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ANALYSIS OF A NON-PROPORTIONALLY DAMPED BUILDING STRUCTURE WITH ADDED VISCOELASTIC DAMPERS

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SUMMARY

If energy dissipating devices, such as base isolators, viscous or viscoelastic dampers, are added to a structure, it turns to so called a nonproportional or a nonclassical damping system, and cannot be analyzed by the efficient mode superposition method based on real valued eigenvalues and mode shape vectors. Although direct integration method provides exact solution for the nonproportional damping system, the time and memory space required for the analysis prevent the method from being used for a practical application. In this research, a non-proportionally damped structure with added viscoelastic dampers are analyzed for earthquake excitations by the complex mode superposition method, and the results are compared with those obtained from the approximate methods such as the direct integration method with matrix condensation, modal strain energy method, and the method neglecting the off-diagonal terms of the transformed damping matrix.

A complex frequency response function is derived for the analysis of a nonproportionally damped structure in a frequency domain. The Kanai-Tajimi ground acceleration spectrum is used to obtain root mean square displacements of the model structure and to verify the effectiveness of the viscoelastic dampers in frequency doamin.

According to the results, the complex mode superposition, with the advantage of using only a few dominant modes, turn out to be very efficient procedure of analyzing the nonproportionally damped structure added with viscoelastic dampers. The direct integration method combined with the matrix condensation technique also provides seismic responses with a reasonable accuracy. It is also found that the discrepancy between the exact solutions and the results from the approximate methods increases as the damping contributed by the addition of viscoelastic dampers increases, and as the dampers are non-uniformly placed.

INTRODUCTION

In the analysis of a structure installed with viscoelastic dampers the modal strain energy method has been generally applied to predict the equivalent damping ratios of the system [Lai et. al., 1995]. The method derives the equivalent damping ratios based on the assumption that the damping is proportional to mass and/or stiffness of the structure system. However the assumption of proportional damping may no longer be valid when the viscoelastic dampers are added to the structure. In this case the direct integration method provides the correct results, but it requires too much computation time and memory space to be applied in practice. There is, however, a reliable alternative procedure for the analysis of the nonproportionally damped structure; the complex mode superposition method which provides exact solution in less time then needed for the direct integration. Compared with the direct integration method, the complex mode superposition has several advantages not only for the efficiency of response evaluation but for the understanding of the modal characteristics of the nonproportionally damped structures.

In this study some of efficient analytical procedures are applied to obtain the seismic response of a nonproportionally damped building structure with added viscoelastic dampers; the complex mode superposition method, direct integration method combined with matrix condensation, modal strain energy method, and the.

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method disregarding the off-diagonal terms of a transformed damping matrix. Special attention has been paid for the derivation of the complex modal superposition procedure, and the reliability of the approximate methods is checked by comparing the approximate solutions with those obtained from the complex mode superposition.

The dynamic behavior of viscoelastic dampers is represented by the Kelvin-Voigt model, in which a spring and a dashpot are connected in parallel. Although more accurate methods of analytical modeling exist, such as based on Boltzmann's superposition principle [Shen et. al., 1995] or on fractional derivative constitutive relationship [Tsai, 1994], they may not be applied on the analysis of large scale structures for their huge computational demands.

With this spring-damper idealization the dynamic system matrices of the structure with added viscoelastic dampers can be obtained by superposing the damper properties to the stiffness and damping matrices of the structure:

$$\mathbf{C} = \mathbf{C}_s + \mathbf{C}_d \tag{1}$$

$$\mathbf{K} = \mathbf{K}_s + \mathbf{K}_d \tag{2}$$

where $\mathbf{C}_s$ and $\mathbf{C}_d$ are the damping matrices of the structure and the added dampers, respectively, and $\mathbf{K}_s$ and $\mathbf{K}_d$ are stiffness matrices of the structure and the dampers, respectively.

EFFICIENT ANALYSIS IN TIME DOMAIN

Complex Mode Superposition Method

The general expression for a dynamic equilibrium equation is

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{p} \tag{3}$$

Where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping, and stiffness matrices of the structure, respectively, and $\mathbf{u}$, $\mathbf{\dot{u}}$, $\mathbf{\ddot{u}}$ are the displacement, velocity, and acceleration vectors of the response, respectively. Also $\mathbf{p}$ is the load vector. For a nonproportional damping system, the general approach to solve the above equation is to reformulate it into the first-order 2n-dimensional equation [Veletos et. al., 1986]:

$$\begin{bmatrix} [\mathbf{0}] & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix} \begin{pmatrix} \mathbf{\ddot{u}} \\ \mathbf{\dot{u}} \end{pmatrix} + \begin{bmatrix} \mathbf{M} & [\mathbf{0}] \\ [\mathbf{0}] & \mathbf{K} \end{bmatrix} \begin{pmatrix} \mathbf{\dot{u}} \\ \mathbf{u} \end{pmatrix} = \begin{pmatrix} [\mathbf{0}] \\ \mathbf{p} \end{pmatrix} \tag{4}$$

The above 2n-dimensional equation is normally expressed by the following simple form:

$$\mathbf{A\dot{y}} + \mathbf{By} = \mathbf{f} \tag{5}$$

The solution of the first-order equation can be obtained by the Laplace transform method:

$$y_m(t) = \frac{1}{a_m} \int_0^t e^{p_m(t-\tau)} Z_m(\tau)\, d\tau \tag{6}$$

$$a_m = \boldsymbol{\phi}_m^T \mathbf{A} \boldsymbol{\phi}_m, \qquad Z_m = \boldsymbol{\phi}_m^T \mathbf{f}$$

where p_m is the eigenvalue of the *m*th mode. Finally, displacements in the physical coordinates can be obtained by the following transformation:

$$\mathbf{u}(t_j) = \Phi\, \mathbf{y}(t_j) \tag{7}$$

Modal Strain Energy Methods

The modal strain energy method (MSE method) conveniently predicts the equivalent modal damping ratios of structures with added viscoelastic dampers through the following equation [Lai et. al., 1995]:

$$\xi_i = \xi_c + \frac{(\eta_d - 2\xi_c)}{2}\frac{\boldsymbol{\phi}_i^T \mathbf{K}_d \boldsymbol{\phi}_i}{\omega_i^2} \qquad (8)$$

where ξ_i and η_d are the damping ratio of the structure in the *i*th mode and the loss coefficient of the damping material, respectively, and $\mathbf{K}_d$ is the stiffness matrix contributed from the dampers. Also, ω_i and $\boldsymbol{\phi}_i$ are the natural frequency and the mode shape vector of the *i*th mode, respecticely, and ξ_c is the damping ratio of the structure itself. The above equation is transformed from its original form to take into account the inherent damping of the structure.

There is another simple but approximate procedure of solving nonproportional damping problems, which is to carry out the mode superposition analysis after neglecting the off-diagonal terms of the damping matrix. In this way only the diagonal terms participate in the analysis.

Direct Integration with Matrix Condensation

To reduce the number of degrees of freedom and the computation time for direct integration, two techniques are applied; rigid diaphragm assumption and the matrix condensation method. In this study the in-plane DOF's of all the nodal points located on the floors are condensed to the three DOF's representing two translational and one rotational degrees of freedom as described in Figure 1.

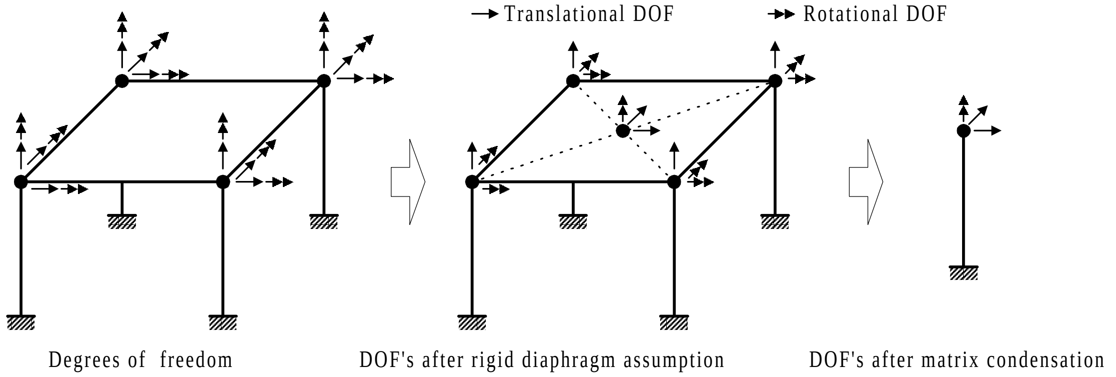

Figure 1: Efficient modeling procedure

The normal procedure for matrix condensation is followed [Paz, 1991], except that the condensed damping matrix of the structure is obtained from the condensed mass and stiffness matrices using the Rayleigh damping method:

$$\mathbf{C}_s^* = \alpha \mathbf{M}_s^* + \beta \mathbf{K}_s^* \qquad (9)$$

where **M*** and **K*** are the condensed mass and stiffness matrices. Finally the condensed damping matrix of the combined system of the structure and the added viscoelastic dampers is obtained by the superposition of the two condensed matrices:

$$\mathbf{C}^* = \mathbf{C}_s^* + \mathbf{C}_d^* \qquad (10)$$

where $\mathbf{C}_d^*$ is the condensed damping matrix contributed from the dampers.

The final expression of the equation of motion of the condensed system is

$$\mathbf{M}^*\ddot{\mathbf{u}}^* + \mathbf{C}^*\dot{\mathbf{u}}^* + \mathbf{K}^*\mathbf{u}^* = \mathbf{p}^* \qquad (11)$$

The above equations are directly integrated using the average acceleration method in time domain to obtain the responses. As the degrees of freedom are greatly reduced as a result of the rigid diaphragm assumption and matrix condensation, the analysis is expected to be carried out more efficiently.

ANALYSIS IN FREQUENCY DOMAIN

For nonproportional damping systems, the response spectrum analysis may not be applicable because the modal participation factors, which determine the contribution of each mode of a proportional damping system to the response, cannot be similarly defined. In this sense it is worthwhile to investigate the behavior of the nonproportionally damped structure subjected to a generalized stationary random ground excitation through the root mean square response obtained in the frequency domain. Igusa et. al. [1984] developed an analytical procedure based on the modal decomposition process to obtain a stationary response of a nonproportionally damped system subjected to a stationary base input. In this study the procedure is developed by deriving the complex frequency response function to analyze a three dimensional building structure with added viscoelastic dampers.

The complex frequency response function can be derived from the unit impulse response function, which in turn can be obtained from the *m*th modal response in time-domain. Thus, from Eq. (6), the unit impulse response function is defined as follows:

$$h_m(t) = \frac{e^{p_m t}}{a_m} \qquad (12)$$

The complex frequency response function of a nonproportionally damped system can be derived from the Fourier transform of the unit impulse response as follow:

$$H_m(i\bar{\omega}) = \int_{-\infty}^{\infty} h_m(t) e^{-i\bar{\omega} t} dt \qquad (13)$$

The above equation is reduced to the following form after some manipulation:

$$H_m(i\bar{\omega}) = \frac{1}{a_m(i\bar{\omega} - s_m)} \qquad (14)$$

The power spectral density function of the response can be expressed as follows:

$$S_q(\bar{\omega}) = \sum\sum \phi_{qm}\phi_{qn} H_m(-i\bar{\omega}) H_n(i\bar{\omega}) S_{p_m p_n}(\bar{\omega}) \qquad (15)$$

where $S_q(\bar{\omega})$ is the power spectrum density function of the qth degree of freedom response, ϕ_{qm} is the qth component of the mth mode vector. $H_m(-i\bar{\omega})$ is the complex frequency response function of the mth mode response, and $H_m(i\bar{\omega})$ is the complex conjugate. Also $S_{p_m p_n}(\bar{\omega})$ is the cross spectral density function of modal load p_m and p_n.

The root mean square (RMS) value of the response can be obtained from the power spectrum density function of the response as follows:

$$\sigma_q = \sqrt{\int_0^{\infty} S_q(\bar{\omega}) d\bar{\omega}} \qquad (16)$$

In this study the earthquake ground acceleration is assumed to be a stationary random process with a zero mean with the power spectral density function given by the following Kanai-Tajimi spectrum:

$$S_{a_g a_g}(\bar{\omega}) = \frac{1 + 4\zeta_g^2 \left(\frac{\bar{\omega}}{\bar{\omega}} \right)^2}{\left[1 - \left(\frac{\bar{\omega}}{\omega_g} \right)^2 \right]^2 + 4\zeta_g{}^2 \left(\frac{\bar{\omega}}{\omega_g} \right)^2} \times S_0 \tag{17}$$

where ζ_g , ω_g and S_0 are the predominant damping coefficient, characteristic ground frequency, and intensity measure. The intensity measure, S_0 can be related to the PGA of the earthquake records based on the following relation [Shinozuka et. al., 1990]:

$$PGA = p_g \sqrt{[\pi\omega_g(\frac{1}{2\zeta_g} + 2\zeta_g)]S_0} \tag{18}$$

where the peak factor, p_g is assumed to be 3.0, which is empirically obtained [Shinozuka et. al., 1990]. The power spectrum density function of force can be obtained by multiplying mass to the above acceleration spectrum, and the above two sided spectrum can be transformed to the one sided one by multiplying two. Finally the power spectrum density function of seismic loads $S_{p_m p_n}(\bar{\omega})$ can be formulated as follows

$$S_{p_m p_n}(\bar{\omega}) = 2\sum_i \sum_j S_{a_g a_g}(\bar{\omega}) m_i m_j \tag{19}$$

where m_i represents the mass in *i*th nodal point associated in the direction of the load.

NUMERICAL RESULTS

One-bay ten-story building structures shown in Figure 2 are analyzed to compare the responses obtained from the complex mode superposition method and from the approximate methods. Viscoelastic dampers are added to the diagonals of each story in the direction of the load. In structure A identical dampers are located on every floor. The shear area of the damping material is doubled in structure B. In structure C the dampers are located only in the lower half stories of the structure. The masses are lumped to each joint and six degrees of freedom (three displacements and rotations along the three perpendicular axes) are considered per node. The first and the second modal damping ratios of the structure without added dampers are assumed to be 1%. The member properties of the model structure and the material properties of the damping materials used in the analysis are based on Zhang et. al. [1989] and they are listed in Table 1 and Table 2, respectively.

Table 3 shows the modal damping ratios obtained from the complex mode superposition method (CMS) and the approximate method, such as modal strain energy method (MSE) and the method that disregards the off-diagonal terms of the damping matrix (APR). According to the results, the error is larger in the MSE method than in the APR method. It also can be noticed that the error grows larger when the dampers are located in the lower half stories of the structure.

The same trend can be found in the top story displacement. Figure 3 through Figure 4 shows the time history of the top story displacements caused by the El Centro S90W(1940) earthquake excitation, obtained from the four different analysis methods. It can be seen that the results from the direct integration with matrix condensation (DIC) are very close to the exact solution obtained from the complex mode superposition. It also can be observed that the results provided by the APR method are closer to the exact solution than those obtained by the MSE method. Also the error increases as the dampers are not placed uniformly through the height. It should be noted that the results provided by the approximate methods forms the lower bound of the exact solutions, mainly because of the prediction of the higher modal damping ratios.

Table 1: Member properties of the model structure

Story	Beams		Columns	
	A (cm^2)	I (cm^4)	A (cm^2)	I (cm^4)
10	143	47800	163	34500
9	143	47800	163	34500
8	143	47800	210	58500
7	143	47800	210	58500
6	143	47800	291	91500
5	218	126100	291	91500
4	218	126100	356	117900
3	218	126100	356	117900
2	218	126100	405	118300
1	218	126100	405	118300

Table 2: Properties of viscoelastic material

Model	G' (ton/m^2)	G'' (ton/m^2)	A (cm^2)	t (cm)
A	250.3	421.4	2000.0	2.0
B			4000.0	

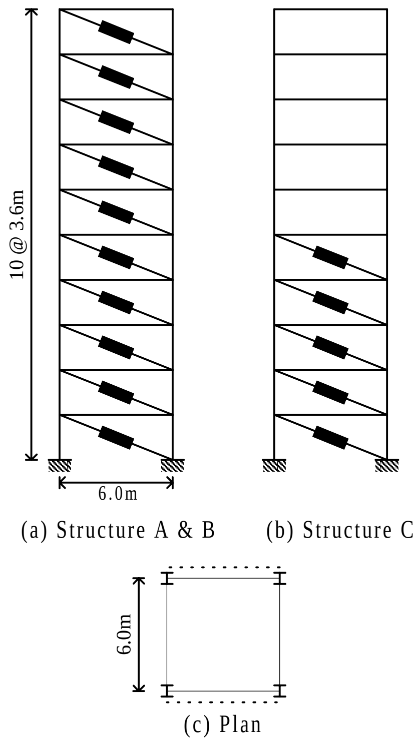

Figure 2: Example Structure

Table 3: Damping ratios of the primary mode obtained from various analysis methods (%)

STRCTURE	MSE(error, %)	APR(error, %)	CMS
A	31.8(25.7)	28.0(10.7)	25.3
B	43.0(52.5)	34.8(23.4)	28.2
C	22.4(135.8)	14.1(48.4)	9.5

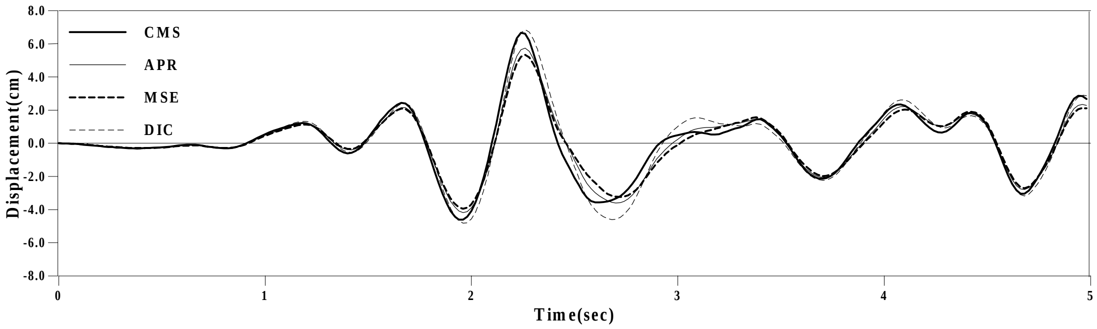

Figure 3: Top floor displacements of structure A

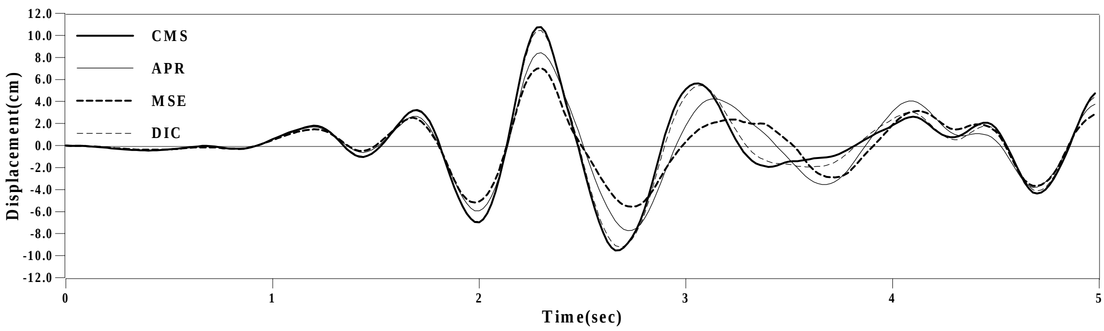

Figure 4: Top floor displacements of structure C

In the frequency-domain analysis, analyses are performed only for the structure A. Three values of the intensity measure, S_0, are obtained from the PGA of the earthquake records of El Centro (S00E, 1940), Taft (N21E, 1952), and Okland City (N26E, 1957), and are listed in Table 4. The predominant damping coefficient, ζ_g and the characteristic ground frequency, ω_g used in Eq. (17) and Eq. (18) are taken to be 28.3rad/sec and 0.6, respectively, which are obtained from firm soil condition [Shinozuka et. al., 1990].

According to the Figure 5, the Kanai-Tajimi spectrum has the largest amplitude around the frequency content of 25rad/sec. In contrast, the peak of the power spectrum of top story displacement moves to around 7□9rad/sec, as shown in Figure 6, reflecting the filtering effect of the structure. The frequency that the peak of the power spectrum occurs moves from around 7rad/sec for no damper case to around 9rad/sec for the case that the dampers are installed on every floor, implying that the stiffness of the structure has increased due to the dampers. Also it can be noticed that the amplitude of the peak decreases significantly, verifying the effectiveness of the dampers. The comparison of the RMS values shown in Table 5 demonstrates the effectiveness more clearly; the RMS displacement is reduced about 85% when the dampers are added to the structure. For comparison, Table 6 shows the RMS responses obtained from the time history analysis, which matches reasonably with the results obtained from the frequency analysis using the Kanai-Tajimi spectrum.

Table 4: PGA and S_0 used in the analysis

	PGA	S_0 (cm^2/sec^3/rad)
El Centro	0.35g	72.32
Taft	0.16g	15.13
Okland City	0.04g	0.91

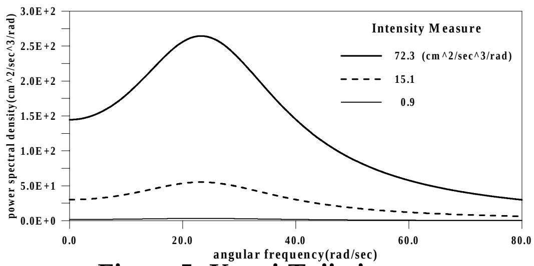

Figure 5: Kanai-Tajimi spectrum

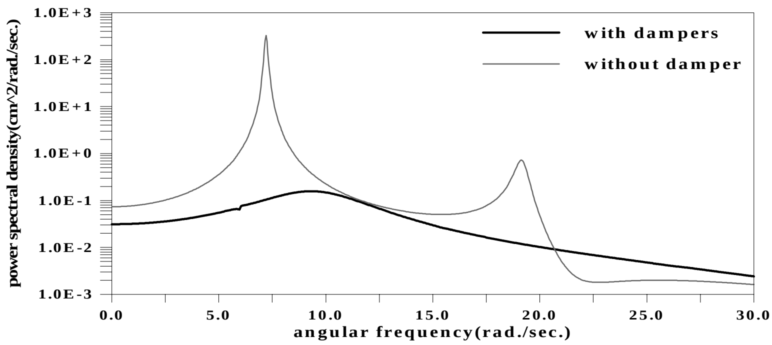

Figure 6: PSD for top story displacement of the structure A

Table 5: RMS top story displacements of the structure A

S_0 (cm^2/sec3/rad)	RMS displacement(cm)
72.32	1.14
15.13	0.52
0.91	0.13

Table 6: RMS top story displacements of the structure A obtained from time domain analysis

Earthquake records	RMS displacement(cm)
El Centro	0.86
Taft	0.43
Ocland City	0.05

CONCLUDING REMARKS

In this study the analytical procedures of complex mode superposition method are derived in time and frequency domain for the analysis of a simple three dimensional building structure with added viscoelastic dampers subjected to an earthquake ground excitation. The analysis results are compared with some of efficient but approximate techniques; such as direct integration with matrix condensation, modal strain energy method, and the method neglecting the off diagonal terms of the damping matrix.

It is observed that for the given model structure the difference between the exact solution obtained from the complex mode superposition method and the ones obtained from the approximate methods increases as the amount of the added damping increases and when the dampers are not uniformly located. The main reason for the difference is the higher modal damping ratios predicted by the approximate methods, which results in lower bound responses.

The complex mode superposition method, although very efficient compared to the direct integration method, requires larger computation time than needed for the conventional mode superposition method because the degrees of freedom are doubled and the analysis is carried out in complex numbers. However, like the conventional mode superposition, only a few predominant modes will satisfy the accuracy of the analysis, which makes this method very efficient and most promising for the analysis of a structure with added viscoelastic dampers. Also efficient is the direct integration method combined with matrix condensation technique. The analytical results turn out to be quite close to the exact solution with a lot of saving in computation time.

REFERENCES

Igusa, T., Kiureghian, D. and Sackman, J. L. (1984), "Modal Decomposition Method for Stationary Response of Non-classically Damped Systems," *Eearthquake Eng. Struct. Dyn.*, Vol. 12, pp. 121-136.

Paz, M., *Structural Dynamics*, Chapman & Hall, 4th Ed.

Shen, K. L. and Soong, T. T. (1995), "Modeling of Viscoelastic Dampers for Structural Application," *J. Eng. Mech.*, Vol. 121, No. 6, pp. 694-701.

Lai, M. L., Chang, K. C., Soong, T. T., Hao, D. S., Yeh, Y. C. (1995), "Full-Scale Viscoelastically Damped Steel Frame," *J. Struct. Engrg*., Vol. 121., No. 10, pp. 1443-1447.

Shinozuka, M., Wen, Y. K., Casciati, F. (1990), "Seismic Damage and Damage-Control Design for RC Frames," *J. Intell. Mater. Syst. and Struct.*, Vol. 1, pp. 476-495.

Tsai, C. S. (1994), "Temperature Effect of Viscoelastic Dampers During Earthquakes," *J. Struct. Engrg.*, Vol.120, No. 2, pp.165-182.

Veletos, A. S., Ventura, C. E. (1986), "Modal Analysis of Non-classically Damped Linear Systems," *Earthquake Engrg. Struct. Dyn.*, Vol. 14, pp. 217- 243. 4.

Zhang, R. H., Soong, T. T. and Mahmoodi, P. (1989), “Seismic Response of Steel Frame with Added Viscoelastic Dampers,” *Eearthquake Eng. Struct. Dyn.*, Vol. 18, pp. 389-396.