$$K_H = \frac{F}{\delta_1(h_1) + \delta_2(h_2) + \varphi_1(h_1)h_\Delta} \tag{9}$$

where $\delta_1(h_1)$ is lateral displacement of the lower bearing, $\delta_2(h_2)$ is that of the upper bearing, $\varphi_1(h_1)= \varphi_2(h_2)$ is rotation angle of the rigid plate.

HORIZONTAL STIFFNESS OF COMPOSITE ISOLATOR

The analyses of the critical buckling load and lateral stiffness of the composite isolator are based on the approximate theory developed by Haringx [1948~49] and Gent [1964] and the recent work by Kelly [1996] and Imbimbo & Kelly [1997] on elastomeric bearings. This is a applied engineering theory in which the steel plate laminate rubber bearing is regarded homogeneous elastic column with equivalent bending stiffness *EI* and shear stiffness *GA*. According to the local coordinate system the isolated body with height within $0<x<h$ for each of the rubber bearing up and down can be described in Fig.5. In case of the reaction moment M_0 and shear force F are known, the horizontal displacement $\delta(x)$ and rotation angle of cross section $\varphi(x)$ at height x was deduced by Imbimbo & Kelly [1997].

$$\begin{cases} \delta_i(x) = -\dfrac{M_{0i}}{P}\cos\alpha_i x + \dfrac{F}{\alpha_i\beta_i P}\sin\alpha_i x + \dfrac{M_{0i}}{P} - \dfrac{F}{P}x \\ \varphi_i(x) = \dfrac{F}{P}\cos\alpha_i x + \alpha_i\beta_i\dfrac{M_{0i}}{P}\sin\alpha_i x - \dfrac{F}{P} \end{cases} \quad (i=1,2) \tag{10}$$

From the integrative equilibrium condition of the whole composite isolator under external and generalized reaction force the following formula can be obtained

$$M_{01} + M_{02} = F(h_1 + h_2 + h_\Delta) + P[\delta_1(h_1) + \delta_2(h_2) + \varphi_1(h_1)h_\Delta] \tag{11}$$

Substituting Eq. (10) into above equation we obtain

$$M_{01}(\cos\alpha_1 h_1 - \alpha_1\beta_1 h_\Delta \sin\alpha_1 h_1) + M_{02}\cos\alpha_2 h_2 = F\left(\frac{\sin\alpha_1 h_1}{\alpha_1\beta_1} + \frac{\sin\alpha_2 h_2}{\alpha_2\beta_2} - h_\Delta\cos\alpha_1 h_1\right) \tag{12}$$

in which

$$\alpha_i{}^2 = \frac{P(P+G_iA_i)}{E_iI_iG_iA_i} \qquad \beta_i = \frac{G_iA_i}{P+G_iA_i} \quad (i=1,2) \tag{13}$$

which are also given by Imbimbo & Kelly [1997].

Substituting the second equation of Eq. (10) into continuous condition of $\varphi_1(h_1)=\varphi_2(h_2)$ the second equation for determination of M_{01} and M_{02} is attainable as follows

$$\alpha_1\beta_1M_{01}\sin\alpha_1h_1 - \alpha_2\beta_2M_{02}\sin\alpha_2h_2 = F(\cos\alpha_2h_2 - \cos\alpha_1h_1) \tag{14}$$

If the lateral force F=0, the above Eq.(13) and Eq. (14) turn into

$$\begin{cases} M_{01}(\cos\alpha_1h_1 - \alpha_1\beta_1h_\Delta\sin\alpha_1h_1) + M_{02}\cos\alpha_2h_2 = 0 \\ \alpha_1\beta_1M_{01}\sin\alpha_1h_1 - \alpha_2\beta_2M_{02}\sin\alpha_2h_2 = 0 \end{cases} \tag{15}$$

Let the determinant of the matrix of the coefficients of Eq. (12) and Eq. (14) equals zero, the equation of critical buckling force is obtained below

$$\tan\alpha_1h_1 = \alpha_2h_2\tan\alpha_2h_2\left(h_\Delta\tan\alpha_1h_1 - \frac{1}{\alpha_1\beta_1}\right) \tag{16}$$

Compared with the corresponding critical force equation of composite isolator without rigid plate given by Imbibe & Kelly [1996] and Zhou et al [1999], Eq. (16) involves an additional term related to h_Δ.

If $F\neq0$ the M_{01} and M_{02} can be obtained from general solution of Eq. (12) and Eq. (14) and then substituting them into Eq. (10), the formula of $\delta_1(h_1)$, $\delta_2(h_2)$ and $\varphi_1(h_1)$ can be obtained, and finally the horizontal stiffness of the composite isolator is attainable by using Eq. (9). Introducing follow non-dimensional variables

$$p = \frac{Ph_2}{\sqrt{E_2I_2G_2A_2}} \qquad \zeta = \frac{h_1}{h_2} \qquad \zeta_\Delta = \frac{h_\Delta}{h_2}$$

$$\xi_b = \frac{E_1I_1}{E_2I_2} \qquad \xi_s = \frac{G_1A_1}{G_2A_2}$$

$$\eta = h_2\sqrt{\frac{G_2A_2}{E_2I_2}}$$

and performing simplifications the following non-dimensional horizontal stiffness is given

$$\frac{K_H(p)}{K_H(0)} = q\frac{\Psi_1\sin p_1\cos p_2 + \Psi_2\sin p_2\cos p_1 - \Psi_1\Psi_2\zeta_\Delta\sin p_1\sin p_2}{2+[\Psi_3\sin p_1 - (1+\zeta)\Psi_2\cos p_1]\sin p_2 - (2\cos p_1 + (1+\zeta)\Psi_1\sin p_1)\cos p_2} \tag{17}$$

where

$$q = \frac{p}{\eta}\left\{1+\frac{\zeta}{\xi_s}+\frac{\eta^2}{12(\zeta+\xi_b)}\left[\frac{\zeta^4}{\xi_b}+\xi_b+4\zeta(1+1.5\zeta+\zeta^2)+12\zeta\zeta_\Delta(1+\zeta+\zeta_\Delta)\right]\right\}$$

$$p_1 = \zeta\sqrt{\frac{p}{\xi_b\xi_s}(p+\eta\xi_s)} \qquad p_2 = \sqrt{p(p+\eta)} \qquad \Psi_1 = \eta\sqrt{\frac{p\xi_s}{\xi_b(p+\eta\xi_s)}}$$

$$\Psi_2 = \eta\sqrt{\frac{p}{p+\eta}} \qquad \Psi_3 = \sqrt{\frac{\xi_s(p+\eta)}{\xi_b(p+\eta\xi_s)}} + \sqrt{\frac{\xi_b(p+\eta\xi_s)}{\xi_s(p+\eta)}} + \Psi_1\Psi_2\zeta_\Delta(1+\zeta+\zeta_\Delta)$$

$$\frac{1}{K_H(0)} = \left(\frac{h_1}{G_1A_1}+\frac{h_2}{G_2A_2}\right)\left[1+\xi_s\eta^2\frac{\zeta^4+\xi_b{}^2+4\zeta\xi_b(1+1.5\zeta+\zeta^2)+12\zeta\xi_b\zeta_\Delta(1+\zeta+\zeta_\Delta)}{12\xi_b(\xi_b+\zeta)(\xi_s+\zeta)}\right] \tag{18}$$

Let $K_H(p)/K_H(0)$=0, an equivalent critical force equation with Eq. (17) be obtained

$$\frac{1}{\Psi_1\tan p_1}+\frac{1}{\Psi_2\tan p_2} = \zeta_\Delta \tag{19}$$

If ζ_Δ=0, Eq. (19) turns into

$$\sqrt{\frac{\xi_s(p+\eta)}{\xi_b(p+\xi_s\eta)}}\tan\left[\zeta\sqrt{\frac{p(p+\xi_s\eta)}{\lambda\xi_b}}\right] = -\tan\sqrt{p(p+\eta)} \tag{20}$$

The relations among non-dimensional horizontal stiffness $K_H(p)/K_H$ (0) of the composite isolator and parameters p, ζ, ξ_b, ξ_s, η are given in Fig.6~9 while ζ_Δ=0.2, 0.1, 0.0. It can be seen from these figures that $K_H(p)/K_H$ (0) is decreased with increasing p and slightly decreased with increasing ζ_Δ. This result shows that the non-dimensional horizontal stiffness in consideration of the influence of the rigid plate is slightly lower than that of without rigid plate and as increasing thickness of the rigid plate, $K_H(p)/K_H$ (0) decreases further more. Remaining other parameters unchanged, $K_H(p)/K_H$ (0) and critical force decreases with increasing ζ as indicated by Fig.6, increases with increasing ξ_b as shown in Fig.7. In addition, Fig.8 and Fig.9 show increasing and decreasing tendency of $K_H(p)/K_H$ (0) and critical force of the composite isolator respectively with increasing ξ_s and η. However the results from Fig.6~9 indicate that the influence of the rigid plate on horizontal stiffness is insignificant if $\zeta_\Delta \leq 0.2$.

HORIZONTAL STIFFNESS OF SERIAL SYSTEM OF RUBBER BEARING AND BEND COLUMN

If one of bearings shown for example the lower in Fig.3 is considered as column, the composite isolator becomes general model of serial system of rubber bearing and column, and the simplified model discussed in previous paragraph is its special case. However the shear deformation of column usually can be neglected and that is the case of ξ_s (or G_1A_1) $\rightarrow\infty$. Thus the parameters involved in Eq. (17) can be simplified as

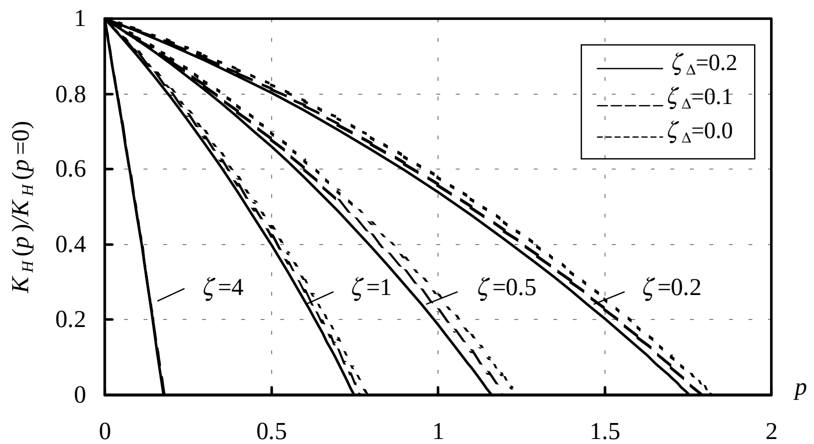

Fig.6 Relation of non-dimensional horizontal stiffness and axial pressure of composite rubber bearing (ξ_s=0.5 ξ_b=0.5 η=1.0)

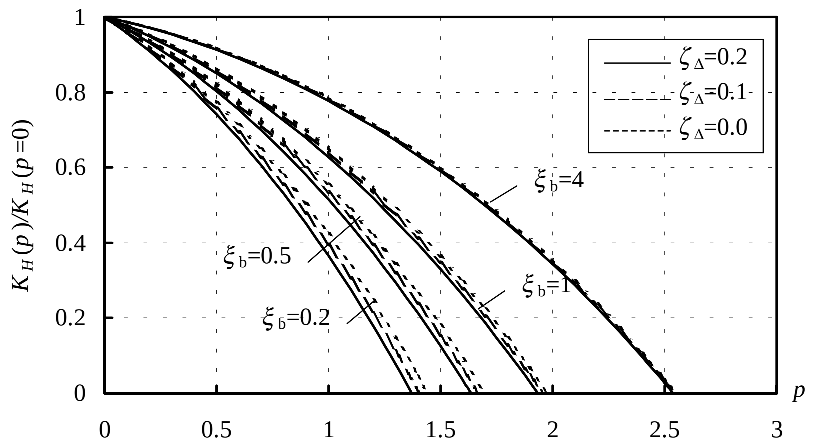

Fig.7 Relation of non-dimensional horizontal stiffness and axial pressure of composite rubber bearing (ξ_s=0.5 ζ=0.5 η=0.5)

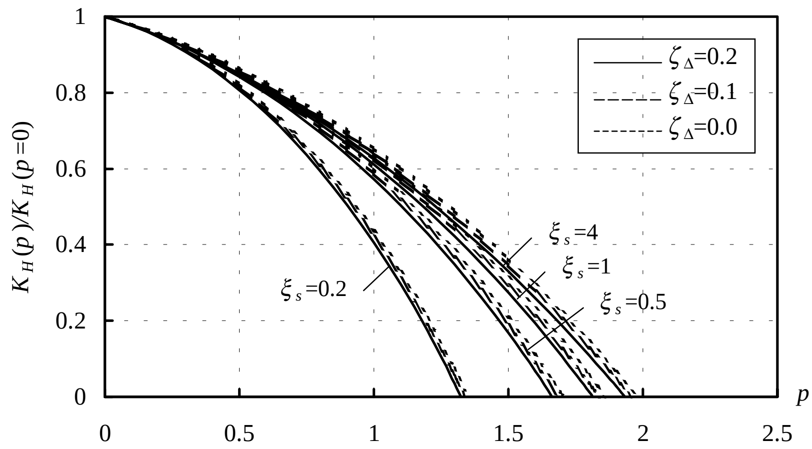

Fig.8 Relation of non-dimensional horizontal stiffness and axial pressure of composite rubber bearing (ξ_b=1 ζ=0.5 η=0.5)

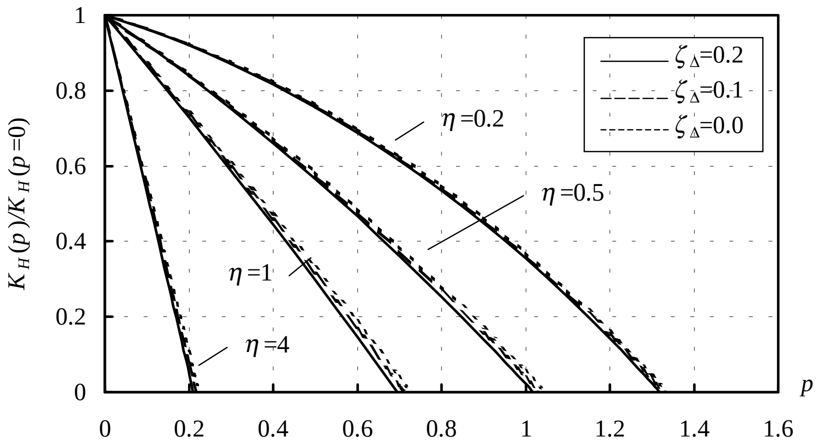

Fig.9 Relation of non-dimensional horizontal stiffness and axial pressure of composite rubber bearing (ξ_s=4 ξ_b=4 ζ=4)

$$\begin{cases} q=\dfrac{p}{\eta}\left\{1+\dfrac{\eta^2}{12(\zeta+\xi_b)}\left[\dfrac{\zeta^4}{\xi_b}+\xi_b+4\zeta\left(1+1.5\zeta+\zeta^2\right)+12\zeta\zeta_\Delta\left(1+\zeta+\zeta_\Delta\right)\right]\right\} \\ p_1=\zeta\sqrt{\dfrac{P\eta}{\xi_b}} \qquad p_2=\sqrt{p(p+\eta)} \\ \Psi_1=\eta\sqrt{\dfrac{p}{\eta\xi_b}} \qquad \Psi_2=\eta\sqrt{\dfrac{p}{p+\eta}} \qquad \Psi_3=\sqrt{\dfrac{p+\eta}{\xi_b\eta}}+\sqrt{\dfrac{\xi_b\eta}{p+\eta}}+\Psi_1\Psi_2\zeta_\Delta\left(1+\zeta+\zeta_\Delta\right) \end{cases} \tag{21}$$

The $K_H(p)/K_H(0)$~p curves calculated by Eq. (17) and Eq. (21) show the relations among $K_H(p)/K_H(0)$ and p, ζ, ξ_b, η of serial system of bearing-column are similar to that of composite isolator but the critical force is slightly increased.

CONCLUSION

1. The approximate formula Eq. (3) is appropriate to larger λ of 4.0 which corresponds the case when shear deformation is dominant in horizontal displacement of rubber bearing, but the proposed Eq. (1) and Eq. (2) are good for various λ weather it is large or small.

2. A close form solution for horizontal stiffness of composite isolator consisting of two rubber bearing in series with rigid plate in between is deduced, and the non-dimensional expression of horizontal stiffness given in this paper is suitable to draw up design diagram. When the thickness of the rigid plate is small compared with the height of one of rubber bearings in composite isolator, for example the height ratio less than 0.2, the influence of the rigid plate on horizontal stiffness is insignificant.

3. The bearing-column serial system can be regarded as a special case of composite isolator that has been deliberately discussed in this paper.

REFERENCE

1. A.N. Gent, Electric Stability of Rubber Compression Springs, Journal of Mechanical Engineering Science, 6(4): 318-326, 1964.

2. J.A. Haringx, On Highly Compressive Helical Springs and Rubber Rod and Their Applications to Free Mountings-Part I, II and III, Philips Reports, 1948-1949.

3. J.M. Kelly, Earthquake-Resistant Design with Rubber, 2nd ed., Springer-Verlag, London, 1996.

4. M. Imbimbo and J.M. Kelly, Stability of Isolators at large Horizontal Displacements, Earthquake Spectra, Vol.13, No.3, August 1997.

5. X.Y. Zhou, M. Han, D.M. Zeng, D.H. Ma, A Simplified Calculation Method of the Horizontal Rigidity Coefficient for the Serial System of Rubber Bearing with Column, Journal of Vibration Engineering, Vol.12, No.2, 1999.6 (in Chines).

6. A.G. Tarics, Composite Seismic Isolator and Method, International Workshop on Use of Rubber Based Bearing for Earthquake Protection of Building, Shantou China, May 17-19, 1994.

7. M. Imbimbo and J.M. Kelly, Stability Aspects of Elastomeric Isolators, Earthquake Spectra, Vol.13, No.3, August 1997.

8. X.Y. Zhou, M. Han, D.M. Zeng, D.H. Ma, Calculation Method of Lateral Stiffness of Combined Rubber Bearing and Serial System of Bearing With Columns, Earthquake Engineering and Engineering Vibration, Vol.19, No.4, 1999.12 (in Chines).

9. X.Y. Zhou, M. Han, D.M. Zeng, D.H. Ma, A Practical Computation Method for Horizontal Rigidity Coefficient of Seismic Isolation Rubber bearing, Building Science, Vol.14, No.6, 1998.12 (in Chines)