INFLUENCE OF BUCKLING OF LONGITUDINAL REBARS IN FINITE ELEMENT MODELLING OF REINFORCED CONCRETE STRUCTURES SUBJECTED TO CYCLIC LOADING

A ATTOLICO[1], S BIONDI[2], C NUTI[3] And M PETRANGELI[4]

SUMMARY

The paper presents the implementation of rebars buckling capabilities into a fibre beam element recently developed by the authors and the results of numerical analyses carried out on few representative concrete structures using this new element.

The fibre model used for the implementation is an equilibrium based beam element using uniaxial constitutive modelling for concrete and steel. The existing uniaxial model for the longitudinal steel fibres has been replaced with a new one with buckling capabilities based on the Menegotto-Pinto constitutive law as modified by Nuti and co-workers to account for rebar buckling in the plastic regime. The model input parameters are the steel properties, the stirrups spacing and the longitudinal rebar diameter. Four hardening rules are used to modify and shift the skeleton branch of the model at each load reversal. Calibration of the model using available test data is presented and the effect of insufficient transverse reinforcement (i.e. large stirrups spacing) on the structural response and ultimate resistance of reinforced concrete bridge piers and frames is investigated.

Two case studies are also presented: the Kobe Hanshin viaduct's pier toppled during the 1995 earthquake characterised by a bending-shear driven compression failure and a large frame representative of Italian civil building designed according to prevailing anti-seismic specification of the sixties and seventies. The results of numerical analysis are discussed in order to evaluate the retrofitting strategies of these seismic sensitive structures.

INTRODUCTION

In seismically resistant design, the behavior of reinforced concrete structural elements subjected to large displacements in the inelastic range is generally considered as influenced by the compressive behavior of the confined concrete and the tensile response of the longitudinal rebars. In the past decades a number of tests were performed on confined concrete specimens (beam and column elements) in order to evaluate strength and ductility of these members, [Mander et al., 1988], [Sheikh et al., 1990], [Madas et al., 1992]. Stress-strain model for concrete subjected to uniaxial compressive loading and confined by transverse reinforcement have been calibrated and used in moment-curvature analysis and in order to obtain design procedures and Code provisions, [Watson et al., 1994].

These stress-strain relationships have also been implemented in the fibre beam elements for the numerical analysis of reinforced concrete beams and columns. This approach is very efficient and a growing number of applications are being carried out with it. The numerical results show a good agreement with test data especially if ductile types of failure mechanisms are considered while less accurate results are obtained when strong strength or ductility decay takes place, [Petrangeli, 1999]. This decay may be due to brittle response of concrete in tension-shear, bond loss or compression softening of both concrete and steel due to rebar buckling. These latter phenomena being correlated as the loss of lateral confinement is amplified by the buckling mechanism.

[1] *Dipartimento di Scienze, Storia dell'Architettura e Restauro, University, Chieti, Italy www.unich.it/dssar*
[2] *Dipartimento di Scienze, Storia dell'Architettura e Restauro, University, Chieti, Italy www.unich.it/dssar*
[3] *Dipartimento di Scienze, Storia dell'Architettura e Restauro, University, Chieti, Italy, Email: nuti@unich.it*
[4] *Dipartimento di Scienze, Storia dell'Architettura e Restauro, University, Chieti, Italy www.unich.it/dssar*

In order to improve the fibre model efficiency shear modelling has been recently introduced in the model [Petrangeli et al., 1999] together with longitudinal rebar buckling capabilities as discussed in the following pages. Contrary to the bending-shear interaction that becomes very important when the shear span ratio is below two, the buckling mechanisms can play a major role also in slender member where it is the major cause for curvature localisation and bending failure.

BUCKLING INFLUENCE IN STRUCTURAL RESPONSE

Reinforcing bars that undergo repeated loading into the inelastic range can be subjected to inelastic buckling. This phenomenon is generally assumed dependent on tie spacing. From a number of experimental studies on inelastic buckling of rebars the following conclusions can be drawn. For low slenderness the compressive monotonic curve essentially coincides with the tensile one and the bar maintains a rectilinear configuration. For medium slenderness a short superposition of stress-strain skeleton is observed in the plastic range while for higher slenderness as soon as the yield point is reached buckling starts and the compressive monotonic curve softens with strength decay. A slenderness ratio (tie spacing/bar diameter) $L/D = 5$ represent the transition point between element subjected to buckling phenomena and those who are not. This result being confirmed both experimentally and by means of numerical simulations, [Monti et al., 1992].

The cyclic behavior is the following: for low tie spacing the compressive cycles envelope substantially coincides with the compressive monotonic curve (i.e. without buckling) although there is an expansion of the hysteresis cycles (isotropic hardening). For higher slenderness a contraction of the hysteresis cycles is observed (isotropic softening), the curvature of the loading branches in compression decreases and the slope of loading branches, after reversals from compression, decreases with increasing cycle amplitude. The tensile cycles envelope is not significantly affected by buckling. After buckling the stiffness of the bar is governed by the flexural response until the complete reversal, [Gomes et al., 1997]. It can be demonstrated that this behavior can be described using four hardening rules dependent on the cyclic plastic work and the maximum strain excursion, [Monti et al., 1992].

Some authors have suggested that buckling of longitudinal reinforcement will occur between ties, with each bar acting as a fixed-fixed column with or without sidesway, [Scribner, 1986]. Consequently, the buckling mode shapes are assumed to take place in a single tie interval as in current codes when provisions is made in order to control effectiveness of crossties confinement. However both experimentally and theoretically it is noticed that longitudinal bars could also buckle over a length greater than one stirrup interval. This type of buckling of longitudinal bars increases strength and ductility decay in structural element due to the strong reduction of concrete confinement, [Papia et al., 1988], [Albanesi et al., 1994].

As a matter of fact, collapse of reinforced concrete structural elements subjected to axial load is strongly dependent if not coincident with the instability of the reinforcing bars. Consequently, the ultimate compressive concrete strain can be defined as the longitudinal concrete strain at which the steel buckling occurs, [Papia et al., 1989]. This criterion is similar to that one which postulates the limit of maximum compressive strain in the concrete core as the strain at which the first hoop fractures. In the first case a mechanical criterion is assumed and equilibrium stability in the plastic range is imposed. In the second one a balance of the strain energy capacity of transverse reinforcement to the strain energy stored in the concrete is imposed and the first hoop fractures event taken as the capacity limit of r.c. element ductility.

FIBRE BEAM ELEMENT

The enhanced fibre model used for this study is based on the fibre beam element developed by the last author over the last few years. The element includes various features from previous fibre elements. Especially the latest fibre beam element with shear modelling, [Petrangeli et al., 1997], seems to provide results that are in good agreement with test data even for squat elements. The principal ingredients of the element are the equilibrium based integrals for the element solution, fixed monitoring sections located at Gauss's points along the element, explicit algebraic constitutive relations for concrete and steel based on the state-of-art formulations.

Unlike the traditional fibre model, the new model has two additional strain fields to be monitored at each cross section, namely the shear strain field and the lateral one. The first one comes explicitly in the element formulation while the second is statically condensed at each section by imposing the equilibrium between transverse steel and concrete.

The longitudinal stress field is therefore found according to the plain section hypothesis while the shear strain field is found using predetermined shape functions. Dependency of the element response on the particular shear shape functions is appreciable although the global response is less sensitive to it. Generally the simpler shear shape functions (linear or parabolic) tend to overestimate the shear deformation in the compression zone at large ductilities when a shift of the shear strain deformation to the tensile zone is observed in real structures [Petrangeli et al., 1999].

Transverse strains and stresses depend on the amount of transverse steel, on the gross section shape and on the concrete dilatancy. This dilatancy is harder to depict especially that due to shear which is mostly localised across the cracks and can be captured by the model only in a smeared sense.

At the present interaction between transverse steel stress-strain response and longitudinal rebar buckling is not implemented, with the longitudinal reinforcement slenderness provided as input data. Interaction between the two mechanisms will be the next step in the element development.

STEEL BUCKLING MODELLING AND NUMERICAL ASPECTS

As stated above four branches can be used to depict the tension and compression skeleton curves: a linear elastic, a yield plateau, a strain-hardening and a post-ultimate stress one. While the monotonic behavior has been studied experimentally in details, the cyclic stress-strain response is still to be fully investigated.

Various approaches have been used to define and calibrate an efficient cyclic stress-strain steel relationship; the Pinto-Menegotto uses an explicit algebraic equation for normalised strain and stress, the Ramberg-Osgood model an implicit algebraic one. More recent developments have put forward other solutions [Dodd et al., 1995]. The explicit equation suggested in the first model has been used in the present work. The original model defines the loading and unloading paths as contained in a bilinear envelope with the monotonic curve defined by the tangent modulus of elasticity at origin and the tangent modulus at hardening.

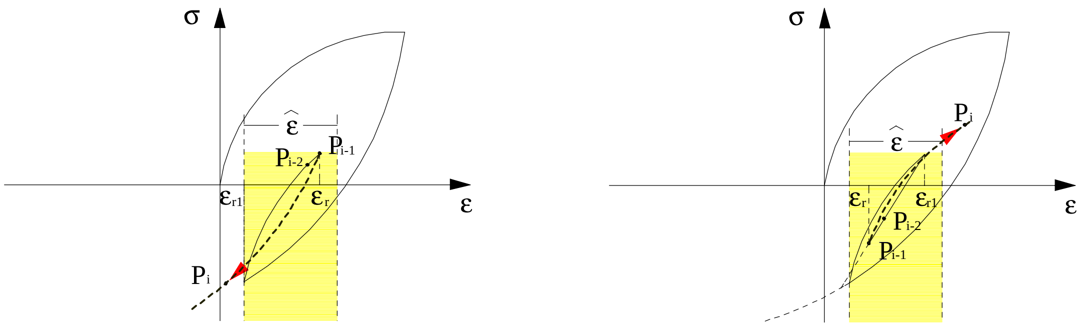


Figure 1. Steel stress-strain cycles: low amplitude reversal modelling

The implemented model consists of an explicit σ-ε relationship for branches between two subsequent reversal points (loading branches). The model parameters are updated at each load reversal (not at each deformation increment). They are controlled by means of four hardening rules and can be easily extended to the case of inelastic buckling. The four hardening rules are defined both in absence of buckling ($L/D < 5$) and in presence of buckling ($L/D > 5$). These rules are a kinematic rule, an isotropic rule, a memory rule and a saturation rule.

The buckling behavior further requires the definition of a compressive to tensile superposition length definition, a softening branch stiffness law and an ultimate compressive softening asymptote rule. These rules permit to analyse both symmetrical and non-symmetrical cyclic strain histories and are computationally very efficient. Numerical results show a good agreement both with experimental results and with numerical results obtained by means of FEM model of a rebar with inelastic buckling, [Mau et al., 1989].

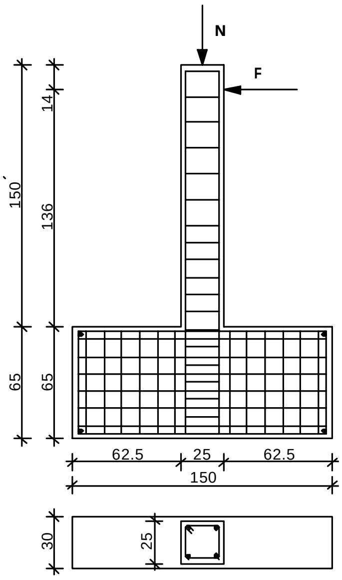
Particular attention has been devoted to the problem of loading and unloading from the skeleton branches. In the seismic analysis of frame structures, fibre stress-strain histories are in fact subjected to numerous loading and unloading, especially those located near the point of contraflexure or the neutral axis. Loading and unloading branches which trespass a certain amplitude are defined by the same algebraic expression used for the skeleton curve but for smaller cycles a simpler expression need to be defined in order to reduce the computational effort and avoid numerical round off error which do arise using the algebraic expression of the skeleton branches.

For small unloading-reloading in fact, the bilinear envelope which define the Menegotto-Pinto becomes too narrow and the curves is not capable to fit in it. In reality this small unloading-reloading are linear elastic and so have been defined in the model. Therefore at unloading the Menegotto-Pinto is calibrated so as to join tangentially with the skeleton curve in the opposite quadrant, but if reloading take place within a certain strain amplitude, an interval is defined where all subsequent loading and reloading are assumed to be linear elastic.

CASE STUDIES

Fibre model calibration

The model has been calibrated using the experimental results of single bent columns tested in the laboratory of the University of Rome. The columns were 1.50m in height with 0.25m square section and 4D16 reinforcing bars (one at each corner) as shown in Fig. 2.. The specimens were subjected to an axial force around 0.16 the squashing load and imposed alternate displacements of the pier top.



SPECIMEN PB3:
Longitudinal rebars: 4D16
Stirrups: D 8 Spacing 13 cm
N = 463 kN
f_c = 46.3 Mpa

SPECIMEN PC2:
Longitudinal rebars: 4D16
Stirrups: D 8 Spacing 9 cm
N = 390 kN
f_c = 39.0 MPa

SPECIMEN PB1:
Longitudinal rebars: 4D16
Stirrups: D 8 Spacing 5 cm
N = 440 kN
f_c = 44.0 Mpa

Figure 2. Specimen geometry, reinforcement arrangements, vertical load and concrete strength

Concrete properties and stirrup spacing for the different specimens are reported in the table above. Longitudinal steel properties were the same with a yield strength of 435 Mpa. Calibration of the steel model parameters were carried out by trial and error except for the yield strength, the initial and hardening modulus and the buckling (softening) modulus.

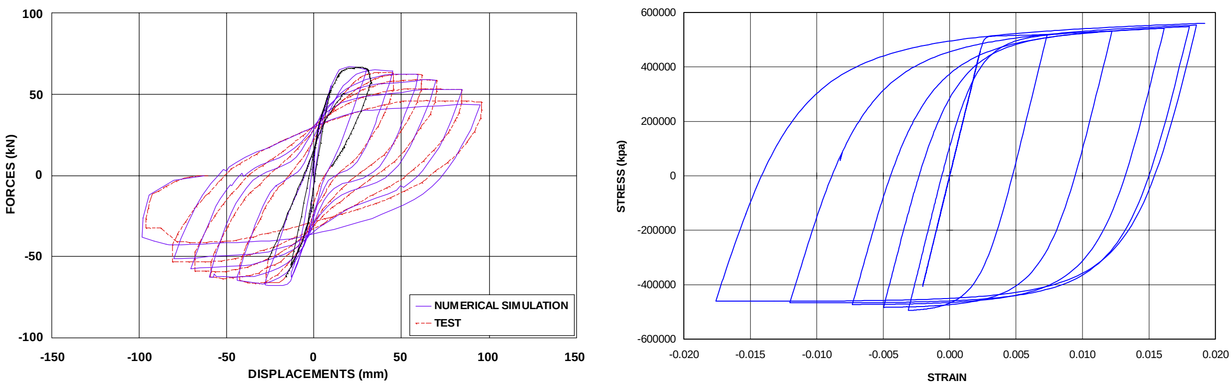


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Figure 3. Specimen PB3: Force-Displacement and Steel Stress-Strain Diagrams

The force displacement response for the PB3 specimen and the corresponding rebar stress-strain history at the column base are reported in Fig. 3. Results for specimen PC2 and PB1 are plotted in Fig. 4.. The model response when increasing the stirrup spacing can be appreciated in Fig. 5. where the results at four numerical analyses of the PB3 specimen with varying stirrup spacing and steel stress-strain model (with or without buckling) are reported.

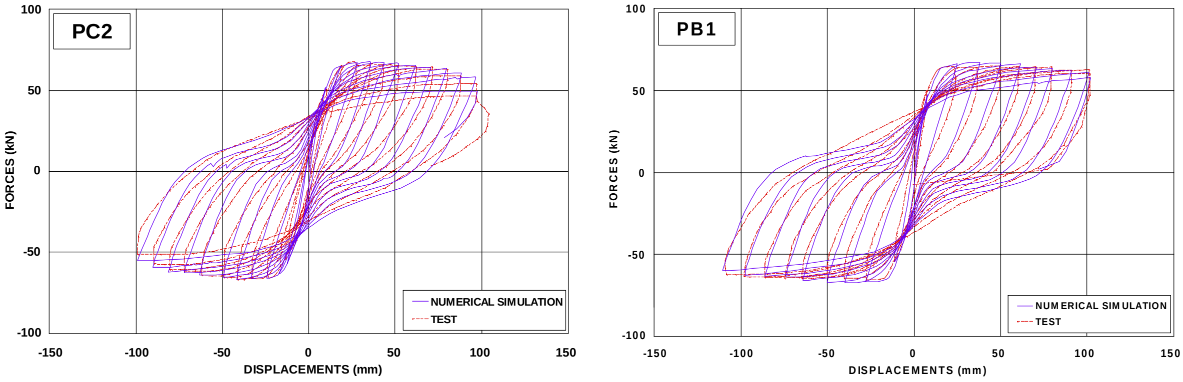


Figure 4. Specimens PC2 and PB1: Force-Displacement and Steel Stress-Strain Diagrams

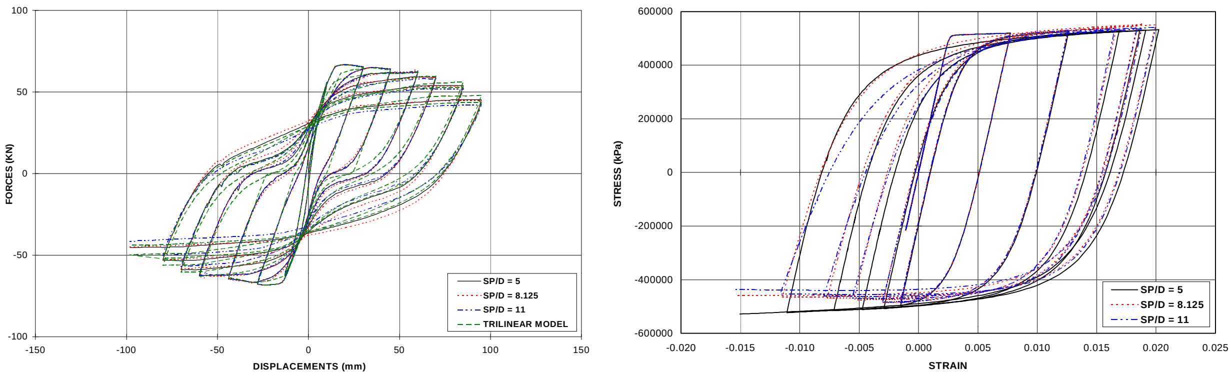


Figure 5. Specimen PB3: Force-Displacement and Steel Stress-Strain Diagrams for Various Steel Models

Hanshin viaduct in Kobe

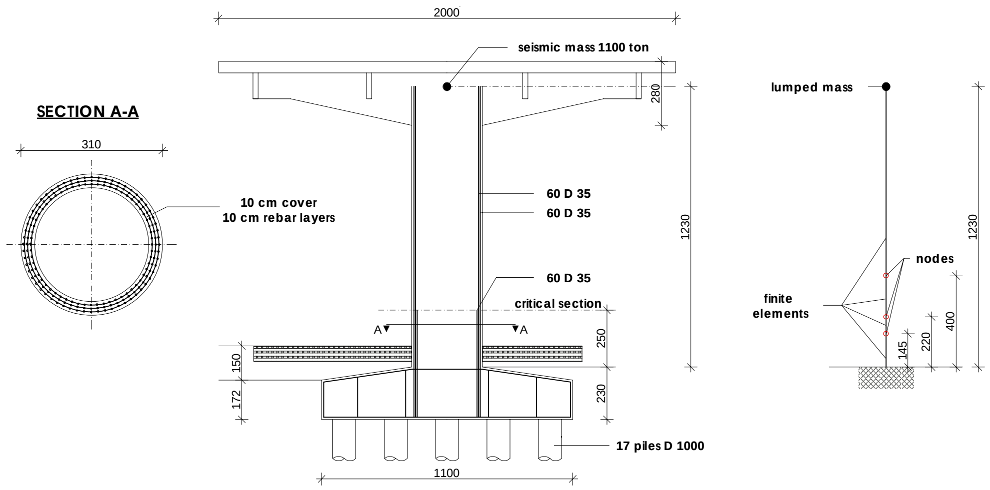
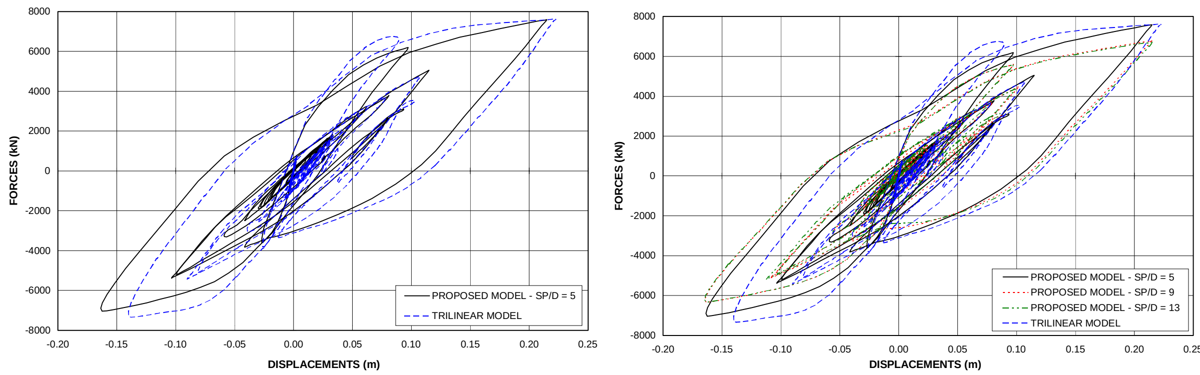


Figure 6. Pier geometry and FEM model

The effect on the failure of the Hanshin viaduct during the Kobe earthquake is investigated. The viaduct made of simply supported prestressed concrete girders resting on single bent circular column toppled due to the pier failure. This failure has been investigated by various authors [Seible et al., 1995, Petrangeli and Pinto, 1997] and although it is not widely recognised to be a compression driven-shear failure initial calculations indicated the shear resistance to be in excess of the bending one. Numerical simulation with the fibre flexural element could non achieve failure neither as concrete crushing or P-Δ topping effect contrary to the shear enhanced model that clearly collapsed due to P-Δ effect following the desegregation of the compressive strut.

Using the new rebar model with buckling capabilities the analyses have been performed once again to investigate the effect of buckling on the flexural response of the viaduct. A single pier has been modelled using the finite element program FIBER with four 2D fibre beam element without shear modelling. A concentrated mass has been placed on top of the pier with a translational and rotational inertia equivalent to one span of the viaduct (M_t = 1100 kN, M_r = 29000 kNm^2). Full Lagrangian formulation (large displacement) has been used. The structure has been subjected to the Kaiyou Weather Bureau accelerogram with PGA = 0.81 g. The reinforced concrete section has been subdivided in 40 concrete fibres and 20 longitudinal steel ones.

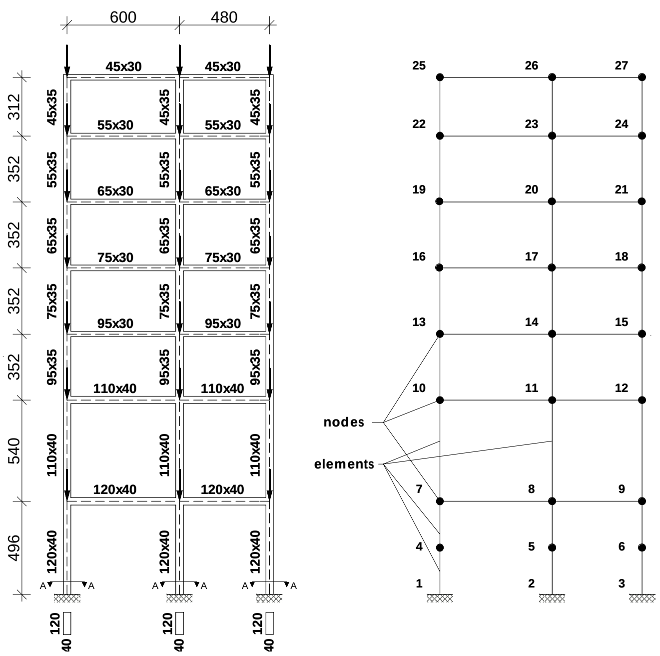


(a) (b)

Figure 7. Force-displacement diagrams: steel stress-strain model and lateral reinforcement ratio influence

In Fig. 7.(a) the comparison between the results obtained with previous (a variable parameter nonlinear with elasto-plastic-hardening monotonic skeleton [Petrangeli et al., 1997]) and actual steel model without buckling is proposed. As shown in Fig. 7.(b) for increasing slenderness ratio, buckling of the longitudinal rebar may have increased the maximum displacement of the viaduct and modified the force-displacement response of the piers but it not sufficient to trigger the structural failure that took place in the real structure.

Large frame of Italian civil building



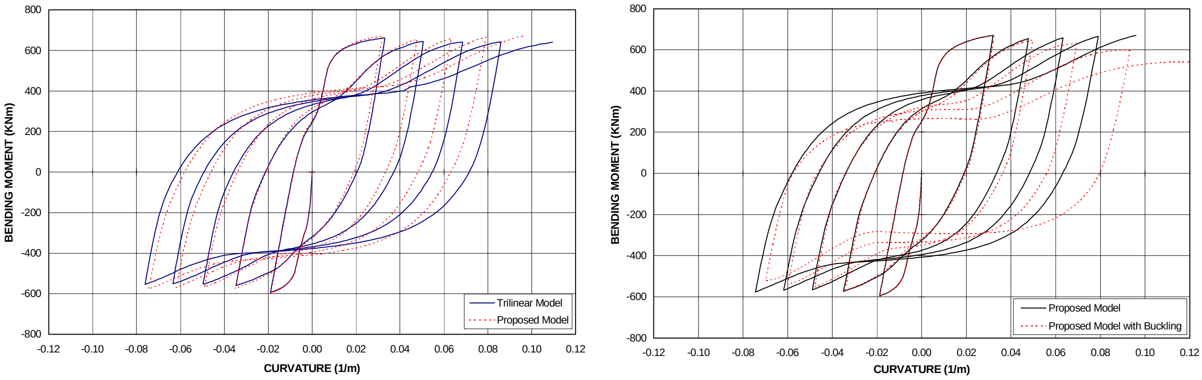
Element	Reinf.	Element	Reinf.
1-4	46D16	**20-23**	8D16
2-5	46D16	**21-24**	8D16
3-6	46D16	**22-25**	6D16
4-7	18D16	**23-26**	6D16
5-8	22D16	**24-27**	6D16
6-9	18D16		
7-10	18D16	**7-8**	Top: 2D12 + 18D16
8-11	22D16	**8-9**	Bott: 2D12 + 8D16
9-12	18D16	**10-11**	Top: 2D12 + 18D16
10-13	18D16	**11-12**	Bott: 2D12 + 8D16
11-14	20D16	**13-14**	Top: 2D12 + 18D16
12-15	18D16	**14-15**	Bott: 2D12 + 8D16
13-16	14D16	**16-17**	Top: 2D12 + 11D16
14-17	18D16	**17-18**	Bott: 2D12 + 5D16
15-18	14D16	**19-20**	Top: 2D12 + 9D14
16-19	12D16	**20-21**	Bott: 2D12 + 8D16
17-20	14D16	**22-23**	Top: 2D10 + 8D12
18-21	12D16	**23-24**	Bott: 2D10
19-22	8D16	**25-26** **26-27**	Top: 2D10 + 7D12 Bott: 2D10

Figure 8. Building frame geometry and reinforcement arrangement

Buckling of longitudinal rebar is very common in r.c. buildings designed according to old Italian norms where stirrup reinforcement was neither properly prescribed nor accurately detailed. Therefore, these buildings that do not suffer from shear problem because of obvious geometrical reasons, do suffer instead from lack of confinement with associated concrete spalling and rebars buckling.

The analysis of the frame depicted in Fig. 8. is intended to investigate these problem on an existing structure. The analysed frame belong to a large Italian hospital located in a seismic area of central Italy. The construction lack of proper anti-seismic detailing especially when it comes to transverse reinforcement contrary to the longitudinal one which is, as it is often the case, particularly abundant.

The frame has been subjected to a push over analysis with an imposed displacement profile linearly increasing with height. In the first cycle a top drift of 2.5% of the frame height has been imposed and increased by 25% at each subsequent cycle.



(a) (b)

Figure 9. Bottom storey lateral column (element 1-4): bending moment-curvature diagrams

In Fig. 9. the moment-curvature response of the base section of first floor column (node 1-4) is reported. This column has a large longitudinal reinforcement (volumetric ratio about 2%) but a low stirrup spacing ratio (L/D = 12.5). In Fig. 9.(a) a comparison between the results obtained with previous and actual steel constitutive model while in Fig. 9.(b) a comparison with or without buckling are proposed.

The reduction of strength and the localisation of curvature is remarkable especially when considering that the analysis has been conducted with the same assigned displacement profile. Also significant is the reduction in the energy dissipation due to buckling of longitudinal rebars at load reversal which increase the pinching in the moment-curvature response.

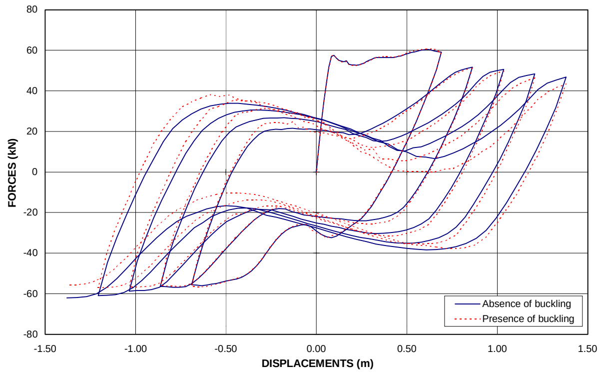


Figure 10. Force-displacement for top storey

The global force-displacement response is reported in Fig. 10. as the total base shear force versus top floor displacement. The frame shows a remarkable pinching with decreasing strength at load reversal due to the pull effect on the column flexural resistance (N-M interaction) typical of tall frame with strong beams as the one under consideration. The effect is remarkable and further amplified by the presence of buckling.

CONCLUSIONS

The implementation of buckling capabilities in a fibre beam element and some preliminary results obtained with the model have been presented. The analyses have been conducted using the purely flexural element so as to investigate the effect of buckling on axial-flexural response alone.

The steel model with buckling capabilities performs satisfactory in all load conditions. The fibre element response is significantly enhanced and the accuracy with respect to test data consistently improved. It is confirmed that the effect of buckling can be relevant in many types of structures, especially old ones lacking of proper hoop confinement and detailing.

These studies are now possible with these new fibres element which are extremely easy to use, robust and fast. In this respect, the implementation of buckling capabilities has been quite straightforward since it did not require additional subroutines but only the modification of the longitudinal steel constitutive behavior. The actual rebar model is even faster and simpler then the previously used steel one as it includes the buckling branches but ignores other details which are not significant when modelling reinforcing bars embedded in concrete.

The effect of buckling and its interaction with the compression-shear behavior need to be investigated as it may prove to be a very interesting field of research. The implementation of the model in the shear enhanced fibre element has already been accomplished but further works need to be done to correlate interactively the 3D constitutive behavior of the concrete fibres and the transverse steel with this new model for longitudinal rebar.

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