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DEVELOPMENT OF A REAL-TIME HYBRID EXPERIMENTAL SYSTEM USING A SHAKING TABLE

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SUMMARY

A hybrid experimental method, in which an actuator-excited vibration experiment and a computer simulation are conducted simultaneously and combined on-line, is being developed as a new seismic experimental method for investigating structural systems. In this report, a method using a shaking table as the actuator is proposed so that the hybrid experimental method can be applied to a secondary structural system attached to a primary structural system. This report also describes a response analysis algorithm for this experimental method and gives the results of prototype experiments using a small shaking table and a digital signal processor and demonstrating the feasibility of this seismic experimental.

INTRODUCTION

For conducting seismic experiments on structural systems, Hakuno et al. Developed a method in which an actuator-excited vibration experiment and a vibration response calculation are conducted simultaneously and combined on-line [Hakuno et al., 1969]. Various researchers have improved this method [Iemura, 1985, Takanashi et al., 1987]. This method has various names and it will be referred to here as the "hybrid experimental method" because it is the hybrid of an excitation experiment using a shaking device (such as an actuator) and a vibration simulation by computer. This hybrid experimental method makes it possible to evaluate the seismic response of a structural system by experimentally exciting only a part of the structure. Seismic experiments can therefore be conducted more economically.

We have been developing "real-time" hybrid experimental systems, which perform the simulation and the experiment in the same time axis and evaluate the precise seismic response of structural systems [Horiuchi, et al., 1996]. In these hybrid experimental methods, the structure under excitation is loaded by driving an actuator according to the relative displacement calculated by a computer. These methods, however, cannot be applied to secondary structures attached to a primary structure (e.g., a mass damper using for controlling the vibration of buildings) because the response of such secondary structures depends on the acceleration of the point of attachment to the primary structure. We therefore developed an experimental method in which a shaking table is used as a shaking device (Horiuchi, et al., 1994), and similar studies have been reported by others [Suwa, et al., 1996, Konagai, et al., 1998].

In this paper, we explain the concept of hybrid experiments using a shaking table and also explain a vibration simulation method we developed for use in this kind of hybrid experiment. We also shows the experimental results, obtained using a prototype system consisting of a small shaking table and a DSP (digital signal processor), that demonstrates the feasibility of this kind of seismic experiment.

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OUTLINE OF HYBRID EXPERIMENT USING A SHAKING TABLE

Let us consider a structural system like that shown in Fig. 1, consisting of a primary system and an attached secondary system. Seismic experiments on such a structure usually use a shaking table in one of the following two ways: to excite the whole structure including the primary structure, or to excite only the secondary structure according to the acceleration record obtained from an earthquake-response simulation of the primary structure. The former method, however, cannot be used when the scale of the primary structure is beyond the capacity of the table, and the latter method does not yield an accurate seismic response when the interaction between the primary and the secondary structures is not negligible. The shaking-table-type hybrid experiment is for investigating a structural system in which a primary structure interacts strongly with a secondary structure. Only the primary structure in Fig. 1 is modeled numerically and its motion is described as follows:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} + \mathbf{q} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are respectively the mass, damping, and stiffness matrices of the numerical model, $\mathbf{x}$ is a relative displacement vector, $\mathbf{f}$ is an external force vector, $\mathbf{q}$ is a reaction force vector generated at the boundary of the actual and the numerical models, and the dots represent differentiation with respect to time. The secondary structure, on the other hand, is excited using a shaking table. As the external forces for calculating the earthquake response of the primary structure, the inertia force $\mathbf{f}$ caused by an earthquake and the measured force $\mathbf{q}$ applied to the table from the secondary structure are used. Naturally, the measured reaction force $\mathbf{q}$ depends on $\ddot{y}_s$, the absolute response acceleration at point s, the point of attachment to the primary structure. The seismic response of the whole structure can therefore be evaluated by repeating the following steps: (1) measure the reaction force $\mathbf{q}$ from the shaking-table-excitation, (2) calculate the relative response acceleration of the primary structure $\ddot{y}_s$ by using the measured reaction-force vector $\mathbf{q}$ and the predetermined external-force vector $\mathbf{f}$, (3) calculate the absolute response acceleration $\ddot{y}_s$ by adding $\ddot{x}_s$ and the ground acceleration $\ddot{z}$, (4) excite the secondary structure with the calculated absolute response acceleration $\ddot{y}_s$ by using the shaking table. It is obvious that the response of the secondary structure attached to the primary structure can be obtained this way because the secondary structure is excited by an acceleration properly reflecting the secondary response. It should be noted that, in the shaking-table-type experiment, excitation in real time is essential for producing acceleration.

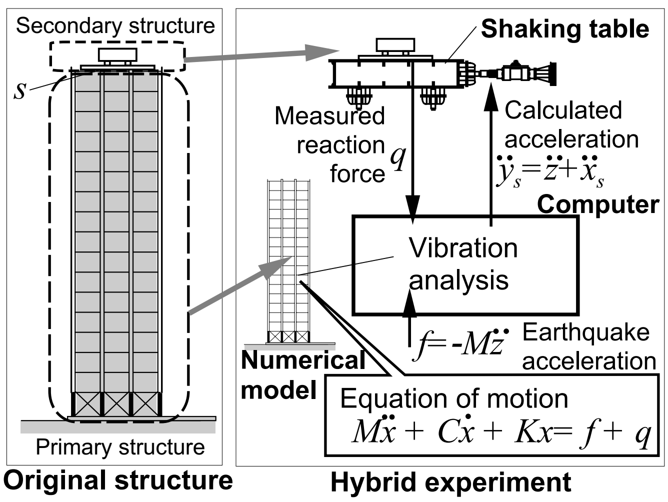

Figure 1: Conceptual view of the hybrid experiment using a shaking table.

SIMULATION OF RESPONSE AND GENERATION OF CONTROL SIGNAL

A hybrid experiment uses a computer to numerically simulate the vibration response and makes use of the simulation results to generate the control signal driving the shaking device. In the actuator-type experiment, "the central difference method" [Takanashi et al., 1987] is usually used for time integration of Eq. (1). Since this method uses information known at a certain time and calculates the displacement after a small time period Δt, it is suitable for the actuator-type experiment because the relative displacement is the quantity loaded on the structure by the actuator. This method, however, is not applicable to the shaking-table-type experiment because the acceleration after the small time period Δt required for this type of experiment cannot be obtained. To solve this problem, we developed a numerical simulation based on the "linear acceleration method" and a control-signal-generation method.

Numerical Simulation Method

By applying the linear acceleration method to the equation of motion written in Eq. (1), we can obtain the following equation:

$$\ddot{\mathbf{x}}_{i+1} = \left\{\mathbf{M} + \frac{\Delta t}{2}\mathbf{C} + \frac{\Delta t^2}{6}\mathbf{K}\right\}^{-1} \left\{\mathbf{f}_{i+1} + \mathbf{q}_{i+1} - \mathbf{C}\left(\dot{\mathbf{x}}_i + \frac{\Delta t}{2}\ddot{\mathbf{x}}_i\right) - \mathbf{K}\left(\mathbf{x}_i + \Delta t\dot{\mathbf{x}}_i + \frac{\Delta t^2}{3}\ddot{\mathbf{x}}_i\right)\right\}, \tag{2}$$

$$\dot{\mathbf{x}}_{i+1} = \dot{\mathbf{x}}_i + \Delta t \frac{\ddot{\mathbf{x}}_i + \ddot{\mathbf{x}}_{i+1}}{2}, \tag{3}$$

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \Delta t\dot{\mathbf{x}}_i + \frac{\Delta t^2}{3}\ddot{\mathbf{x}}_i + \frac{\Delta t^2}{6}\ddot{\mathbf{x}}_{i+1}, \tag{4}$$

where Δt is the time step for calculation, the subscript i indicates the value at time t_i, and $t_{i+1} = t_i + \Delta t$. Therefore, by utilizing the linear acceleration method —that is, Eqs. (2), (3), and (4)— we can calculate the vibration response at time t_{i+1} from the displacement, velocity, and acceleration at time t_i and the external force at time t_{i+1}; that is, $\mathbf{f}_{i+1} + \mathbf{q}_{i+1}$. These formulas, however, cannot be applied to the shaking-table-type hybrid experiments directly because $\mathbf{q}_{i+1}$ is a reaction force caused by the absolute acceleration of the attachment point s at time t_{i+1}. That is, $\ddot{y}_{s_{i+1}} = \ddot{x}_{s_{i+1}} + \ddot{z}_{i+1}$, and thus functions of $\ddot{x}_{s_{i+1}}$ are found on both sides of Eq. (2). In addition, the dynamic characteristics of the shaking device should be taken into consideration in this type of experiment. Electrohydraulic servo-controlled actuators are usually used for driving a shaking table, and these actuators have a response delay. In other words, the shaking table produces the required acceleration a little later than the control signal is inputted. The control signal of the shaking table therefore has to be generated so that acceleration $\ddot{y}_{s_{i+1}}$ can be approximately produced at time t_{i+1} by the shaking table.

The simplest way to have the shaking table produce the approximate acceleration of the attachment point s, $\ddot{y}_{s_{i+1}}$, by is to make the control signal by making use of the calculated value of $\ddot{x}_s$ at time t_i. In this case, the calculated acceleration is produced by the shaking table one calculation time step later. In addition, the dynamic characteristics of the shaking table cause an additional delay.

The influence of these delays on the experimental error is determined by considering the free vibration of the structural model shown in Fig. 2. This structural system consists of a primary structure, which is a single-degree-of-freedom system of mass M, and a secondary structure, which is a rigid mass m. Since $\ddot{z} = 0$ so that $\ddot{y} = \ddot{x}$, the equation of motion of the primary structure becomes

$$M\ddot{x} + Kx = q = -m\ddot{x}', \tag{5}$$

where $\ddot{x}'$ is the acceleration actually produced by the shaking table. By assuming a sinusoidal response (because free vibration is considered), we can be obtain the following equation:

$$x' = A\sin\omega(t-\Delta t) = \cos\omega\delta t\left(A\sin\omega t\right) + \omega\sin\omega\delta t\left(-A\sin\omega t/\omega\right) \approx \ddot{x} + \omega^2\delta t\dot{x}, \tag{6}$$

where δt is the delay time to obtain the required acceleration, and $\ddot{x} = A\sin\omega t$ ($\omega^2 = K/(M+m), \omega\delta t << 1$). By substituting this equation into Eq. (5), we obtain

$$\left(M+m\right)\ddot{x} + (m\omega^2\delta t)\ddot{x} + Kx = 0. \tag{7}$$

The influence of the delay in producing the acceleration is thus found to be additional damping. This damping increases the experimental error, so the delay should be compensated.

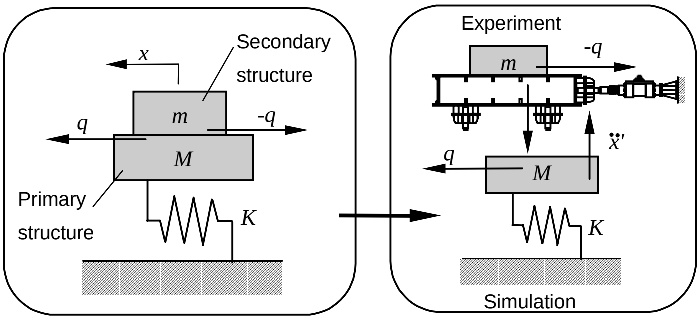

Figure 2: Experiment of a single-degree-of-freedom system

Generation of Control Signal

The acceleration after the supposed delay time is predicted by making use of the calculated acceleration and the control signal supplied to the shaking table is generated by using this predicted acceleration. The predicted acceleration $\tilde{\ddot{x}}$ is obtained through the following equation:

$$\tilde{\ddot{x}} = \sum_{k=0}^{n} a_k \ddot{x}^{(k)} , \tag{8}$$

which is the predicted value by extrapolating according to an n th-order polynomial function. Here $\ddot{x}^{(k)}$ s are the accelerations calculated at time $k \times \delta t$ prior, n is the order of prediction, and a_k is a constant depending on the prediction order (Table 1). The 0th order prediction means the control signal is generated without prediction; in other words, the calculated value at time t_i is used to generate the control signal for time t_{i+1}. By supplying this control signal, we can obtain the table acceleration and can then measure the reaction force. Since this reaction force is an approximate value of $\mathbf{q}_{i+1}$, the vibration response at the time t_{i+1} can be calculated using Eqs. (2), (3), and (4). It should be noted that the values used for Eq. (8) are the results of Eq. (2). This calculation procedure is equivalent to conducting the convergence once in a simulation of nonlinear vibration response.

Stability and Error of the Experiment

The stability and the error of the experiment using the numerical simulation discussed above are of interest from the viewpoint of application to seismic experiments. These are discussed in detail elsewhere [Horiuchi et al., 1999] by considering a rigid-mass secondary system. The results of that discussion can be summarized as follows.

When considering a single-degree-of-freedom system with a mass of M as a primary structure and a rigid mass of m as the structure under excitation, the proposed numerical simulation method for the shaking-table-type hybrid experiment has a stability criterion in terms of the ratio μ, that is the ratio of the mass of the secondary structure m to the mass of the primary structure M. This criterion depends on the prediction order as listed in Table 2, in which it can be seen that a higher-order prediction results in less stability.

The error of the experiment with n th-order prediction is proportional to the $(n+1)$ -th power of Δt. Therefore, a higher prediction order gives a smaller experimental error. Since Δt is equivalent to the prediction time δt here, it can be concluded that the error caused by numerical simulation in the shaking-table-type hybrid experiment can be made similar to that in the linear-acceleration method by making the prediction time small enough.

As discussed above, the prediction order has contrasting effects on the stability and the error. The prediction order should therefore be decided considering the trade-off between the stability and the error.

Table 1: Coefficients for prediction.

Order n	a_0	a_1	a_2
0	1	–	–
1	2	–1	–
2	3	–3	1

Table 2: Stability criterion for shaking-table-type hybrid experiments.

Prediction order	0th	1st	2nd
Stable criterion	$\mu < 1$	$\mu < \frac{1}{3}$	$\mu < \frac{1}{7}$

DEMONSTRATION EXPERIMENT

Purpose and Method of Experiment

We conducted demonstration experiments by using an actual shaking table in order to show the feasibility of this type of experiment under experimental errors caused by actually driving a shaking table and measuring a reaction force. The experimental system consisted of a computer, a shaking table, and a reaction force-measuring device (Fig. 3). Two types of structure under excitation were used. One was a rigid mass, which is the simplest secondary structure. The other was a single-degree-of-freedom system, which may appear in the actual seismic experiments.

Experimental Setup

Control computer

The control computer for performing numerical simulation and generating the control signal was a DSP (TMS320C30). This DSP can output a control signal for a shaking device through a D/A converter and input a reaction force though an A/D converter as well as iterate a numerical simulation process in every given time step. In these experiments, the time step was 0.5 ms both for control and calculation, so the control frequency was 2000 Hz. It should be noted that this DSP requires one time step for outputting a control signal generated by using the calculation results. This required time has the equivalent influence on experimental results to that of the delay in producing acceleration by a shaking table.

Shaking table

When this experimental method is used to evaluate a large-scale structure, the shaking table has to have a loading capacity of several tons. The small shaking table shown in Fig. 3, however, was used in the present work because the purpose of the experiments was simply to demonstrate the feasibility of the method. The table was supported by a linear guide and driven by a hydraulic actuator with a load capacity of 15 kN and a stroke of ± 100 mm. The dynamic characteristics of this shaking table are shown in Fig. 4 in the form of a transfer function from the control signal to the obtained acceleration. Even though the gain increases a little as the frequency increases, it is almost flat from 0 to 20 Hz. The phase tends to be delayed. This can be explained by the fact that the delay exists in producing acceleration.

Reaction-force measuring device

The reaction force-measuring device must support the weight of the structure under excitation and must measure the reaction force caused by the motion of the structure precisely. Therefore, in the device used in these experiments, as shown in Figs. 3 and 5, the base is supported with rubber bearings and a load cell is placed between the base and the table in order to measure the reaction force. The capacity of this measuring device was measured in the following way. A rigid mass of 3.25 kg was fixed on the 3.75-kg base and the acceleration of the base was compared with the measured reaction force. The transfer function of these signals is shown in Fig. 6. In the frequency range from 0 to 20 Hz, the mass on the measuring device (7.0 kg) was measured precisely and the phase delay was small. It is therefore confirmed that the capacity of this device is good enough for conducting a shaking-table-type hybrid experiment.

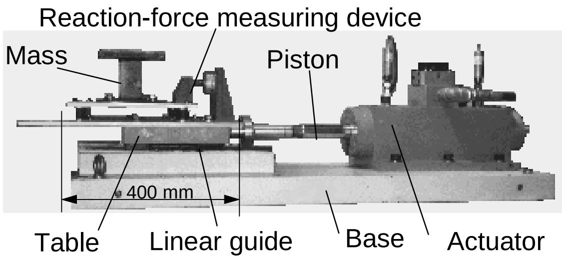

Figure 3: Experimental setup.

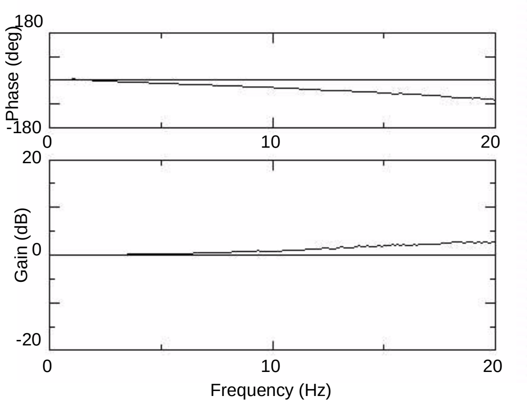

Figure 4: Dynamic characteristics of the shaking table.

Results for a Rigid Secondary System

Experiments were conducted by considering a single-degree-of-freedom primary structure consisting of a mass and a spring and a rigid-mass secondary structure. The total mass and the spring stiffness of the primary structure were assumed to be 50 kg and 17.8 kN/m, and the actual mass of the secondary structure was 3.75 kg. The natural frequency of the total system was thus 3.75 Hz. A damping coefficient of 18.8 Ns/m was added to the numerical model so that the damping ratio of the total structure became 0.01.

When an actual shaking table is used, not only the delay inherent in the numerical simulation but also that caused by the dynamic characteristics of the shaking table should be compensated. In these experiments, the shaking table delay was evaluated to be 4.5 ms, which was the delay at the natural frequency of the considered structure (3.75 Hz) obtained from the transfer function shown in Fig. 4. The sum of this 4.5-ms delay and the 1-ms computer-related delay was compensated by the prediction using Eq. (8). The second-order prediction was used in these experiments.

It should be noted that the reaction force q used for the numerical simulation was obtained by measuring the base acceleration a_B and subtracting the inertia force from the measured reaction force q' as follows:

$$q = q' - m_B a_B , \tag{9}$$

where m_B is the mass of the base.

The experimental results are summarized as follows. The transfer function obtained from the hybrid experiment under the condition equivalent to the base random excitation is shown in Fig. 7, where the analytical result for the whole structure is also shown for comparison. Even though data fluctuation caused by experimental noise is observed, the experimental result (indicated by the solid line) agrees well with the analytical one (indicated by the dotted line). It was therefore confirmed that this experimental system can evaluate vibration response precisely. Next, the time history obtained experimentally under the base excitation by a three-period 3.5-Hz sinusoidal wave is shown in Fig. 8, in which also analytical result is shown for comparison. The two results are (except for some experimental noise) almost identical.

Results for a Flexible Secondary System

Experiments were also conducted by considering both the primary structure and the secondary structure are single-degree-of-freedom systems. The secondary structure had a mass of 6.1 kg, a stiffness of 7600 N/m, and thus a natural frequency of 5.6 Hz. The other conditions —including the parameter of the primary structure in the numerical model, the delay compensation, and the reaction force measurement— were the same as in the experiments discussed in Section 4.3.

The experimental results are summarized as follows. The transfer function obtained from the hybrid experiment under the condition equivalent to the base random excitation is shown in Fig. 9, where the analytical result of the two-degree-of-freedom structure is also shown for comparison. Again, the experimental result (solid line) agrees well with the analytical one (dotted line). And the time history obtained experimentally under the base excitation by an earthquake record is almost identical to the analytical result (Fig. 10). The experimental results show a relatively large vibration even when the input acceleration is almost zero. This was caused by the noise in the measurement of the reaction force, which became excitation force for the numerical model. This effect, however, is negligible when the structural response is large enough.

CONCLUSIONS

This paper described, for seismic experiments with a secondary structure attached to a primary structure, a shaking-table-type hybrid experimental method and a numerical simulation algorithm for this method. Demonstration experiments using prototype experimental system consisting of a small shaking table and a DSP confirmed the feasibility of this experimental method. The results obtained can be summarized as follows:

There is a stability criterion that depends on the ratio of the masses of the secondary and primary structures. This criterion also depends on the order of acceleration prediction, and a higher-order prediction results in less stability. As for the error, on the other hand, a higher-order prediction gives a smaller error. The prediction order should therefore be decided considering the trade-off between stability and accuracy.

Demonstration experiments were conducted using a prototype experimental system consisting of a small shaking table and a digital signal processor. The experimental results agreed well with the numerical results. The feasibility of this experimental method was therefore demonstrated.

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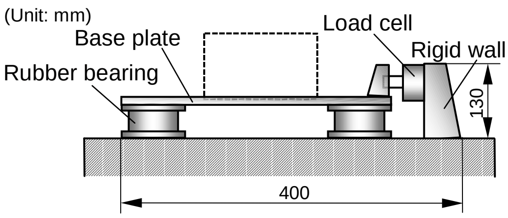


Figure 5: Schematic of reaction-force measurement. device

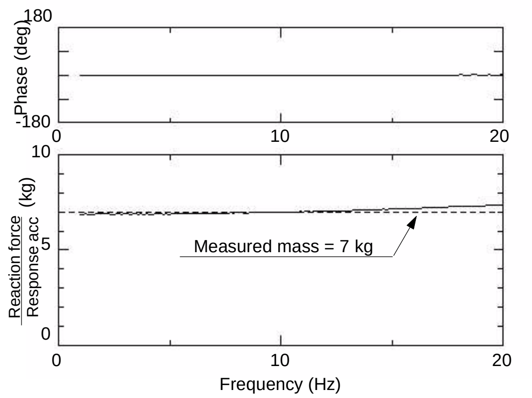


Figure 6: Dynamic characteristics of reaction-force measurement.

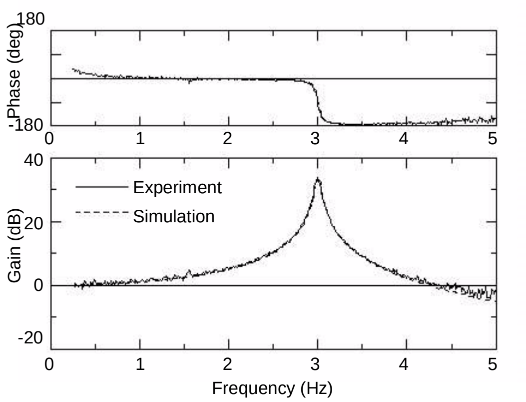


Figure 7: Acceleration transfer-function obtained for a rigid secondary system.

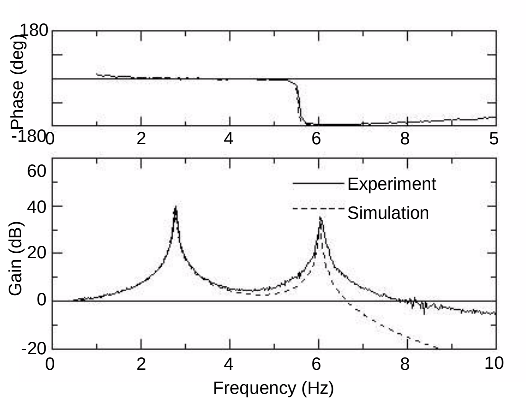
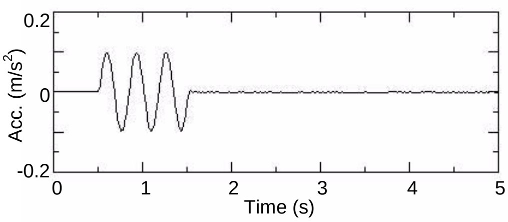
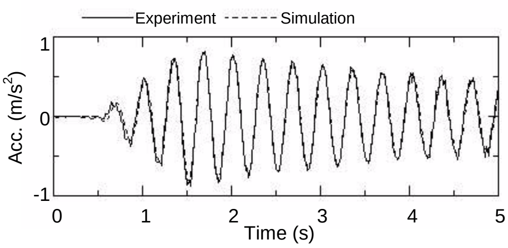


Figure 9: Acceleration transfer-function obtained for a SDOF secondary system.

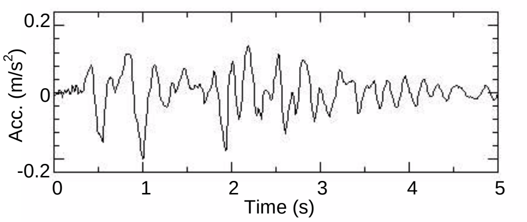


(a) Input acceleration

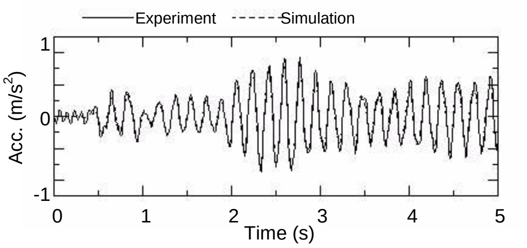


(b) Response acceleration

Figure 8: Time history obtained for a rigid secondary system.



(a) Input acceleration



(b) Response acceleration

Figure 10: Time history obtained for an SDOF secondary system.