DYNAMIC ANALYSIS OF LIQUID-TANK-SOIL SYSTEM USING BE-FE-BE COUPLING

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SUMMARY

This study is numerically simulating the behavior of a base-isolated liquid storage tank under seismic excitation. In order to consider the liquid-structure-soil interaction, boundary elements, finite elements and boundary elements (BE-FE-BE) coupling method is adopted, which combines the versatility of shell finite elements for the tank with the efficiency of boundary elements for liquid and soil. The base-isolation system is idealized using effective springs and dampers and it is included in the entire system by connecting the liquid-structure and foundation-soil. The numerical results are compared with the reference solutions for the respective interaction problems to assure the validity and accuracy of the developed analysis technique. Then, an earthquake-response analysis is carried out to demonstrate the applicability of the developed technique.

INTRODUCTION

The dynamic response of liquid storage tanks under seismic ground motion is different from that of common structures such as buildings or bridges. It is well known that the difference of the response is caused by the effect of hydrodynamic pressure. A number of the studies have been performed numerically and analytically to count this effect. In those studies, it is usually assumed that the structure is connected rigidly to the ground even though the soil-structure interaction is a very important factor that affects the response of the whole system. In addition, the base-isolation system is used in practice to reduce the damage due to seismic motion. Therefore, for precise analysis, it is necessary to consider the whole system by solving the three dimensional liquid-structure-soil interaction including the base-isolation system.

In this study, the structure part is modeled using the degenerated shell finite elements that can model heterogeneous materials and arbitrary shape of the structure. The liquid contained in the tank is assumed to be inviscid, incompressibe and irrotational ideal fluid to simplify the formulation. Based on these assumptions, the liquid is modeled by internal boundary elements, which can reduce the size of global matrices by discretizing only the surface of a liquid volume. The soil region is modeled by external boundary elements. To obtain the dynamic fundamental solutions for the half-space, the numerical integration algorithm over wavenumbers is investigated in the frequency domain using the generalized reflection and transmission coefficients [Kim et al., 1999]. Through these procedures, the motion of the base-isolated liquid storage tank on the half-space is analyzed combining the three parts by BE-FE-BE coupling.

MODELING OF FLUID, STRUCTURE, AND SOIL REGION

Modeling of the fluid region with boundary elements

The fluid in the tank is assumed to be inviscid and incompressible and the flow of the fluid is presumed to be irrotational and time harmonic. According to these, the governing equation of the fluid may now be expressed in the form of Laplace equation.

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$$\nabla^2\phi(\mathbf{x},t)=0 \tag{1}$$

Where, ϕ : the velocity potential, $\mathbf{x}=(x,y,z)$: the position vector.

In order to solve the Eq. (1), the boundary element method is applied and the obtained boundary integral equation is expressed in matrix form as follows.

$$[H]\{\phi\}=[G]\{q\} \tag{2}$$

Here, $[H]$ and $[G]$ are boundary element coeffient matrices for the velocity potential vector(ϕ) and the flux vector(q), respectively.

The boundary element equation must be converted into the form of the finite element equation for coupling. Using the boundary conditions at the free surface and interface between fluid and structure, the hydrodynamic pressure on the tank wall can be obtained as following form.

$$\{P_t\}=\omega^2\rho[B_I]^{-1}[B_{II}]\{u_t\} \tag{3}$$

The matrices $[B_I]$ and $[B_{II}]$ are given by

$$[B_I]=[H_{tt}]+\omega^2[G_{tf}][D]^{-1}[H_{ft}]-g[H_{tf}][D]^{-1}[H_{ft}] \quad [B_{II}]=[G_{tt}]+\omega^2[G_{tf}][D]^{-1}[G_{ft}]-g[H_{tf}][D]^{-1}[G_{ft}] \tag{4}$$

where, $[D]=g[H_{tt}]-\omega^2[G_{ff}]$ and the subscript t denotes the nodes on the tank wall and the subscript f denotes the nodes for the free surface of liquid. In order to convert the hydrodynamic pressure into equivalent nodal force of the finite element, the shape function, $[N]$ is introduced here. Then the hydrodynmic pressure can be expressed as the finite element form as follows

$$\{F(\omega)_p\}=\omega^2[M^{BE}]\{u_t\} \tag{5}$$

where, $[M^{BE}]=\rho[N][B_I]^{-1}[B_{II}]$: the mass matrix of the boundary elements for the fluid

Modeling of the tank with finite elements

The governing equation of motion for the tank structure subjected to ground excitation can be expressed in the matrix form as follows.

$$[M^{FE}]\{\ddot{\overline{u}}\}+[C^{FE}]\{\dot{\overline{u}}\}+[K^{FE}]\{\overline{u}\}=\{\overline{F}\} \tag{6}$$

Where, $[M^{FE}]$: the mass matrices of the finite elements

$[K^{FE}]$: the stiffness matrices of the finite elements

$[C^{FE}]$: the damping matrices of the finite elements

By performing the Fourier transform, Eq. (6) is rewritten in the frequency domain as the following form.

$$[[K^{FE}]+i\omega[C^{FE}]-\omega^2[M^{FE}]]\{u(\omega)\}=\{F(\omega)\} \tag{7}$$

Where, $\{F(\omega)_e\}=\{F(\omega)\}=\{F(\omega)_e\}+\{F(\omega)_p\}$: the generalized force matrix

$\{F(\omega)_e\}$: the external force matrix and $\{F(\omega)_p\}$: the hydrodynamic force matrix

Modeling of the soil region with boundary elements

When the finite portion of half-space is occupied by the foundation, the layered half-space is created by imposing the equilibrium and compatibility conditions for the displacements over the entire surface of foundation, ¥Ĺ. Since the horizontal flat surface is stress free, the traction of Green's functions automatically vanish on ¥Ĺ. Following equation is established by using the quadratic shape function at each element.

$$u_{ip}(\boldsymbol{x}) = \sum_{k=1}^{N} t_{jk} \left[\int_{¥Ã} u^{*}_{ji}(\boldsymbol{x}_s ; \boldsymbol{x}) d¥Ã \right] \qquad (i, j = x, y, z) \tag{8}$$

Where $\boldsymbol{x} = (x, y, 0)$ and $\boldsymbol{x_s} = (x_s, y_s, 0)$ represent the source point and the observation point located on the traction-free surface, respectively. The function u^{*}_{ji} represents the *j-th* component of the displacement at point $\boldsymbol{x}$ due to a concentrated point force at point acting in the *i-th* direction and t_j represents the *j-th* component of the surface traction. The value u_{ip} represents the *i-th* component of the displacement of the *p-th* element, and t_{jk} represents the *j-th* component of the surface traction in the *k-th* surface element. Eq. (8) can be written conveniently in a matrix form as

$$\{u\} = [G]\{t\} \tag{9}$$

where, the matrix $[G]$ represents the force-displacement relationship among the elements of ¥Ĺ and is called as compliance matrix for the flat surface ¥Ĺ. To match the traction by boundary elements with the nodal force by finite elements for coupling, the matrix $[N]$ is introduced. Below equation is obtained by considering the applied nodal force on the foundation and displacement equilibrium requirements.

$$[N]^T [G]^{-1} [N]\{u\} = [S_s^{BE}]\{u\} = \{P_{ext}\} \tag{10}$$

Where, $[S_s^{BE}]$: the impedence matrix of the boundary elements at soil

Therefore, the relationship between external force and nodal displacement is established.

FORMULATION OF THE WHOLE SYSTEM

To solve the problem such as liquid storage tanks interacting with fluid and soil, it is necessary to have the solution to the governing equation of the whole system. Therefore, the governing equations of each region are coupled using the equilibrium and compatibility conditions at the interfaces. The coupling procedure consists of following three steps.

First, fluid and structure modeling are combined, and obtained the following equation.

$$\left(-\omega^2 \begin{bmatrix} M_{ff}^{FE} + M_{ff}^{BE} & M_{ft}^{FE} \\ M_{tf}^{FE} & M_{tt}^{FE} \end{bmatrix} + i\omega \begin{bmatrix} C_{ff}^{FE} & C_{ft}^{FE} \\ C_{tf}^{FE} & C_{tt}^{FE} \end{bmatrix} + \begin{bmatrix} K_{ff}^{FE} & K_{ft}^{FE} \\ K_{tf}^{FE} & K_{tt}^{FE} \end{bmatrix} \right) \begin{Bmatrix} u_f \\ u_t \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{11}$$

Second, structural and soil modeling are combined, and obtained the following equation.

$$\left[-\omega^2 [M_s^{FE}] + i\omega [C_s^{FE}] + [S_s^{BE} + K_s^{FE}] \right] \{u_s\} = \{F_s\} \tag{12}$$

Where, $\{u_s\}$: the displacement of the foundation

$\{u_g\}$: the ground motion

$[F_s]$: the earthquake force applied to the foundation through the ground

Third, the liquid storage tank is connected with the foundation and soil by the base-isolation system. As a result, the governing equation of the whole system is represented as follows

$$\left(-\omega^2\begin{bmatrix} M_s^{FE} & 0 & 0 \\ 0 & M_{ff}^{FE}+M_{ff}^{BE} & M_{ft}^{FE} \\ 0 & M_{tf}^{FE} & M_{tt}^{FE} \end{bmatrix} + i\omega\begin{bmatrix} C_s^{FE}+C_b & -C_b & 0 \\ -C_b & C_{ff}^{FE}+C_b & C_{ft}^{FE} \\ 0 & C_{tf}^{FE} & C_{tt}^{FE} \end{bmatrix} + \begin{bmatrix} S_s^{BE}+K_s^{FE}+K_b & -K_b & 0 \\ -K_b & K_{ff}^{FE}+K_b & K_{ft}^{FE} \\ 0 & K_{tf}^{FE} & K_{tt}^{FE} \end{bmatrix}\right)\begin{Bmatrix} u_s \\ u_f \\ u_t \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \\ 0 \end{Bmatrix} \quad (13)$$

where, K_b : the stiffness matrix of the base-isolation system

C_b : the damping matrix of the base-isolation system

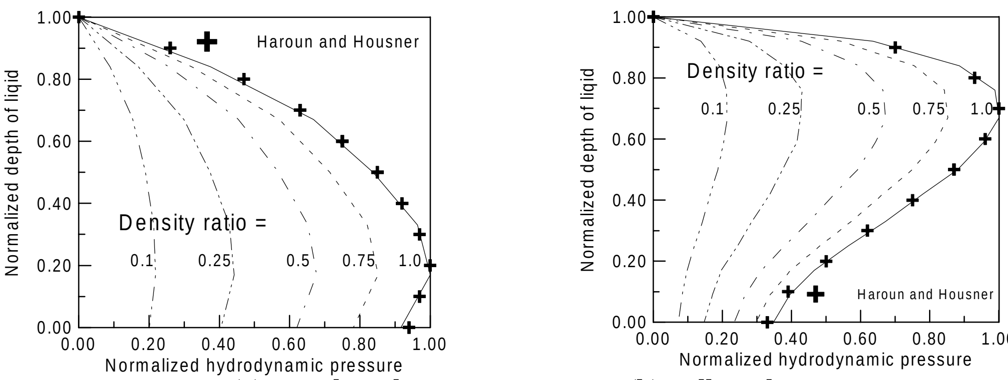
NUMERICAL ANALYSIS

Verification of fluid-structure interaction

To verify the validity of the developed program, the natural frequencies and corresponding mode shapes are compared with the analytical solutions under axisymmetric models in circumferential directions [Haroun and Housner, 1981]. The analyses are carried out on the broad and tall tanks. The dimensions of broad tank are *r*(radius)=18.30m, *h*(height)=12.20m, and *t*(wall thickness)=0.0254m. And the dimensions of tall tank are *r*=7.32m, *h*=21.96m, and *t*=0.0109m. The same material properties are used for two types of tanks; Young's modulus E=206.80 GPa; Poisson's ratio ν =0.3; and density of structures $\rho_s = 7{,}840$ kg/m^3. The broad and tall tanks are filled with liquid, of which density is taken to be 1,005kg/m^3. Table 1 shows the results obtained by present study and the referenced results. Fig. 1 and Fig. 2 present the mode shapes for comparison. Fig. 3 shows the hydrodynamic pressure distributions acting on the wall. The present study results are well matched with the results of Haroun and Housner[Haroun and Housner, 1981] at the dominant circumferential mode(cosθ).

Table 1: Natural frequencies of full broad/tall tanks for three-dimensional analysis

Broad Tank				Tall Tank			
3-D Mode Number	Present Study (rps)	Circumferential Mode Number	Axisymmetric Result (rps)	3-D Mode Number	Present Study (rps)	Circumferential Mode Number	Axisymmetric Result (rps)
9	14.13	cos6θ	13.88	1	3.51	cos5θ	3.46
11	17.05	Cos5θ	16.90	2	4.02	cos6θ	3.77
14	20.93	Cos4θ	20.79	3	4.06	cos4θ	4.08
16	26.08	Cos3θ	26.01	5	5.92	cos3θ	5.96
25	32.59	Cos2θ	32.61	7	10.30	cos2θ	10.37
31	38.72	Cosθ	38.83	20	22.25	cosθ	22.35



(a) Broad Tank **(b) Tall Tank**

Fig. 3: Fundamental hydrodynamic pressure distributions for different liquid density ratios

Verification of soil-structure interaction

In order to verify the developed procedures, responses of a massless rigid foundation on layered half space are compared with those of Luco's[Luco, 1974]. Fig. 4 illustrates the analyzed model. The material properties of layered half space are given in Table 2. The comparisons of vertical stiffness(K_{vv}) and vertical compliance(C_{vv}) for layered media are shown in Fig. 5. This indicates that the developed method matched well with Luco's results[Luco, 1974].

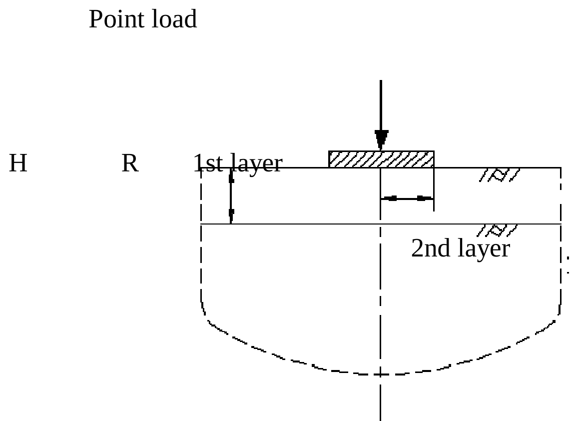
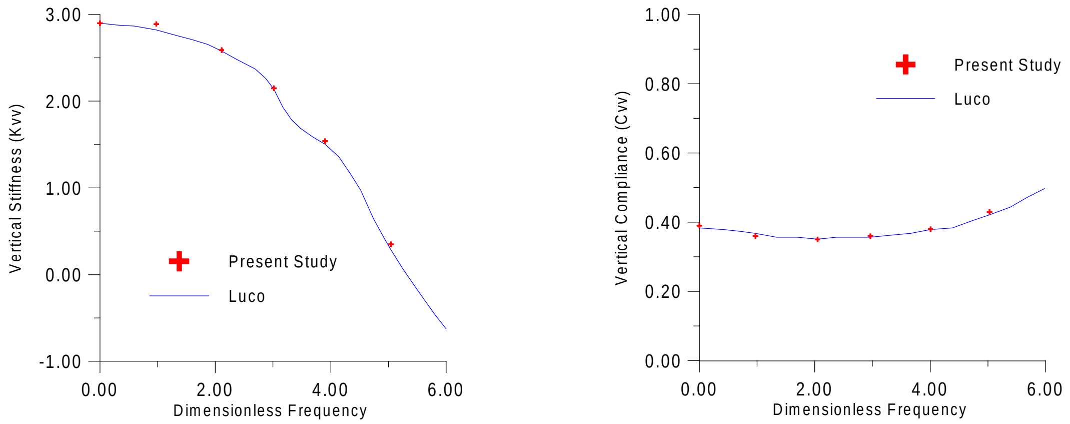


Fig. 4: Massless rigid foundation on layered soil

Table 2: Material properties of multi-layered half space

Properties		**1st layer**	**2nd layer**
Young's Modulus	E (Mpa)	1.76	12.5
Poisson's ratio	ν(-)	0.3	0.25
Density	ρ (kg/m^3)	1700	2000
Ratio of shear wave velocity	-	2.5	



(a) Vertical stifness **(b) Vertical compliances**

Fig. 5: Comparisons of compliances in layered media

Application of fluid-structure interaction

(1) Effect of different liquid density

The distributions of the hydrodynamic pressure for the different liquid density ratios(liquid to water density) are shown in Fig. 3. The normalized hydrodynamic pressure increases in proportion to liquid density. This is because the effect of added mass by liquid-structure interaction increases as the liquid density increases. Also, the location of the maxium hydropressure dose not change much regardless of the liquid density.

(2) Effect of different liquid level

The effect of liquid level is investigated by free vibration analysis. The dimensions of the tank are r =7.32m, h=21.96m, and t =0.0254m. The material properties for the tank are the same as those of tall tank. Fig. 6 indicates that the natural frequencies of the full tank is 35-45% of the empty tank and the natural frequencies do not vary linearly as the liquid filling ratio. This is because the effect of the inertia force that is added by considering liquid-structure interaction increases as liquid filling ratio increases. Note that when the liquid filling ratio is less than 0.2, this has little influence on the tank behavior. Thus, the effect of the liquid-structure interaction is negligible in case the liquid filling ratio is less than 0.2.

(3) Effect of tank wall thickness

The effect of wall thickness is investigated by free vibration analysis. The dimensions of the tank are r=29.18m and h=52.92m. The tank is filled with water at level h'=31.74m. The material properties for the tank are Young's modulus E=39.20 GPa; Poisson's ratio ν =0.18; and density of tank ρ_s =2,500 kg/m^3. The thickness of the tank wall varies 0.0292m, 0.15m, 090m, 2.0m, and 2.92m. Fig. 7 shows that the natural frequencies of the tank increase as the tank wall thicknesses increase. The increment of the tank wall thickness affect the stiffness of whole system. Therefore, the effect of liquid-structure interaction decreases.

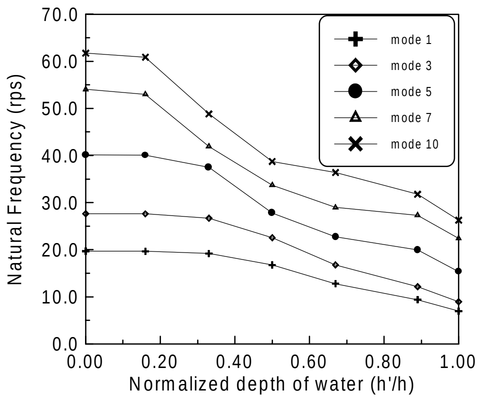


Fig. 6: Effect of different liquid filling ratios

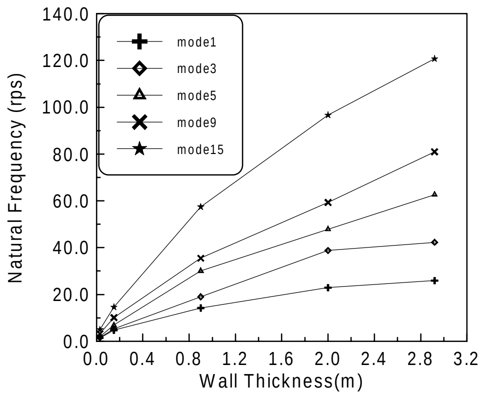
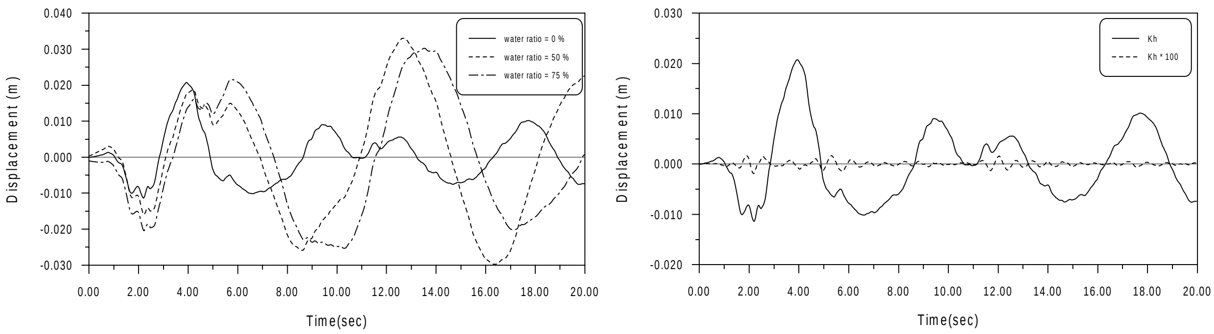


Fig. 7: Effect of tank wall thickness

Time-history response using FFT

An earthquake-response analysis for the liquid storage tank was carried out using the N-S component of the El-Centro earthquake as the input ground motion. The maximum ground acceleration is taken to be 0.319g. The obtained solution in frequency domain is recomposed into the time domain using inverse FFT. The dimensions for the tank are r=29.63m, h=52.95m, wall and bottom thickness=0.73m, and roof thickness=0.90m. The material properties for the tank are Young's modulus E for the wall=31.0 Gpa and for the bottom=20.0 Gpa; Poisson's ratio ν =0.20; and density of tank ρ_s = 2,548 kg/m^3. Water density is taken to be 1,005 kg/m^3. The material properties for the soil are the velocity of shear wave C_S =8,222.6 m/sec; density of soil ρ =2,000 kg/m^3. The stiffness of the base isolation system is taken to be 3.30×10^5 kg/m. Fig. 8(a) shows that the relative displacements between foundation and tank bottom are delayed when the liquid filling ratios increases. It is because the liquid affects the system by the form of additional mass. Fig. 8(b) shows the relative displacements between foundation and tank bottom decrease when the stiffness of isolator increases.



(a) For different liquid level **(b) For lifferent lase-lsolator**

Fig. 8: Relative displacements for different base-isolator between foundation and tank bottom

CONCLUSION

In this paper, a three-dimensional numerical model has been developed to analyze the base-isolated liquid storage tank on layered ground. This is performed by coupling the boundary elements for liquid and soil region and the finite elements for the tank. The liquid-structure interaction is verified by comparing obtained results with solutions conducted by Haroun and Housner for both broad and tall tank. The several parametric studies are performed considering the liquid-structure interaction. Also, the soil-structure interaction is verified by the Luco's result. Then the whole system is analyzed by BE-FE-BE coupling method. By those analyses, following results are obtained.

(1) As the liquid density increases, the hydrodynamic pressure acting on wall increses. Also, the change of the liquid density dose not affect the location of the maximum hydropressure.

(2) As the liquid level increases, the natural frequencies of the liquid storage tank decrease. Also, the effect of the liquid-structure interaction is negligible when the liquid filling ratio is less than 0.2.

(3) The increment of the tank wall thickness cause the stiffness increment of the whole system. Therefore, the natural frequencies of the liquid storage tank increase as the tank wall thickness increases.

(4) The behavior of the liquid storage tank on the layered half-space is analyzed successfully using developed analysis program. In this study, the response for the liquid level and base-isolator stiffness is investigated. The obtained results show the reasonable tendency of the behavior. Later, more parametric studies on the various ground conditions including the layered soil will be performed.

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