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EARTHQUAKE RESPONSE ANALYSES OF AN EMBEDDED STRUCTURE UTILIZING AN AVERAGED HORIZONTAL SOIL STIFFNESS

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SUMMARY

In this paper, an averaged horizontal soil stiffness that can take dynamic behavior of the surface soil surrounding the embedded structure into account is presented. Assuming that the side surface of the foundation embedded into the surface soil mediums deforms as a triangle shape along the longitudinal axis, the averaged horizontal soil stiffness per unit thickness for a whole side surface of the foundation is defined. Approximate formulae of the static value and the frequency-dependent function of the averaged horizontal soil stiffness are proposed. A simplified analytical method of sway-rocking soil spring model in which the shear deformation by rocking motion is evaluated by the averaged soil stiffness and sway and rocking motions are represented by the Novak's stiffnesses is also proposed. The approximation and applicability of the proposed method are discussed by comparing with the results from the 3-dimensional analysis, the original 3-dimensional thin layer approach.

INTRODUCTION

It is well known that the laterally axial and rotational stiffness functions for a horizontal soil layer [Novak *et al.* 1978, 1980] represent the dynamic characteristics of side soil for an embedded structure, as illustrated in Fig.1, Fig. 2(a) and 2(c). The authors proposed the shear stiffness to consider the coupling of the foundation stiffnesses related to the side surface of the embedded structure, as shown in Fig. 2(b) [Ikeda *et al*. 1992]. Usually, the bottom of the embedded structure is often sustained by the soil medium that is harder than the side surface one. In this case, the oscillation of the side surface medium strongly affects the response of the structure [Harada *et al.* 1981]. We derived the laterally axial, shear and rotational stiffnesses corresponding to the above three modes by using a single thin layer supported on a dashpot mat which represents the characteristics of a half-space. Especially, we presented the averaged soil stiffness for a whole side surface of the embedded structure related to the mode of (b) in Fig. 2 [Shimomura *et al.* 1996]. An approximate formula for the static value of the averaged soil stiffness is defined by using the impedance ratio of the supported soil to the side surface medium and the aspect ratio of the embedded depth to the radius of the cross section of the structure. Another approximate formula for radial inhomogeneous soils neighboring the embedded structure is also given [Veletsos et al. 1988].

This paper focuses the shear mode caused by the rocking motion that affects the response of the embedded structure supported by harder soil medium than side surface one. A simplified analytical method of sway-rocking soil spring model in which the shear deformation by rocking motion is evaluated by the averaged soil stiffness and sway and rocking motions are represented by the Novak's stiffnesses is presented. The Novak's stiffness functions in the proposed method that depend on frequencies are utilized. The approximation and applicability of the proposed method are discussed by comparing with the results from another sway-rocking soil spring model where the above three modes represented by the Novak's stiffness functions and the 3-dimensional analysis, the original 3-dimensional thin layer approach [Tajimi *et al*. 1976, 1977].

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ASSUMPTION OF ANALYSIS AND DERIVATION OF AVERAGED SOIL STIFFNESS

Assumed that the embedded structure that has a circular or an equivalent circular cross section is rigid body and sustained by a half-space medium that is evaluated as the dynamic soil stiffness functions of a rigid circular foundation resting on the free surface. The dynamic soil stiffness functions are given as follows,

$$k_H = \frac{8GR}{2-\nu}\left(1 + i0.6\frac{\omega R}{V_s}\right) \tag{1}$$

$$k_R = \frac{8GR^3}{3(1-\nu)}\left(1 + i0.3\left[\frac{\omega R}{V_s} - 0.56\right]\right) \tag{2}$$

ω is the circular frequency, R is the radius of the cross section and V_s is the shear velocity of the half-space. When $\omega R / V_s \le 0.56$, the imaginary part of Eq. (2) assumes to be zero. The side surface soil of the embedded structure is evaluated as the homogeneous or inhomogeneous soil stiffnesses.

We consider that the soil model which consists of a single thin layer of thickness H is supported by the dashpot mat. Assuming the massless rigid body of height H and radius R is embedded in this model, the relationship between the discrete subgrade reaction of the soil and the displacement on 1 and 2 nodal surfaces in Fig.3 is given as follows. Here, it is assumed the single thin layer is homogeneous medium.

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \frac{\pi GH}{6}\left(\begin{matrix} F(\alpha_1,\beta_1,R)\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} + g(C)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + h(C)\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}\right) \\ + F(\alpha_2,\beta_2,R)\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - g(C)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - h(C)\begin{bmatrix} 2 & 1 \\ 2 & 5 \end{bmatrix}\right) \end{matrix} \right) \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{3}$$

where $C = s\omega R / V_s$, $s = V_{sB} / V_s$, $\varsigma = V_s / V_p$, V_{sB}, V_{so} = shear wave velocity of dashpot and a single thin layer, V_p = compressive wave velocity.

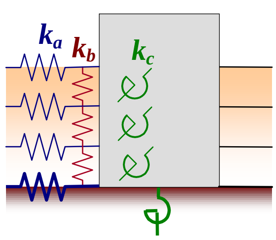


Figure 1: Sway-roking model with k_a, k_b and k_c for embedded structure

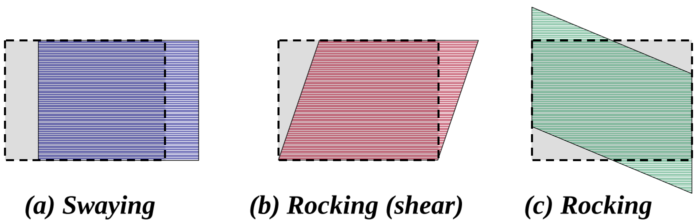


Figure 2: Deformation of rigid body

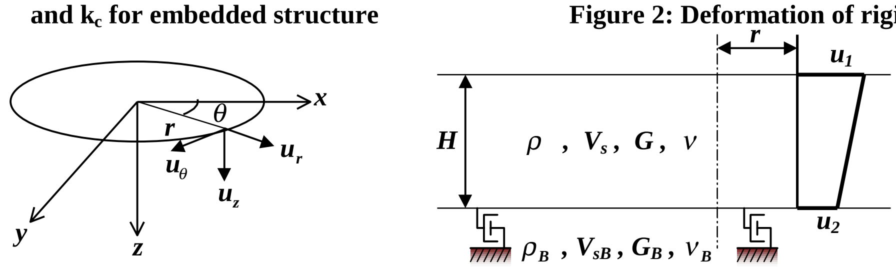


Figure 3: Analysis model and coordinate system

Furthermore,

$f(z)=[(1+iz/3)^2-iz/3]^{1/2}$

$f_1(z)=iz/3-f(z)$, $\quad f_2(z)=iz/3+f(z)$, $\quad g(z)=1/f(z)$ $\quad h(z)=iz/3f(z)$,

$$\beta_1=\frac{1}{H}[(\frac{\omega H}{V_s})^2-6(1+f_1(C))]^{1/2},\ \alpha_1=\varsigma\beta_1,\ \beta_2=\frac{1}{H}[(\frac{\omega H}{V_s})^2-6(1+f_2(C))]^{1/2},\ \alpha_2=\varsigma\beta_2 \tag{4}$$

$$F(\alpha,\beta,r)=\beta r\frac{H_2^{(2)}(\alpha r)H_1^{(2)}(\beta r)+H_2^{(2)}(\beta r)H_1^{(2)}(\alpha r)/\varsigma}{H_2^{(2)}(\beta r)H_0^{(2)}(\alpha r)+H_2^{(2)}(\alpha r)H_0^{(2)}(\beta r)}$$

where $\mathrm{Im}.(\beta_1)<0, \mathrm{Im}.(\beta_2)<0$ and $H_\nu^{(2)}(*)$ =Hankel function of the 2nd kind of the ν -th order.

Novak Soil Spring

In the case when s converges at 0, functions of $g(C)$ and $h(C)$ would converge 1 and 0, respectively. So, Eq. (3) yields to the following relationship the nodal force and displacement vectors when the rigid body is subjected to a unit horizontal translation $Ue^{i\omega t}$

$$\begin{bmatrix}P_1\\P_2\end{bmatrix}=\frac{\pi GH}{2}F(\varsigma\frac{\omega}{V_s},\frac{\omega}{V_s},R)\begin{bmatrix}1&1\\1&1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}U \tag{5}$$

The Novak's soil spring is derived from the above expression as follows.

$$k=\frac{P_1+P_2}{UH}=\frac{(K_{11}+K_{12}+K_{21}+K_{22})}{H}=2\pi GHF(\varsigma\frac{\omega}{V_s},\frac{\omega}{V_s},R) \tag{6}$$

HORIZONTAL SUBGRADE REACTION DUE TO ROCKING MODE

The nodal surface force vector caused by rocking mode of rotational angle $\Theta e^{i\omega t}$ can be given from Eq. (5) as follows.

$$M=\begin{Bmatrix}H\\0\end{Bmatrix}^T\begin{bmatrix}P_1\\P_2\end{bmatrix}\Theta=\begin{Bmatrix}H\\0\end{Bmatrix}^T\begin{bmatrix}K_{11}&K_{12}\\K_{21}&K_{22}\end{bmatrix}\begin{bmatrix}H\\0\end{bmatrix}\Theta \tag{7}$$

Therefore, the moment about the bottom rotation axis by the horizontal resistance reaction caused by rocking mode of rotational angle Θ can be given by the next expression.

$$M=\int_0^H k\Theta z^2dz=\frac{k\Theta H^3}{3}=K_{11}H^2\Theta \tag{8}$$

Here, an approximation of the averaged horizontal soil spring that expresses the horizontal resistance of the side surface soil to the rocking motion is defined by the next expression.

$$\bar{k}=3K_{11}/H \tag{9}$$

Figure 4 depicts the averaged horizontal soil stiffness functions per unit depth with the impedance ratio $s=V_{sB}/V_{so}$, where V_{sB},V_{so} : the shear velocity of the supported and the side surface medium. The value of Poisson's ratio, ν is equal to 0.4, the impedance ratio, s is taken equal to 1,2,4 and the aspect ratio R/H is equal to 1. The longitudinal and lateral axes of Fig.4 are represented by the non-dimensional stiffness $\bar{k}/G$ and the non-dimensional circular frequency ω/ω_g (where ω_g is the natural circular frequency of the side surface soil), respectively. Fig.4 also shows the averaged horizontal soil stiffness per unit depth from Novak's one.

Approximate Formula of Static Value of Averaged Horizontal Soil Stiffness

An approximate formula of the static value of the averaged horizontal soil stiffness is presented. The static value of the averaged horizontal soil stiffness function is assumed to be the maximum value of the real part of the stiffness function. The approximate formula is expressed as Eq. (10)

$$\bar{k}_0 = 4G(1+3R/H)s^{1/8} \tag{10}$$

In case of homogeneous side surface soil, the impedance ratio $s = V_{sB}/V_{so}$ and the aspect ratio of the embedded structure R/H are utilized as parameters in Eq. (10). In Eq. (10) G means the shear modulus of the side surface medium. Fig. 5 shows the static values of the averaged horizontal soil stiffness functions from Eqs. (9) and (10) with the aspect ratio of the embedded structure R/H as parameter. The longitudinal and lateral axes of Fig.5 are represented by the non-dimensional stiffness $\bar{k}_0/4G$ and the impedance ratio $s = V_{sB}/V_{so}$, respectively. In the range of the parameters, Eq. (10) gives good results to approximate the curves from Eq. (9).

An approximate formula of the static value of the averaged horizontal soil stiffness for considering the inhomogeneity of the side surface soil versus $\Delta R/R_i$ is given the following equation with $s = V_{sB}/V_{so}$, R/H and $\alpha = V_{si}/V_{so}$ as parameters.

$$\bar{k} = \bar{k}_0(1+\Delta R/R_i[\alpha^{3/2}-1]) \tag{11}$$

where $\Delta R = R_o - R_i$, R_o:distance from the center of the embedded structure to the outer boundary of the soil neighboring the structure, R_i: the radius of the structure, V_{si}, V_{so}: shear velocity in zone I and O. Fig. 6 depicts the static values of the averaged horizontal soil stiffness functions for a case of inhomogeneous side surface soil with G_i/G_o as parameter when R/H is equal to 1, ν is taken equal to 0.4, s is equal to 2. The lateral axis means $\Delta R/R_i$, while the longitudinal one represents the non-dimensional stiffness $\bar{k}/4G_o$. The approximate formula would give appropriate static values for $\Delta R/R_i < 0.5$ for engineering purposes.

Approximate Formula for Frequency-Dependent Function of Averaged Soil Stiffness

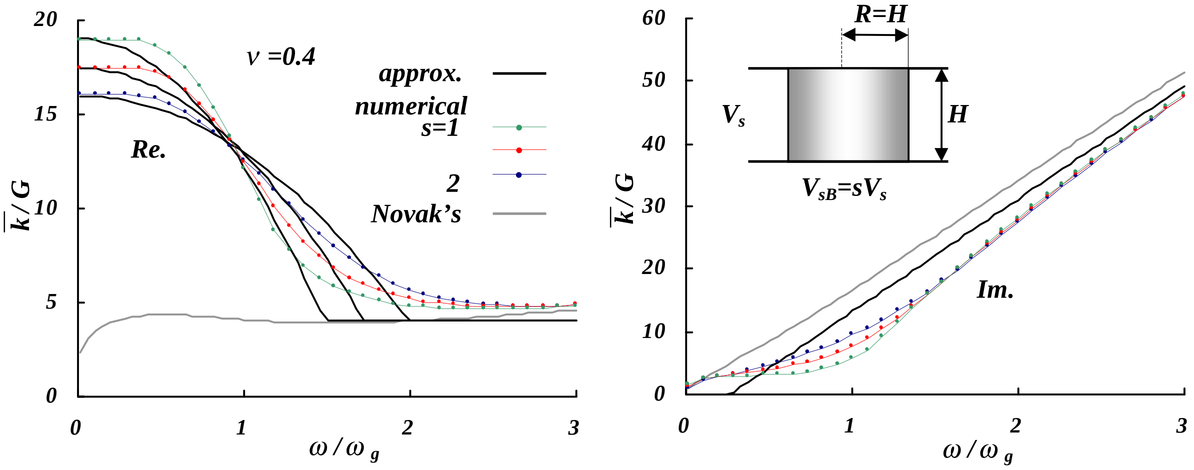
Referring to the curves from Eq. (9) in Fig. 4, an approximate formula of the averaged horizontal soil stiffness functions depending on frequencies is estimated by using the method of least squares for a case of $R/H = 1$ and $\nu = 0.4$. The approximate formula is shown Eq. (12) with $s = V_{sB}/V_{so}$ as a parameter.

$$\bar{k}_s = 4G[\beta\{1-\eta(\omega/\omega_g)^2\} + i\{5(\omega/\omega_g)-3\}] \tag{12}$$

which for $\mathrm{Re}.(\bar{k}_s) < 4G_o$ gives $\mathrm{Re}.(\bar{k}_s) = 4G_o$ and $\mathrm{Im}.(\bar{k}_s) < 0$ gives $\mathrm{Im}.(\bar{k}_s) = 0$, and β means $(1+3R/H)s^{1/8}$. Fig. 4 also depicts the averaged horizontal soil stiffness functions from Eqs. (9) and (12) with $s = V_{sB}/V_{so}$ as a parameter. The averaged horizontal soil stiffness functions from Eq. (9) have an inflection point, while those from the approximate formula are inadequate to describe such a point. In the range of ω/ω_g, approximations of the proposed formula are acceptable for practical purposes.

SEISMIC RESPONSE ANALYSIS OF AN EMBEDDED STRUCTURE

We propose a simplified analytical method of sway-rocking soil spring model considering the oscillation of the side surface medium, in which the shear deformation by rocking motion is evaluated by the averaged soil stiffness and sway and rocking motions are represented by the Novak's stiffnesses. Confirming the applicability of the proposed method, comparisons among the results from the proposed method, another sway-rocking soil spring model utilizing the Novak's stiffnesses for all modes and the 3-dimensional analysis, the original 3-dimensional thin layer approach, are carried out. Fig.7 shows an analysis model that is a rigid body with circular cross section embedded in a uniform or stratified medium and subjected to the incident shear wave. The values of aspect ratio of the embedded structure and Poisson's ratio of soil are taken equal to 1 and 0.4, respectively. The complete expressions of equations of motion of rigid body are given by



(a) Real part of stiffness functions **(b) Imaginary part of stiffness function**

Figure 4: Averaged horizontal stiffness in comparing with numerical results vs approximate ones (Poisson's ration ν=0.4, aspect ratio R/H=1)

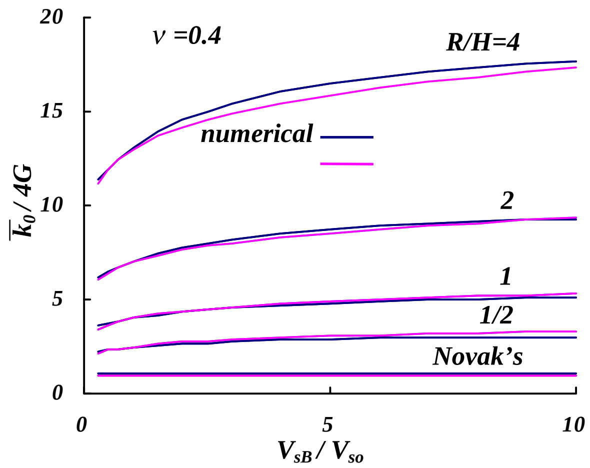


Figure 5: Static averaged horizontal stiffnesses by proposed method and resulting formula for homogeneous surface soil

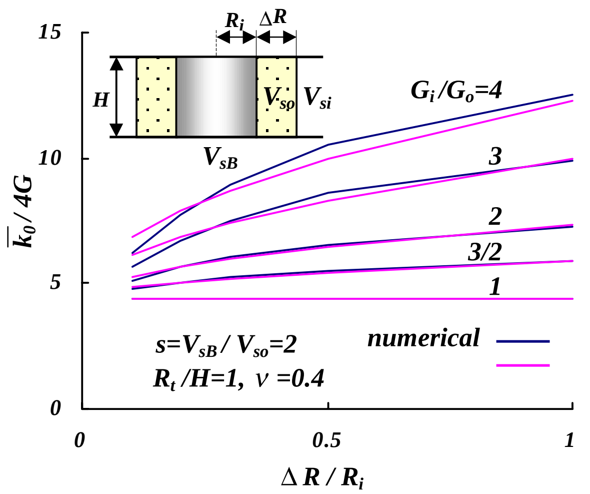


Figure 6: Static averaged horizontal stiffnesses by proposed method and resulting formula for inhomogeneous surface soil

$$\begin{bmatrix} m_b & m_b H_G \\ m_b H_G & m_b H_G^2 + I_b \end{bmatrix} \begin{bmatrix} \ddot{u}_f \\ \ddot{\phi}_f \end{bmatrix} + \begin{bmatrix} k_{HH} & k_{HR} \\ k_{RH} & k_{RR} \end{bmatrix} \begin{bmatrix} u_f \\ \phi_f \end{bmatrix} = \begin{bmatrix} \{1\}^T \{p\} \\ \{\hat{H}\}^T \{p\} \end{bmatrix} = \begin{bmatrix} Q \\ M \end{bmatrix} \tag{13}$$

where, m_b: mass of the rigid body, H_G: height from the bottom to the center of gravity of the rigid body, I_G: moment of inertia referred to the principal axis through the center, $\{\hat{H}\}$:height vector from the bottom of the rigid body to each nodal surface, u_f :absolute horizontal displacement of the rigid body, ϕ_f :rotational displacement of the rigid body, $\{p\}$:the driving force vector. $k_{hh}, k_{HR} (= k_{RH})$ and k_{RR} are sway, coupling of sway and rocking, and rocking impedance functions, respectively.

$$k_{HH} = k_H + \{1\}^T [k_a]\{1\}$$

$$\begin{aligned} k_{RR} &= k_R + \{\hat{H}\}^T [k_s]\{\hat{H}\} + \{1\}^T [k_c]\{1\} \\ k_{HR} &= k_{RH} = \{\hat{H}\}^T [k_a]\{1\} \\ \{p\} &= [k_a']\{U^*(i\omega)\} \end{aligned} \tag{14}$$

$$[k_a] = diag.(k_{a1}, k_{a2}, \cdots, k_{an-1}, k_{an}),\ [k_s] = diag.(k_{s1}, k_{s2}, \cdots, k_{sn-1}, k_{sn}) \tag{15}$$

$$[k_c] = diag.(k_{c1}, k_{c2}, \cdots, k_{cn-1}, k_{cn}),\ [k'_a] = diag.(k_{a1}, k_{a2}, \cdots, k_{an-1}, k_{an} + k_H)$$

where $\{U^*(i\omega)\}$:absolute displacement vector at the free field. k_H, k_R are the dynamic soil stiffness functions of a rigid circular foundation resting on the free surface defined by Eqs. (1) and (2). The Novak's laterally axial $\bar{k}_a$ and rotational $\bar{k}_c$ stiffnesses per unit depth that depend on frequencies are defined as follows,

$$\bar{k}_a = 4G[1 - (\frac{\omega R}{V_s})^2 f_1(\nu) + i(\frac{\omega R}{V_s}) f_2(\nu)] \tag{16}$$

$$\bar{k}_c = \pi G R^2 [1 - 0.4(\frac{\omega R}{V_s})^2 + i(\frac{\omega R}{V_s} - 0.22)] \tag{17}$$

which for $\mathrm{Im}(\bar{k}_c) < 0.0$ gives $\mathrm{Im}.(\bar{k}_c) = 0.0$ and for $\omega R / V_s > 1.0$ gives $\mathrm{Re}.(\bar{k}_c) = 0.6\pi G R^2$. k_a, k_s, k_c which are expressed as discrete stiffnesses are shown as follows,

$$k_{aj} = \bar{k}_a (H_{j-1} + H_j)/2,\ k_{sj} = \bar{k}_s (H_{j-1} + H_j)/2,\ k_{cj} = \bar{k}_c (H_{j-1} + H_j)/2 \tag{18}$$

NUMERICAL RESULTS AND DISCUSSIONS

In the following analyses, uniform and two layered soil models are considered. It is assumed that the embedded depth of the rigid body (E) is 10m, shear velocity of the soil (V_{so}) is 500m/s, mass density of the soil (ρ) is 1.8t/m^3, Poisson's ratio of the soil (ν) is 0.4 and hysteretic damping (h) is 0.05. In a case of two layered soil model, it is assumed that the velocity of the surface layer (V_{so}) is 250m/s and that of the supported medium (V_{sB}) is 500m/s. The height (H_F), the height of the center of gravity (H_G), the radius (R), and mass density (ρ_F) of rigid body are 20m.,10m,10m, 2.4t/m^3, respectively.

Uniform Soil Model

Fig.8 (a) and (b) show the sway and rocking impedance functions at bottom of the rigid body obtained by the proposed method and the 3-dimensional thin layer approach. The longitudinal axes are identical to the non-dimensional stiffnesses $k_{HH}/[8GR/(2-\nu)], k_{RR}/[8GR^3/3(1-\nu)]$, while the lateral axis corresponds to frequencies (Hz). The rocking impedance functions computed by using the Novak's stiffness are also shown in Fig. 8(b). It can be seen that the real part of the sway impedance function from the proposed method is underestimated, on the other hands, the imaginary part is overestimated by comparing with that computed with the 3-dimensional analysis, while trends of frequency dependency of the impedance functions obtained by the proposed method are similar to the exact solution. By comparing with the rocking impedance functions computed from the proposed method and another method using the Novak's stiffness, the imaginary part of the Novak's stiffness is closer than that of the averaged horizontal soil stiffness, while the averaged horizontal soil stiffness compensates the difference of the real part between the 3-dimensional analysis and the Novak's stiffness. Fig. 8(c) depicts amplification factors from the free surface at the free field to the center of gravity of the rigid body. Agreement between the 3-dimensional results and the proposed ones is fairly good.

Two Layered Soil Model

Fig. 9(a) and (b) shows the sway and rocking impedance functions computed with the proposed method and the 3-dimensional thin layer approach. Fig. 9(b) includes the rocking impedance functions computed by using the Novak's stiffness. Comparing with the results in Fig. 8(a) and (b), it is indicated that not only imaginary but also real parts of the impedance functions estimated by the proposed method agree better with the 3-dimensional results than those by the Novak's stiffness. Amplification factors from the free surface at the free field to the top of the rigid body are depicted in Fig. 9(c). It can be seen that the proposed method gives more conservative results than those by the Novak's stiffness.

CONCLUSIONS

Approximate formulae of the static value for cases of homogeneous and inhomogeneous soil mediums and the frequency-dependent function for a case of homogeneous soil of the averaged horizontal soil stiffness that can take dynamic behavior of the surface soil surrounding the embedded structure into account are proposed. As a result, those proposed formulae are acceptable for engineering purpose. A simplified analytical method of sway-rocking soil spring model in which the shear deformation by rocking motion is evaluated by the averaged soil stiffness and sway and rocking motions are represented by the Novak's stiffnesses is also proposed. The approximation and applicability of the proposed method are examined by comparing with the results from the original 3-dimensional thin layer approach. Agreement of not only sway and rocking impedance functions but also amplification factors of the rigid body obtained from the proposed method and the 3-dimensional approach is fairly good. It is indicated that the approximate formulae and the simplified method of sway-rocking model proposed in this paper can be utilized in preliminary analysis to evaluate the dynamic behavior of the side surface soil and embedded structure.

REFERENCES

1. Harada, T., Kubo, K. and Katayama, T. (1981), "Dynamic soil-structure interaction analysis by continuum formulation method", *Report of the Institute of Industrial Science, The University of Tokyo*, Vol.29, No.5, pp1-56.
2. Ikeda, Y., Tajimi, H. and Shimomura, Y. (1992), "A simplified method of obtaining interaction stiffnesses associated with the embedment of structures", *Proc. 10th WCEE*, pp.1525-1530.
3. Novak, M., Nogami, T. and Aboul-Ella, F. (1978), "Dynamic soil reactions for plane strain case", *Engrg Mech. Div., ASCE*, Vol.104, No.EM4, pp953-959.
4. Novak, M. and Shete, M. (1980), "Approximate Approach to Contact Effects of Piles," *Special Technical Publication on Dynamic Response of Pile Foundation* ; Analytical Aspects, ASCE.
5. Shimomura, Y. and Ikeda, Y. (1996), "A Simplified Seismic Response Analysis of An Embedded Cylindrical Structure", *Proc. 11th WCEE*, Ref. No.85.
6. Tajimi, H. and Shimomura, Y. (1976), "Dynamic analysis of soil-structure interaction by the thin layered element method", *Trans. of AIJ*, Vol.243, pp41-51 (in Japanese).
7. Tajimi, H., Minowa, C. and Shimomura, Y. (1977), "Dynamic response of a large-scale shaking table foundation and its surrounding ground", *Proc. 7WCEE*, Vol.4, pp61-66.
8. Veletsos, A. S. and Dotson, K.W. (1988), "Horizontal Impedance for Radically Inhomogeneous Visco-elastic Layers," *EESD*, Vol.16, pp947-966.

Table 1: η in Eq. (12) depending on $s = V_{sB} / V_{so}$

$s=V_{sB}/1V$	η
1	0.19
2	0.26
4	0.36

Table 2: $f_1(\nu)$ and $f_2(\nu)$ in Eq.(15) depending on ν

ν	$f_1(\nu)$	$f_2(\nu)$
0.25	0.00	2.20
0.33	0.00	2.40
0.40	0.02	2.70
0.45	0.11	3.00

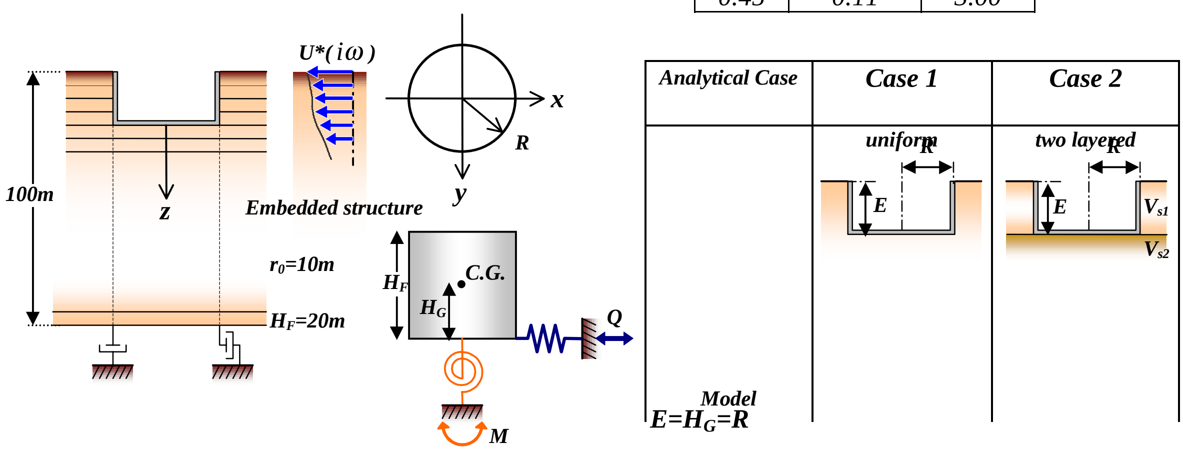
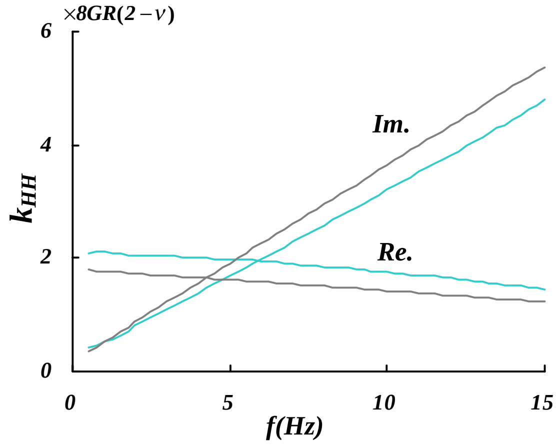
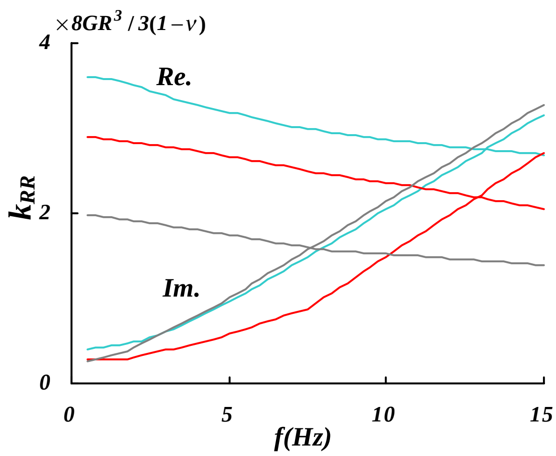


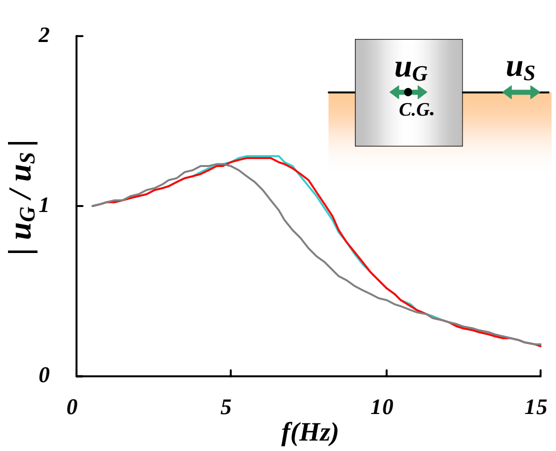
Figure 7: Mathematical model and properties of embedded structure and soil



(a) Sway impedance functions

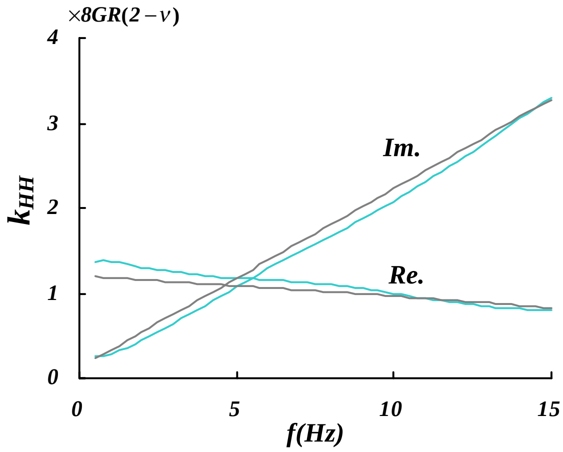


(b) Rocking impedance functions

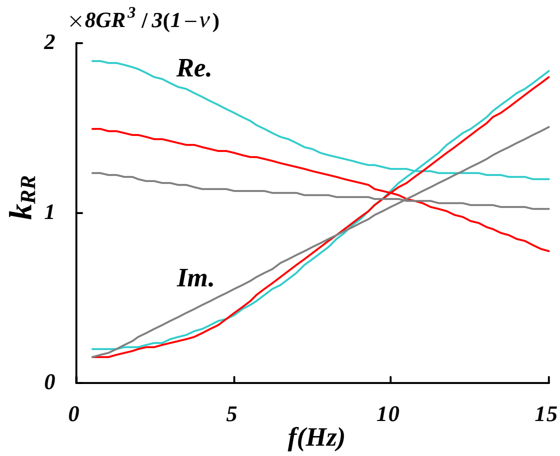


(c) Frequency response amplification from free surface at free field to mass center of rigid body

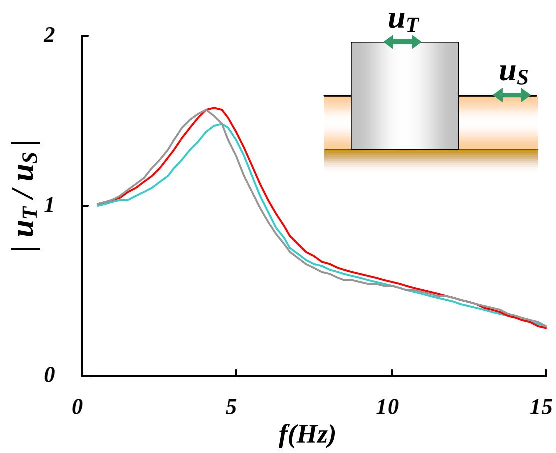
Figure 8: Numarical results for uniform soil



(a) Sway impedance functions



(b) Rocking impedance functions



(c) Frequency response amplification from free surface at free field to top of rigid body

Figure 9: Numarical results for two layered soil