THE ANALYTICAL ACCURACY OF AUTHORS' THREE MACROSCOPIC MODELS OF RC FRAMED SHEAR WALLS USING FIVE HUNDRED AND SEVENTY THREE SPECIMENS

Makoto MAKOTO[1], Masao TAKEHARA[2], Norikazu ONOZATO[3] And Michika OKABE[4]

SUMMARY

In this paper the analytical accuracy of the authors' three macroscopic models, which are referred to the original, the simplified, and the elasto-plastic models, for evaluating the maximum strength of RC framed shear walls is discussed. The analyses using these three macroscopic models are executed for five hundred and seventy three specimens, conducted in Japan from 1970 to 1995. And their results were compared with the observed maximum strengths in experiment. The comparisons show that the analytical accuracy of the authors' three macroscopic models is excellent compared with that of using the other models in Japan for large range of the various parameters of the specimens.

INTRODUCTION

In Japan, RC framed shear walls have been designed according to Standard for Structural Calculation of RC Structures of AIJ, in which their maximum strength is evaluated as smaller value between the shear strength carried by reinforcing bars of the wall panel and the summation of shear strength carried by concrete of the wall panel and shear strength of the side columns. However, this evaluated maximum strength does not correspond to the observed failure behaviors of RC framed shear walls in experiments. As an experimental formula for evaluating the maximum strength Hirosawa's formula [AIJ,1990] is widely used, but the physical meaning is not clear. On the other hand, analytical studies using macroscopic models have been carried out. As typical models Shohara's model [Shohara, R.,et al., 1984] and AIJ's model, which is adopted in Design Guideline for Earthquake Resistant Reinforced Concrete Building Based on Ultimate Strength Concept of AIJ, are widely used.

In the previous papers, we also proposed the original and the simplified models for evaluating the only maximum strength, and the elasto-plastic model for evaluating strength and deformation. Although these macroscopic models may well evaluate the maximum strength, but the analytical accuracy for many and various RC framed shear walls is not yet clarified. From this point of view, this paper aims to clarify the analytical accuracy of the authors' three macroscopic models using five hundred and seventy three specimens of R.C framed shear walls conducted in Japan from 1970 to 1995.

2. OUTLINES AND ANALYTICAL PROCEDURES OF MODELS

2.1 Original Model [Mochizuki,M., *et al*., 1990]

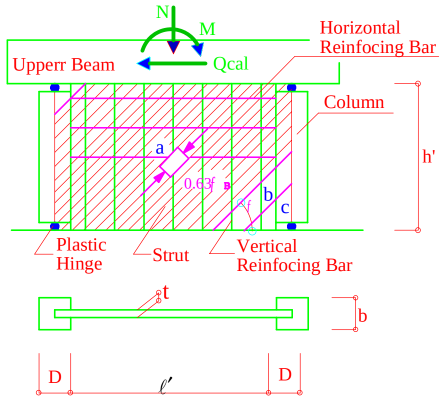
This study, in any case of the authors' three models, treats RC framed shear walls with one span-one story. In order to introduce the effects of the upper stories a moment, and a vertical and a horizontal forces acting on the upper beam are considered as external loads. Figure 1 shows the original model. The original model consists of upper and lower beams with large sectional area, two side columns ,which are sufficiently reinforced to assure not to fail in shear, compressive struts ***a***, ***b***, and ***c*** with the same inclination angle θ deg., and vertical and

[1] *Kohgakuin University, 1-27-2, Nisisinnjuku, Shinjuku-ku, Tokyo, Japan,e-mail:dt56056@ccs.kogakuin.ac.jp*
[2] *Tokyo Architecture School, 11-1, Kikuicyo Shinjuku-ku, Tokyo, Japan*
[3] *Kohgakuin University, 1-27-2, Nisisinnjuku, Shinjuku-ku, Tokyo, Japan,e-mail:dt56056@ccs.kogakuin.ac.jp*
[4] *Kohgakuin University, 1-27-2, Nisisinnjuku, Shinjuku-ku, Tokyo, Japan,e-mail:dt56056@ccs.kogakuin.ac.jp*

horizontal reinforcing bars. Each member of the original model is assumed to be under the following conditions at the maximum strength.

1) Upper and lower beams are rigid, and they do not fail.
2) Bending moments at the both ends of the side columns are expressed by the yield curve of column, and those at the middle part of the side columns are on or inside of the yield curve. The equations expressing the yield curve are quoted from Standard for Structural Calculation of RC Structures of AIJ.
3) Compressive struts ***a*** are under yielding, and their yield strength is taken as $0.63\sigma_B$ based on the slip failure strength of the wall panel proposed by the authors. Horizontal stress component of compressive struts ***b*** is in equilibrium with the stress of horizontal reinforcing bars because the parts of the side columns crossing the bars are under flexural yielding. But the stress of the compressive struts ***b*** is not larger than that of the compressive struts ***a***. Compressive struts ***c*** are ignored because the parts of the side columns crossing the compressive struts ***c*** are under tensile yielding.
4) All vertical and horizontal reinforcing bars are under tensile yielding.

The maximum strength of the original model is evaluated by considering only equilibrium conditions based on the lower bound theorem of the limit analysis. The equilibrium conditions reduce to nonlinear simultaneous equations, and must be analyzed by the iteration method. The required equilibrium conditions for the analysis and the method to determine the positions and widths of the compressive struts ***a***, ***b***, and ***c*** are explained in detail in [Mochizuki, M., et al., 1990]. Figure 2 shows the analytical flow for the original model with a specific inclination angle θ of the compressive struts. Here, the solution Q satisfying the statically admissible stress condition is evaluated as the summation ($Q=\Sigma Qc+Qw$) of the shear force (ΣQc) of the two side columns and the horizontal component (Qw) of axial forces of all the compressive struts at the top face of the lower beam. The maximum value among the calculated solutions by varying the value of inclination angle of the compressive struts is the maximum strength ($Qcal$) of the original model based on the lower bound theorem.



N : External Vertical force
M : External Moment
Qcal : External Horizontal force

Figure 1: Original model

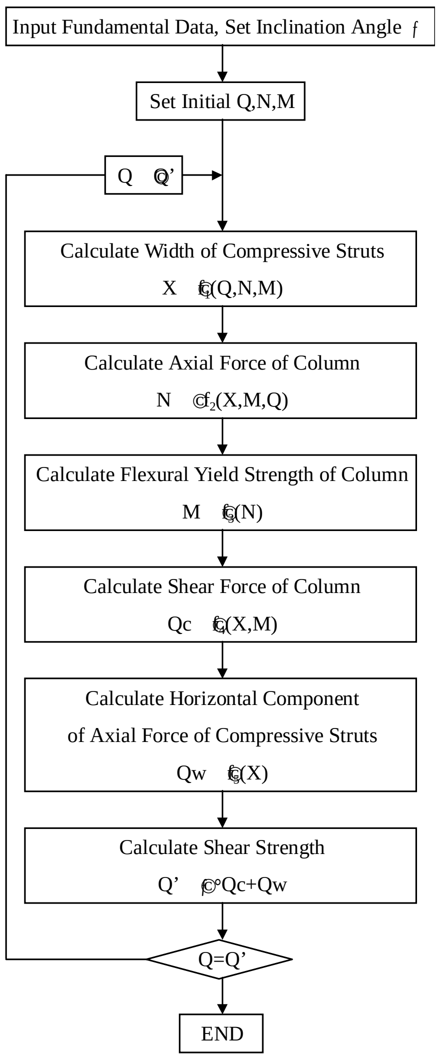
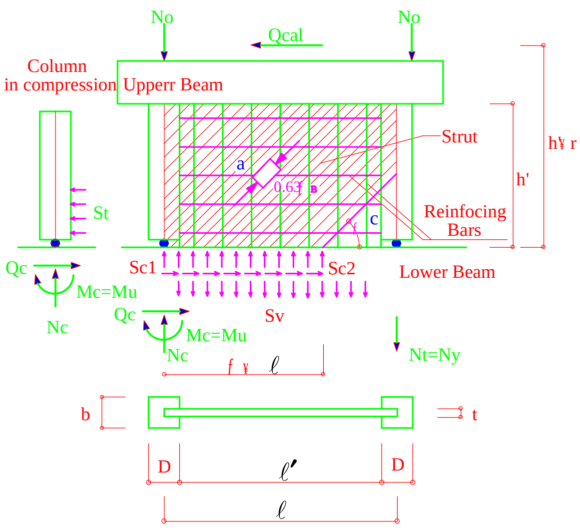


Figure 2: Analytical flow

2.2 Simplified Model [Mochizuki,M., *et al*., 1991]
The simplified Model is proposed to avoid the iteration method for the analysis of the original model. The simplified formulae for evaluating the maximum strength by simple calculation are deduced from this simplified model. Figure 3 shows the simplified model and its assumed stress distributions. The model consists upper and lower beams, two side columns, compressive struts ***a*** and ***c***, and vertical and horizontal reinforcing bars. Each member of the simplified model is assumed to be under the following conditions at the maximum strength.

1) Upper and lower beams are rigid, and they do not fail.
2) Compressive struts ***a*** are under yielding, and their yield strength is taken as $0.63\sigma_B$.
3) Compressive strut ***c*** are ignored because the part of the side columns crossing the compressive struts ***c*** is under tensile yielding.
4) All vertical and horizontal reinforcing bars are under tensile yielding.
5) Bending moment and shear force at the bottom end of the side column in tension are negligible.



Sc1 : Shear stress distributing on lower beam by struts

Sc2 : Vertical stress distributing on lower beam by struts

Sc3 : Horizontal stress distributing on column by struts

Sh : Horizontal stress distributing on column by reinforcing bars

St : Horizontal stress distributing on column by wall panel

Sv : Vertical stress distributing lower beam by reinforcing bars

σ_B : Compressive strength of concrete cylinder

${}_g\sigma_y$: Yield strength of longitudinal bar of column

${}_s\sigma_{hy}$: Yield strength of horizontal reinforcing bar of wall panel

${}_s\sigma_{vy}$: Yield strength of vertical reinforcing bar of wall panel

Figure 3: Simplified model

6) Bottom end of the side column in compression is under flexural yielding.

Unknown quantities in the assumed stress distributions of Figure 3 are the horizontal width $\xi \cdot \ell$ of the compressive struts ***a*** and the axial force *Nc* at the bottom end of the side column in compression. The flexural yield strength of the side column in compression at the bottom end may be evaluated from the formula of column if the axial force is known. The two unknown quantities are obtained from only two equilibriums of moment at the both bottom ends of the side columns. The simplified formulae for evaluating the maximum strength of RC framed shear walls are summarized in the flow of numerical calculation of Table 1. In Table 1, *Qcal* is the maximum shear strength of the framed shear wall , *Qw* is horizontal component of axial force of all the compressive struts at the top face of the lower beam, and *Qc* is the shear force of the side column in compression. At the initial step of the flow the first approximate values *Mu** and *Qc** are introduced, but *Qcal* is evaluated using the second approximate values of *Mu* and *Qc* without further interaction.

Table 1: Flow of numerical calculation

$$Mu^* = \frac{Ny \cdot D}{2} \qquad (1)$$

$$Qc^* = \sqrt{2Mu^* \cdot St} \quad \left(\textit{provided} \quad Qc^* \geq \frac{2Mc^*}{h'}\right) \qquad (2)$$

$$\xi = \alpha + \sqrt{\alpha^2 + \frac{Sv}{Sc2} + 2\,\frac{\alpha' \cdot Qc + N0 + Ny(D/2\ell + l)}{Sc2 \cdot \ell}} \qquad (3)$$

$$Qw = Sc1 \cdot \xi \cdot \ell \qquad (4)$$

$$Nc = \alpha' \cdot Qc^* + (\alpha + \xi/2 - 1)Qw + Sv \cdot \ell/2 + N0 - Ny \cdot D/2\ell \qquad (5)$$

$$\left.\begin{array}{ll} Mu = (0.4 Ny \cdot D + 0.12 D \cdot \sigma_B)\left(\dfrac{b \cdot D \cdot \sigma_B + Ny - Nc}{0.6 b \cdot D \cdot \sigma_B + Ny}\right) & (Nc > 0.4 b \cdot D \cdot \sigma_B) \\ Mu = 0.4 Ny \cdot D + 0.5\, Nc \cdot D\left(1 - \dfrac{Nc}{b \cdot D \cdot \sigma_B}\right) & (0.4 b \cdot D \cdot \sigma_B \geq Nc > 0) \\ Mu = 0.4 D(Ny + Nc) & (0 \geq Nc) \end{array}\right\} \qquad (6)$$

$$Qc = \sqrt{2Mu \cdot St} \qquad (7)$$

$$Qcal = Qc + Qw \qquad (8)$$

$$\left.\begin{array}{ll} \theta = 25^\circ \times h'/\ell' + 25^\circ & (h'/\ell' \leq 0.8) \\ \theta = 45^\circ & (0.8 \leq h'/\ell' \leq 1.2) \\ \theta = 25^\circ \times h'/\ell' + 15^\circ & (1.2 \leq h'/\ell' \leq 1.8) \\ \theta = 45^\circ & (1.8 \leq h'/\ell') \end{array}\right\} \qquad (9)$$

Where,

$\alpha = -h' \cdot r/\ell$,

$\alpha' = +h' \cdot r/\ell$

$Ny = A_g \cdot {}_g\sigma_y$

$Sc1 = 0.63\sigma_B \cdot t \cdot \cos\theta \cdot \sin\theta$

$Sc2 = 0.63\sigma_B \cdot t \cdot \sin^2\theta$

$Sc3 = 0.63\sigma_B \cdot t \cdot \cos^2\theta$

$Sv = Psv \cdot {}_s\sigma_y \cdot t$

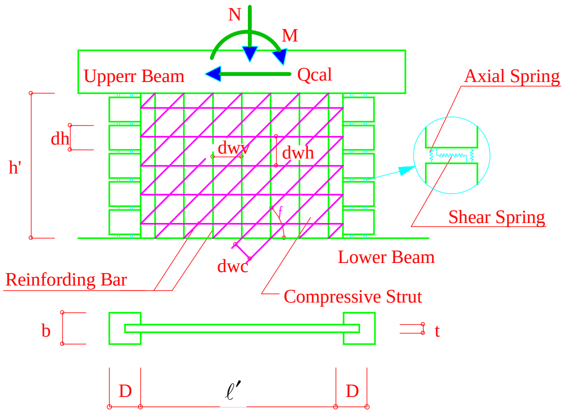
$Sh = Psh \cdot {}_s\sigma_y \cdot t$

$St = Sc3 - Sh$

$(\textit{provided} \;\; St \geq 0)$

2.3 Elasto-Plastic Model [Onozato, N., *et al.*, 1990]
The original and simplified models mentioned above evaluate only the maximum strength of RC framed shear walls. But, the force and displacement relationship is also necessary in design of RC framed shear walls. The elasto-plastic model is proposed by introducing the constitutive laws of each member of the model. Figure 4 shows the elasto-plastic model of RC framed shear walls. The elasto-plastic model consists of upper and lower beams, two side columns, compressive struts, and vertical and horizontal reinforcing bars. The inclination angle of compressive struts is the same as the angle obtained in the analysis by the original model. Each member of the elasto-plastic model has the following properties (Figure 5).

1) Upper and lower beams are rigid, and they not fail.
2) Columns are transposed with rigid elements, elasto-plastic axial springs, and elastic shear springs. The axial springs are located at each center of the longitudinal reinforcing bars in compression and tension, and their strengths and stiffnesses are expressed as follows,



dwc	: Width of strut
dwh	: Divided vertical width of wall panel
dwv	: Divided horizontal width of wall panel
dh	: Divide height of column
Ec	: Elastic modulus of concrete
Es	: Elastic modulus of reinforcing bar
G	: Shear modulus of concrete
ε_o	: Strain at maximum strength of concrete
ε_u	: Ultimate strain of concrete

Figure 4: Elasto-plastic model

Tensile range:

Strength $$_cN_{nt} = \frac{A_g ¥_g fĐ_y}{2} \quad (1)$$

Stiffness $$_cK_{nt} = \frac{E_s ¥ A_g}{2dh}$$

Compressive range:

Strength $$_cN_{nc} = \frac{A_g ¥_g \sigma_y + b ¥ D ¥ \sigma_B}{2} \quad (2)$$

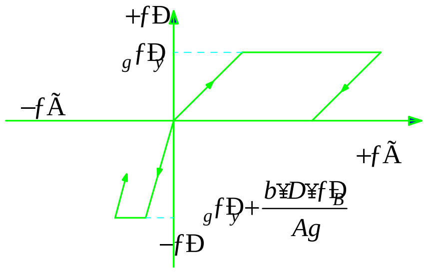
Stiffness $$_cK_{nc} = \frac{E_s ¥ A_g + E_c ¥ b ¥ D}{2dh}$$

The stiffness of the shear spring is expressed by Equation (3) and the shear spring dose not fails.

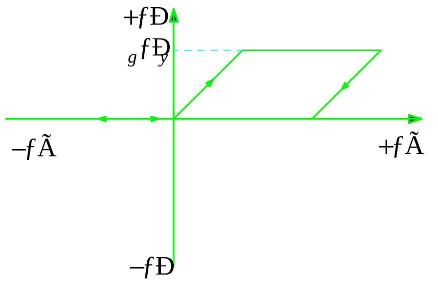
$$cKc = \frac{G ¥ b ¥ D ¥ cKn}{dh ¥ cKnc} \quad (3)$$

Where, *cKn/cKnc* is the reducing coefficient of stiffness, and *cKn* is the average stiffness of the axial springs located in compressive and tensile ranges at the same time. This reducing coefficient is based on the consideration of the extension of horizontal cracks. The vertical and horizontal reinforcing bars are reinforcing bars included in divided wall panel of which the width is *dw*. Their strength and stiffness are expressed by Equations (4) and (5), respectively as follows,

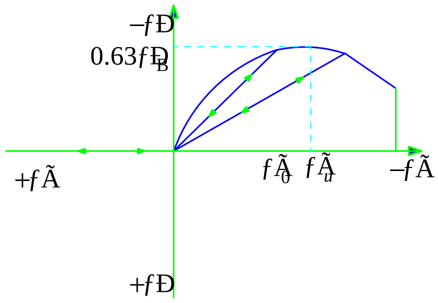
Strength $$\left.\begin{aligned} {}_vN_t &= Psv \cdot {}_s\sigma_{vy} \cdot dwv \cdot t \\ {}_hN_t &= Psh \cdot {}_s\sigma_{hy} \cdot dwh \cdot t \end{aligned}\right\} \quad (4)$$



a) Axial spring



b) Reinforcing bar



c) Compressive strut

Figure 5: Constitutive laws (4)

$$\text{Stiffness} \quad \left. \begin{aligned} {}_{v}K_t &= Psv \cdot Es \cdot dwv \cdot t/h' \\ {}_{h}K_t &= Psh \cdot Es \cdot dwh \cdot t/\ell' \end{aligned} \right\} \tag{5}$$

3) Compressive struts are subjected to the stress-strain relationship by S, Popovics [Popovic,S., 1971], in which σ_B is modified into 0.63 σ_B as follows,

$$\sigma = \frac{n \cdot \zeta}{n - 1 + \zeta^{n}} \cdot \sigma'_B \qquad \begin{aligned} \sigma'_B &= 0.63\sigma_B \ , \quad & n &= 0.57 \times 10^{-2} \cdot \sigma_B + 1 \\ \zeta &= \varepsilon / \varepsilon_o \ , \quad & \varepsilon_o &= 4.29 \times 10^{-4} \cdot {\sigma_B}^{0.25} \end{aligned} \tag{6}$$

Where, 0.63 σ_B is the effective compressive strength of the wall panel and is the same as the yield strength of the compressive struts of the original and simplified models. The ultimate strain ε_u is taken as 0.003. This value was decided from the elasto-plastic analysis of the framed shear walls using parameter ε_u as the value which generated the best compatibility between the envelope curves of experiment and analysis. The analysis is executed by the incremental method using the observed properties of materials.

The main features of the authors' three macroscopic models are summarized as follows,

1) The resisting mechanism of RC framed shear walls is not divided into a truss and an arch resisting mechanisms.
2) The coefficient of effective compressive strength of the wall panel takes as the constant value of 0.63, not depending on the value of σ_B.
3) The shear resisting capacity of the side column is directly introduced.

3. SPECIMENS FOR ANALYSES

The number of specimens for analyses is five hundred and seventy three, in which one hundred and thirty six specimens were conducted by the authors and the other specimens were by other research groups in Japan. The specimens were quoted from the Summaries of Technical Papers of Annual Meeting of AIJ and Proceedings of JCI published from 1970 to 1995. All the specimens were selected by the following criteria.

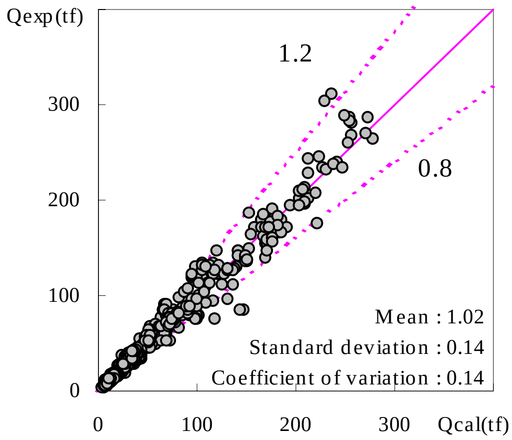
1) Configuration of specimen is framed shear wall or H or Box typed wall panel structure,
2) Upper and lower beams have large sectional area and sufficient reinforcement.
3) Specimen is subjected to a cyclic horizontal or a cyclic diagonal load.
4) Compressive strength of concrete is larger than σ_B =150kgf/cm^2
5) Observed maximum strength is larger than 5.0tf
6) Shear failure of side column does not occur until the maximum strength.
7) Ratio of two maximum strengths in positive and negative loading directions is not larger than 1.2

The ranges of main parameters of the selected specimens are as follows,

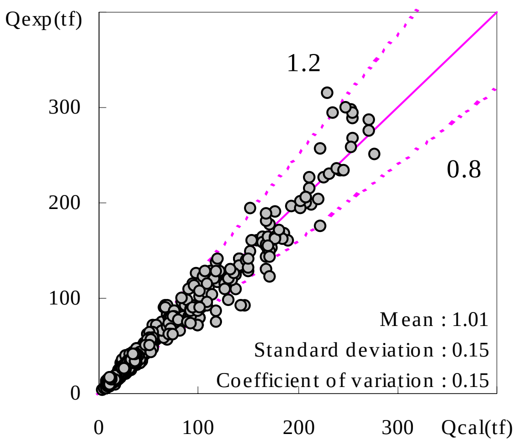
1)	Number of story	: $N = 1 \sim 4$
2)	Compressive strength of concrete	: $\sigma_B = 150 \sim 1400$ kgf/cm^2
3)	Gross longitudinal reinforcement ratio of column	: $Pg = 1.0 \sim 8.5$ %
4)	Shear reinforcement ratio of wall panel	: $Ps = 0.2 \sim 3.0$ %
5)	Aspect ratio of wall panel	: $\kappa = \frac{h'}{l'} = 0.2 \sim 3.0$
6)	Ratio of height of inflection	: $M/Q(\ell + D) = 0.2 \sim 3.0$

4. ANALITICAL RESULTS

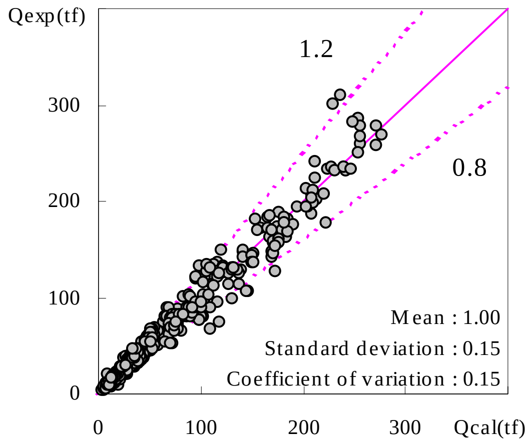
The analyses were executed for the selected five hundred and seventy three specimens using the authors' three models. And Shohara's model, AIJ's model, and Hirosawa's formula were also used for comparison. In the case that two maximum strengths in positive and negative loading directions were recorded in the papers two maximum strengths were treated as different sample points of the specimen. Then, the number of total sample points is seven hundred and thirty, in which one hundred and seventy are by the authors' and five hundred specimens and sixty are by the other groups. In this paper the analytical accuracy of the models are discussed on the sample points of the specimens. In the analyses the observed dimensions and properties of the specimens



(a) Original model



(b) Simplified model



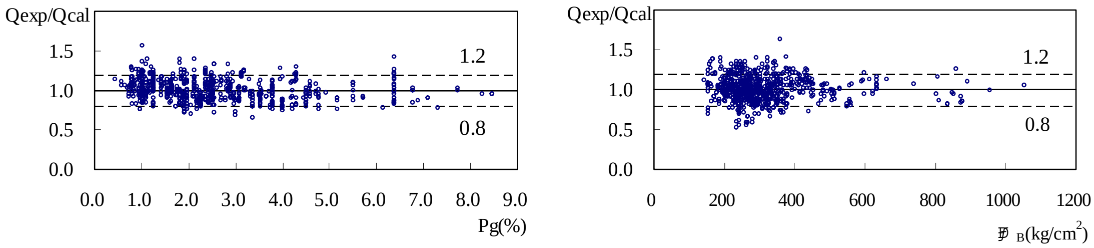
(c) Elast-plastic model

Figure 6: Comparisons of observed and calculated maximum strengths

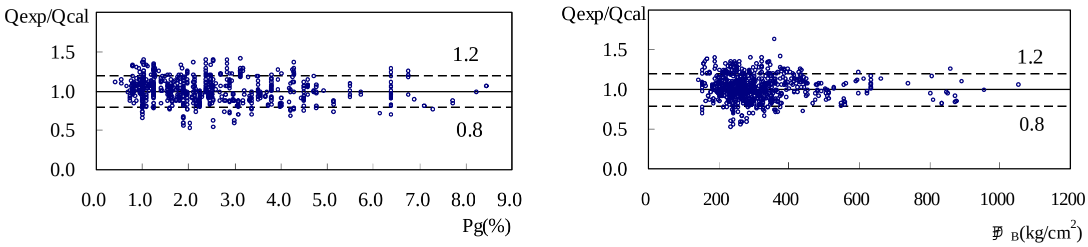
Table 2: Analytical accuracy 1

Model	Original model	Simplifid model	Elasto-plastic model	Shohara's model	AIJ's model	Hirosawa's formula
Mean	1.01	1.01	1.00	1.31	1.18	1.27
Standard deviation	0.14	0.15	0.15	0.29	0.26	0.37
Coefficient of variation	0.14	0.15	0.15	0.22	0.22	0.29

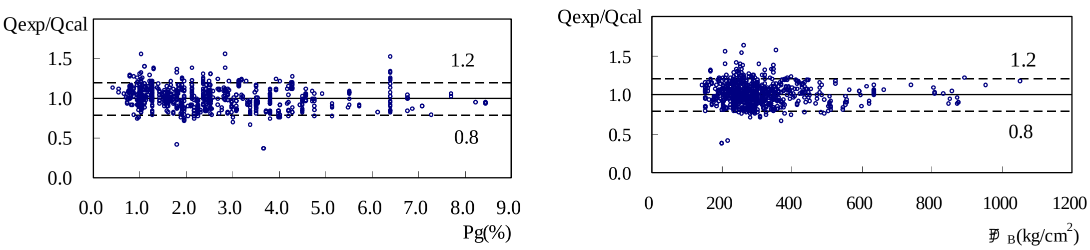
were considered. Figure 6 shows the comparisons of the observed and calculated maximum strengths for the original, the simplified, and the elasto-plastic models. And Table 2 shows the analytical accuracy of the authors' models which is defined as the ratio of the observed maximum strength to the calculated maximum strength. The values of mean, standard deviation, and coefficient of variation of the analytical accuracy are 1.01, 0.14 and 0.14 for the original model, 1.01, 0.15 and 0.15 for the simplified model, and 1.00, 0.15 and 0.15 for the elasto-plastic model. Table 2 shows also the analytical accuracy of Shohara's and AIJ's models, and Hirosawa's formula. Figure 7 shows the relationships between the main parameters : compressive strength of concrete σ_B, gross longitudinal reinforcement ratio of column, and analytical accuracy of the authors' three models. These figures show that the analytical accuracy of the authors' three models is excellent for the large range of two parameters.



(a) Original model

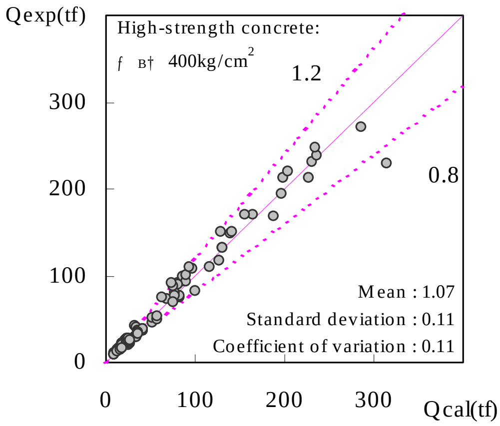


(b) Simplified model



(c) Elasto-plastic model

Figure 7: Relationships between Qexp/Qcal and main parameters



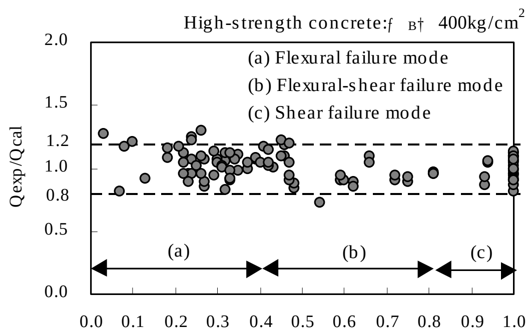


Figure 8: Comparison of Qexp and Qcal **Figure 9: Relationship between Qexp/Qcal and ξ**

Figure 7: Relationships between Qexp/Qcal and main parameters

In general it is said that the coefficient ν of effective compressive strength of concrete tends to small value as the compressive strength of concrete is becoming higher. In AIJ's model the coefficient of compressive strength of concrete is specified as the following equation.

$$\nu = 0.7 - \sigma_B / 2000 \tag{6}$$

Table 3: Analytical accuracy 2

Model	All specimens		Specimens with high-strength concrete	
	0.8(%)	1.2(%)	0.8(%)	1.2(%)
Original model	4.82	8.92	4.12	3.09
Simplifid model	5.38	10.62	0.93	6.54
Elasto-plastic model	7.08	7.79	6.19	4.12
Shohara's model	6.37	37.82	13.40	30.92
AIJ's model	2.12	33.43	8.25	41.24
Hirosawa's formula	6.37	52.12	12.37	50.51

High-strength concrete F_B 400 kgf/cm^2

However, the authors' three models treat the coefficient of effective compressive strength of concrete as the constant value of 0.63. Figures 8 and 9 show the comparison of Qexp and Qcal using the simplified model and the relationship between Qexp/Qcal and the coefficient ξ of effective width of the compressive struts ***a***, respectively. In the figures the results of the specimens with high strength of concrete $\sigma_B \geq 400$ kgf/cm^2 are plotted, and the number of the sample points is one hundred and seven. The figures show that the simplified model using the constant value 0.63 as the coefficient of effective compressive strength of concrete is adequate to evaluate the maximum strength of RC framed shear walls with high strength of concrete. But this dose not mean that the coefficient is constant value, and is not depending on compressive strength of concrete. Table 3 is the percentages of the sample points of which Qexp/Qcal is larger than 1.2 and smaller than Qexp/Qcal is smaller than 0.8 to the total sample points. This table shows also that the authors' three models are more excellent compared with the other models.

5. CONCLUSION

In this paper the analyses were executed for five hundred and seventy three specimens of RC framed shear walls, which were conducted form 1970 to 1955 in Japan, using the authors' three macroscopic models for evaluating the maximum strength of RC framed shear walls and other macroscopic models published in Japan. The analytical results show that the analytical accuracy of the authors' three model is more excellent compared with the other models for large range of the various parameters of RC framed shear walls.

6. REFERENCES

Architectural Institute of Japan (1990), "Ultimate strength and deformation capacity buildings in seismic design", pp390-403.

Mochizuki, M., and Onozato, N. (1990), "Macro model of multistory framed shear walls and its analytical method", Concrete Research and Technology, Vol.1, No.1, pp121-132.

Mochizuki, M., and Onozato, N. (1992), "Simplified formula for estimating maximum strength of multistory framed shear walls and discriminant of their failure modes", Proceedings of Earthquake Engineering , Tenth World Conference, pp4321-4328.

Mochizuki, M., Takehara, M., and Onozato, N. (1990), "Slip shear strength of shear walls surrounded with reinforced and stiffened frame", Journal of Structural and Construction Engineering (Transactions of AIJ), No.416, pp79-89.

Onozato, N., and Mochizuki, M. (1990), "Elasto-plastic analysis of framed shear walls using macro model", Proceedings of the Japan Concrete Institute, Vol.12, No.2: pp569-574.

Popovics, S. (1971), "Factors affecting the elastic deformation of concrete", Proc. of the International Conference in Materials, Kyoto, Vol.4, pp172.

Shohara, H., and Kato, B. (1984), "Ultimate strength of Multi-shear walls", Journal of Structural and Construction Engineering (Transaction of AIJ), No.343, pp24-31.