A RATIONAL MODEL FOR REINFORCED CONCRETE MEMBRANE ELEMENTS SUBJECTED TO SEISMIC SHEAR

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SUMMARY

A model to predict the load-deformation response of reinforced concrete membrane elements subjected to seismic shear is presented. The model combines equilibrium, strain compatibility and simple constitutive relationships into a general, rational formulation. Rather than treating cracked concrete as a single homogeneous material, the model treats the concrete between the cracks separately from the cracks themselves. This approach allows the model to capture the important characteristics that define the complex reverse-cyclic response, such as the orientations of principal stresses and strains, and the concrete stress-strain response. To properly simulate the reverse-cyclic response, the model allows the orientation of principal stresses to vary significantly from the orientation of principal strains.

The proposed model assumes that the cracks are evenly distributed and at fixed orientations. The deformations at the cracks are represented by a set of crack strains which are defined by compatibility with the reinforcement strains and a constitutive relationship for the closing of the cracks. The concrete between the cracks is modelled by a simple, unsoftened parabolic stress-strain function. Unlike existing monotonic models, the proposed model does not require an empirical compression softening function to account for the large strains that occur when the cracks widen. The model actually provides a rational explanation of the softening phenomenon: the crack strains have a compression component which becomes very significant once the weaker reinforcement yields. The occurrence of concrete compressive stresses in the direction of tensile strains is also explained by the explicit modelling of the crack deformations. Unlike other models that have been proposed for reinforced concrete subjected to reverse-cyclic shear, no complex empirical functions are used to mimic the response. The proposed model shows excellent agreement with experimental results.

INTRODUCTION

A well known objective of the seismic design of reinforced concrete structures is to avoid brittle shear failures. Of particular concern is the significant part of the infrastructure which was built before our understanding of seismic shear was sufficiently advanced to achieve this objective. This was made apparent by recent earthquakes, such as the Loma Prieta and Northridge earthquakes in California and the Kobe earthquake in Japan, where serious deficiencies in even relatively new structures were brought to light. With public safety at stake and limited funds available for retrofitting, the ability to properly assess the capacity of existing structures and prioritize the repair work is very important.

A typical approach to the design of new structures is to use a conservative strength model to design the transverse shear reinforcement and to limit the shear stress. This is reasonable for structures where the response is primarily flexural and thus capacity design principles can be applied, however, there are many situations where this is not appropriate. For shear dominated structures with elements such as short columns, squat walls or coupling beams, the maximum shear demand can not be limited by simply controlling the flexural strength. In

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these cases, a simple conservative shear strength model is not sufficient. The shear stiffness of individual elements can also significantly influence the response of structures that are thought to be flexure dominated. For example, analysing a tall building while ignoring shear deformations (i.e., assuming infinite shear stiffness) will often indicate very large bending moment reversals and correspondingly large shear forces in the basement walls, due to the rigid interconnection with floor slabs. However, including a realistic shear stiffness in the analysis often eliminates the bending moment reversals and significantly reduces the shear forces, resulting in an entirely different design condition, or in a different predicted failure mode. It is clear, also, that if sophisticated analysis methods such as dynamic time-history or static push-over are to be reasonably accurate, they must include a good model for shear stiffness and its degradation for a variety of load cases.

Fig. 1(a) illustrates the wide variation in shear stiffness which can occur in a reinforced concrete element subjected to reverse-cyclic shear. The uncracked shear stiffness of this membrane element was about G = 12,000 MPa. After diagonal cracking, the stiffness reduced to about G = 1200 MPa, or one-tenth of the uncracked stiffness. After six cycles of loading where yielding occurred in the y-direction reinforcement, the effective stiffness was further reduced to about G = 600 MPa, one-twentieth of the uncracked stiffness. Not surprisingly, designers are faced with questions such as what shear stiffness value is appropriate, to what extent will diagonal cracking reduce the shear stiffness, and does yielding of the transverse reinforcement constitute failure or simply a further softening of the element.

Cyclic shear models currently available are either simple envelope-type models which relate shear strength to ductility demand, or they consist of a set of empirical or semi-empirical rules to model the shear hysteresis loops, sometimes tied to a rational monotonic model used to represent the envelope of the cyclic response. These models, developed mostly for bridge columns, are not sufficiently general, or rational, to apply to a wide range of elements, or to a variety of loading conditions.

A long-term research project is being conducted at The University of British Columbia to develop rational methods for the design and assessment of reinforced concrete subjected to reverse-cyclic shear. Emphasis has been placed on developing practical tools which can be implemented by designers. The first phase of this work [Adebar et al., 1995] involved the testing of numerous beam and column elements to study the interaction of axial load, flexural ductility and degradation of seismic shear response. Another phase of the work consisted of analysing data from membrane elements subjected to cyclic shear in order to develop an understanding of the fundamental mechanisms involved in the seismic shear response of reinforced concrete [Gérin and Adebar, 1999]. Recently, a simple rational model was developed for the response of membrane elements subjected to seismic shear.

EXPERIMENTAL STUDY

The results from membrane element tests conducted at the University of Toronto were studied to identify the fundamental characteristics of the response of reinforced concrete to seismic shear, and to clarify the differences between cyclic and monotonic shear responses. The results from a membrane element subjected to reverse-cyclic shear, specimen SE8 [Stevens et al., 1991], and from a membrane element subjected to monotonic shear, specimen PP1 [Meyboom, 1987], are used to illustrate some of the key findings of this study.

The elements were 1600 mm square by 285 mm thick, with a 3:1 ratio of x-direction to y-direction reinforcement. Specimen SE8 had 3% reinforcement in the x direction and 1% in the y direction and a concrete cylinder strength of 37 MPa. Specimen PP1 had 1.8% reinforcement in the x direction and 0.66% in the y direction and a concrete cylinder strength of 27 MPa. The specimens were instrumented with displacement transducers at various orientations to measure average strains over a 1200 mm square central area. Fig. 1(a) shows the shear stress versus shear strain response of specimen SE8. The first complete load cycle caused cracking in two directions. During the next four cycles of loading, to an applied shear stress of +/− 4.5 MPa, the response was essentially linear elastic. The element was then cycled between +/− 5.75 MPa, causing yielding in the y-direction reinforcement at each peak. The element failed by crushing of the concrete at the seventh yielding load cycle.

The details of the cyclic shear response are examined more closely by comparing the results from one cycle of reverse-cyclic loading (Fig. 2) with the results from the monotonic test. The orientation of the principal average strains (the strain angle) and of the principal average stresses (the stress angle) are examined in Figures 3 and 4; the angles are measured from the x-direction to the minimum principal stress or strain value. In the monotonic element (Fig. 3), the stress angle is initially at 45 degrees since the element is subjected to pure shear. After diagonal cracks form, the stress angle drops to about 40 degrees, at which point the stress and strain angles are approximately equal. As loading increases, the stress angle remains approximately constant until the *y*-direction reinforcement yields at a shear strain level of about 4.5 millistrain. At this point, the difference between the stress angle and the strain angle, which has been slowly decreasing, is about 5 degrees. After the reinforcement yields, both angles reduce with the difference between the two remaining approximately constant.

Fig. 4(a) shows the stress and strain angles for specimen SE8 during a single cycle of loading. At large values of shear stress and shear strain, the variation between the stress and strain angles is very similar to the monotonic case (i.e., about 6 degrees). However, at low levels of shear stress and strain, the angles differ by up to 30 degrees. This large deviation between the principal angles is an important characteristic of the cyclic response of reinforced concrete.

The relationship between the principal concrete compression stress and the minimum principal strain (i.e., the standard concrete compression stress-strain relationship) is examined in Fig. 5 for both the response of PP1 to monotonic loading and the response of SE8 during one cycle of reverse-cyclic loading. In the monotonic case, the relationship begins at the origin and is approximately parabolic; a typical response. In the reverse-cyclic case, there are residual strains and stresses (i.e., the relationship is offset from the origin) attributed to the previous cracks not closing completely. When the loading is increased from the point of zero shear stress, the minimum principal strain becomes tensile even though the minimum concrete stress is increasingly compressive. The resulting bulge in the curve is a significant deviation from the typical stress-strain relationship of concrete. As loading further increases, the relationship returns to the more typical parabolic shape.

ANALYTICAL MODEL

Following the study of experimental data [Gérin and Adebar, 1999], a model for membrane elements subjected to seismic shear was developed. The following description of the model focusses on two parameters: the principal angles and the concrete stress-strain response. Further details of the proposed model are presented elsewhere [Adebar and Gérin, in press].

Vecchio and Collins [1986] presented the Modified Compression Field Theory (MCFT) which uses fundamental strain compatibility assumptions, equilibrium and stress-strain relationships to predict the complete load-deformation response of membrane elements subjected to monotonic shear. The theory treats cracked concrete as a single homogenous material, and thus is formulated in terms of average stresses and average strains. It uses stress-strain relationships for cracked concrete that include compression strain softening and tension-stiffening.

In the reverse-cyclic shear case, the presence of cracks in two directions, and the opening and closing of these cracks during load cycles, results in a much more complex behaviour that cannot be modelled using the concept of a homogenous cracked-concrete material. In the model presented here, the cracks in two opposite directions and the concrete between the cracks are modelled as separate components. The approach used to model the cracks is relatively simple: the cracks are assumed to be uniformly distributed and spaced close enough that deformations at the cracks can be described in terms of crack strains. This smeared crack approach is reasonable for elements with sufficient, well distributed reinforcement. For each of the two directions of loading, the orientation of the cracks is assumed fixed in the direction normal to the orientation of the principal concrete tensile stresses prior to cracking.

A fundamental strain compatibility assumption of the proposed model is that, in the *x* and *y* directions, the sum of the concrete strain and the strain due to the crack deformations equals the average strain of the reinforcement. This assumption is analogous to the strain compatibility assumption in the MCFT in which the average strain of the cracked concrete is assumed to be equal to the average strain of the reinforcement. When there are diagonal cracks in only one fixed direction, as in the monotonic case and in the cyclic case when the cracks in one of the directions have closed, it is assumed that the normal component of the crack strains that is parallel to the cracks is zero. This assumption coupled with the compatibility requirement in the two reinforcement directions defines the complete Mohr's circle of strain representing the deformations of the cracks (Fig. 7). As shown by the Mohr's circle, when the crack deformations include a shear component in the fixed crack direction (crack slip) there is a principal compression strain associated with the crack deformations. This compressive strain plays a very important role in the direction of principal average strains and in the softening of concrete; these issues are discussed further below. The shear deformation of the cracks results mainly from unequal quantities of reinforcement in the *x* and *y* directions. When the weak direction reinforcement yields, the shear deformation increases dramatically, significantly increasing the principal compressive strain.

When there are two sets of diagonal cracks open at the same time, which occurs during the transition from one direction of loading to the other, many different combinations of crack deformations will satisfy the strain compatibility requirements. Thus a constitutive relationship for the closing cracks is used to determine the crack deformations. This relationship, which is compared with experimental data in Fig. 6, relates the normal stress across the crack with the normal strain (crack width). Also shown in Fig. 6 is a similar relationship proposed by Matsuzaki et al. [1989].

Since the MCFT treats cracked concrete as a homogenous material, it assumes that the orientation of the principal stresses coincides with the orientation of the principal strains. As shown in Fig. 3, this assumption is reasonable for monotonic loading, and yields good results, but is not suitable for reverse-cyclic loading. In the proposed model, the concrete between the cracks is treated as the homogenous material and therefore the directions of the principal concrete stresses and principal concrete strains coincide. No assumption is made about the principal direction of the crack strains, but rather it is inherently defined by the strain calculation method described above. Generally, the direction of the principal crack strains does not coincide with the direction of the principal concrete stresses. Also, since the concrete compressive strains are much smaller than the crack strains, the latter have a greater influence on the direction of the total average strains. This approach yields a principal strain angle which agrees extremely well with the experimental results [Fig. 4(b)].

While a complex approach is used to relate strain directions to stress directions, a simple model is used for the principal concrete stress direction. Prior to any reinforcement yielding, the orientation of the principal concrete stress is assumed to be constant, which as shown in Fig. 3, is a reasonable assumption. After the weak direction reinforcement yields, the direction of the principal concrete stress and the direction of the principal average strain are assumed to rotate equally, i.e. the difference between the two principal angles is assumed to remain constant after yielding. The result of these two assumptions can be seen in Fig. 4(b).

In monotonic shear the concrete tensile stresses play an important role and must be accounted for. However, including concrete tensile stresses after cracking (i.e., tension-stiffening) results in an added complexity. In the case of reverse-cyclic shear, the concrete tensile stresses degrade after the first few post-cracking cycles of loading, and therefore it is not necessary to include a rigorous model for post-cracking tensile stresses. This means a simple model can be used to predict the direction of principal concrete stresses in the elastic range: the principal angle can be calculated from the relative stiffness of the reinforcement in each direction and of the concrete in compression.

Existing monotonic models that treat cracked concrete as a homogenous material include compression softening of the concrete to account for the large strains that occur when the cracks widen. Since the proposed model considers the cracks explicitly, an empirical compression softening model is not needed; in fact, the model presented here provides a rational explanation of the softening phenomenon. While there are some micro-cracks and transverse tensile strains that soften the concrete between the primary cracks, it is believed that this slight softening may be neglected.

The response of element PP1, which was subjected to monotonic shear, is shown in Fig. 8(a) in terms of the concrete principal compression stress versus the minimum principal average strain. The experimental response "flattens" when the weaker *y*-direction reinforcement yields at a principal compression stress of about 7 MPa. Also shown in the figure is the unsoftened, parabolic response of concrete as determined from a cylinder test: clearly it is very different from the response of diagonally cracked concrete in a membrane element. For this reason, Vecchio and Collins developed a stress-strain relationship for concrete where the increase in maximum principal average strain softens and weakens the concrete. As seen in the figure, the Vecchio and Collins model gives a much better prediction than the unsoftened response. In the proposed model, the unsoftened parabolic response is used for the concrete between the cracks. The predicted response in terms of minimum principal average strain shown in Fig. 8(a) is obtained by adding the strains due to the cracks to the unsoftened concrete strains. As describe previously, and seen in the figure, the crack strains have a compression component which becomes very significant once the weaker reinforcement yields. This rational explanation is consistent with some of the early descriptions of the softening phenomenon by Collins [1978].

The concrete response of specimen SE8, in terms of principal compressive stress versus minimum principal average strain, is shown in Fig. 8(b) for the loading segment of the single cycle. Unlike the monotonic response, there is a "bulge" in the cyclic response at low compression stress values. While others have used empirical functions to mimic this complex concrete response, the model proposed here automatically captures this effect and provides an explanation of it. At low concrete compressive stresses, there are small compressive strains from the concrete between the cracks, in accordance with the well known parabolic relationship between concrete stresses and strains. There are also additional strains from the diagonal cracks that are opening, but the largest strains are from the diagonal cracks in the opposite direction that have not yet closed and have a large tensile strain component. These cracks remained open at the end of the previous unloading phase to accommodate plastic strains in the weaker reinforcement that have accumulated during yielding. As loading increases, these cracks eventually close and the response becomes similar to the monotonic case. Note that the residual stresses in the concrete are not included in the model, and hence the prediction starts at zero stress while the experimental data does not. These residual stresses and strains do not effect the stiffness or the response at high stress levels and therefore, to keep the model simple, are not included.

As seen in Figure 8, the concrete response in the proposed model agrees well with the observed response, for both monotonic and cyclic tests. The complete predicted shear stress versus shear strain response for specimen SE8 is shown in Fig. 1(b), with the details of the single cycle appearing in Fig. 3. Both show excellent agreement with the experimental results.

CONCLUSION

A rational model for membrane elements subjected to seismic shear was presented. It combines equilibrium, strain compatibility and constitutive relationships to predict the complete load-deformation response of the elements. Rather than treating cracked concrete as a single homogeneous material, the model treats the concrete between the cracks separately from the cracks themselves. This allows the orientation of the concrete stresses to deviate from the orientation of the principal average strains. It also allows tensile average strains to occur with compressive concrete stresses, in the low stress range. A simple, unsoftened parabolic stress-strain relationship is sufficient to model the concrete between the cracks and still capture the compression softening which is observed in diagonally cracked concrete. The model actually provides a rational explanation for the softening effect. The proposed model shows good agreement with the experimental data for each of the constitutive parameters, such as the principal angles and the concrete stress-strain response. As a result, the shear stress versus shear strain hysteresis loops are well predicted, with pinching and yielding similar to the observed response.

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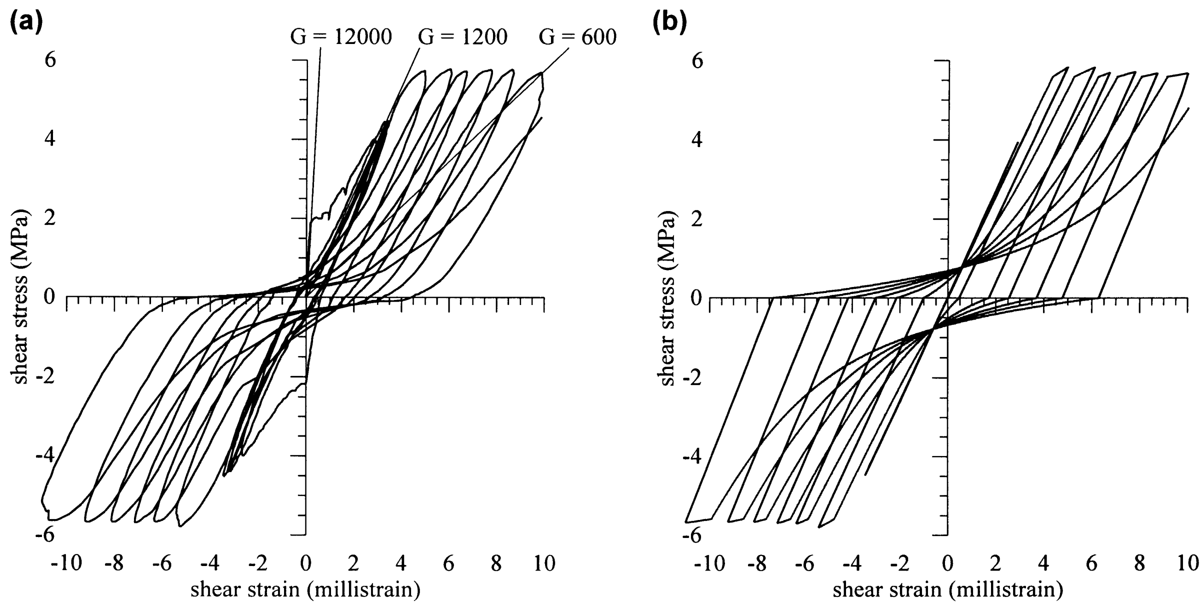


Fig. 1 - Shear response of specimen SE8: (a) experimental; (b) predicted

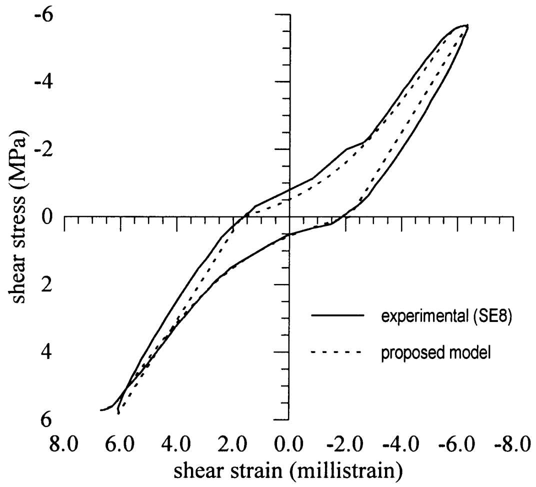


Fig. 2 Shear response, single cycle

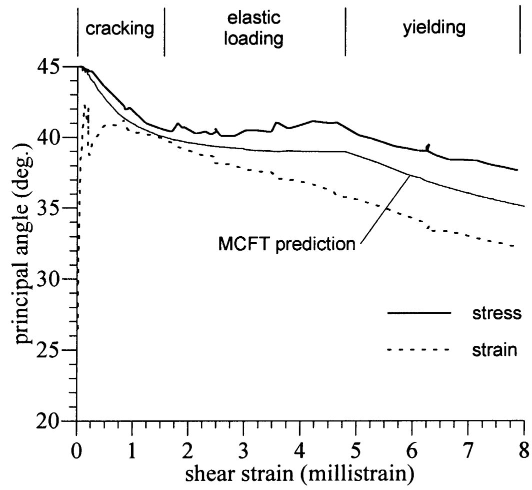


Fig. 3 - Principle angles, monotonic case (PP1)

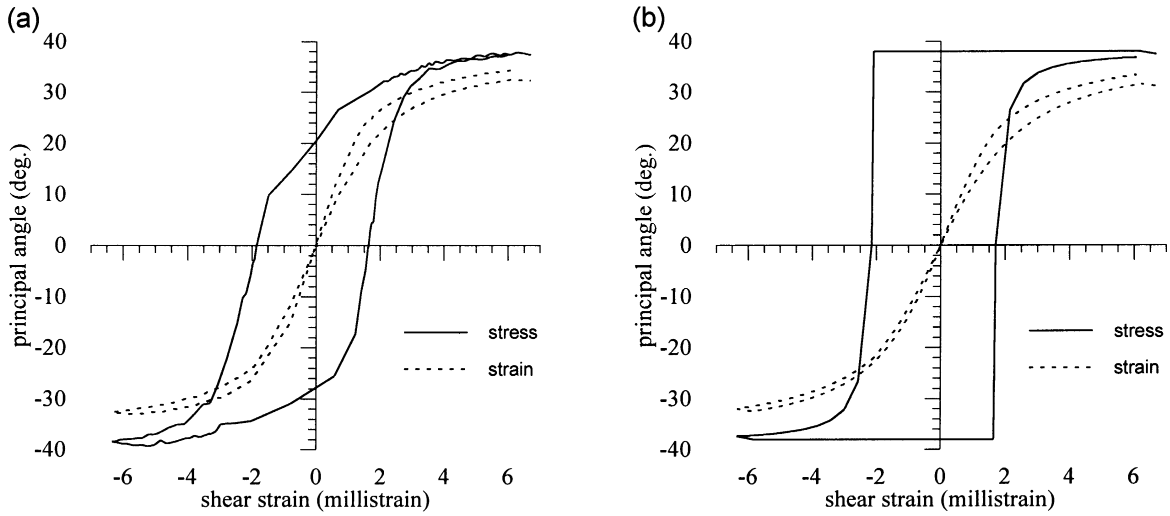


Fig. 4 - Principle angles, cyclic case: (a) experimental (SE8); (b) proposed model

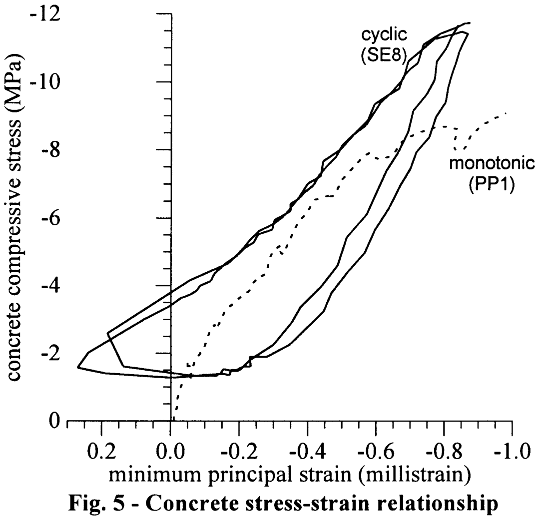


Fig. 5 - Concrete stress-strain relationship

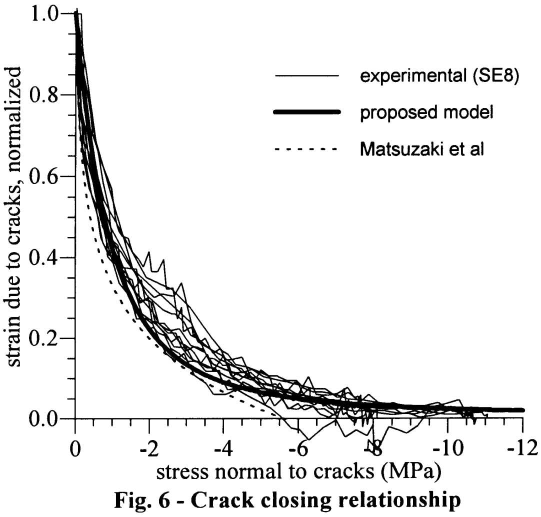


Fig. 6 - Crack closing relationship

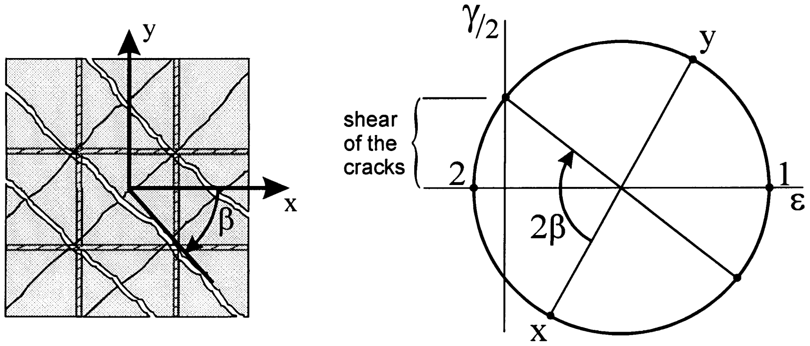
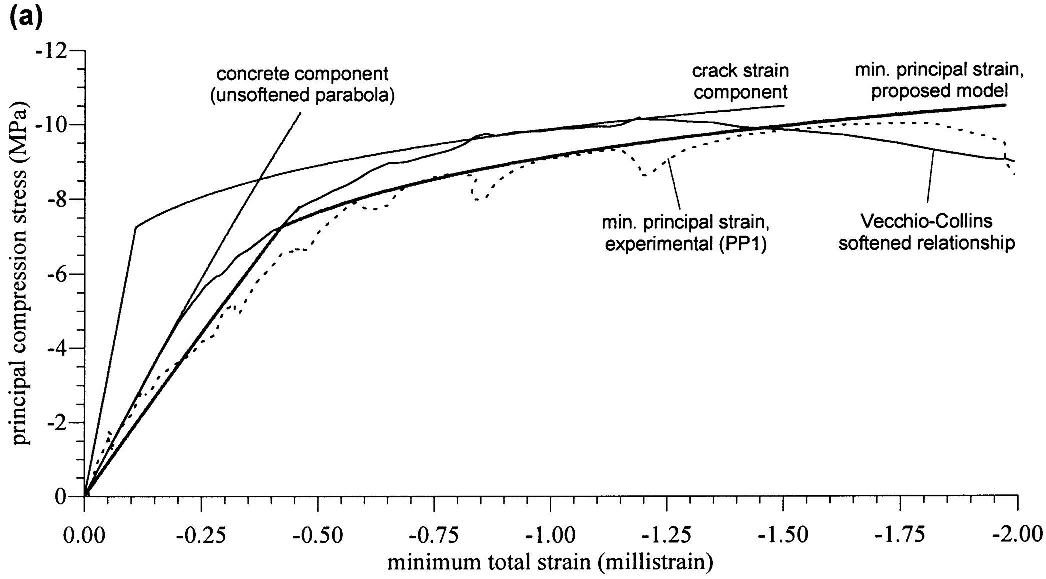


Fig. 7 - Strains due to deformations at the cracks



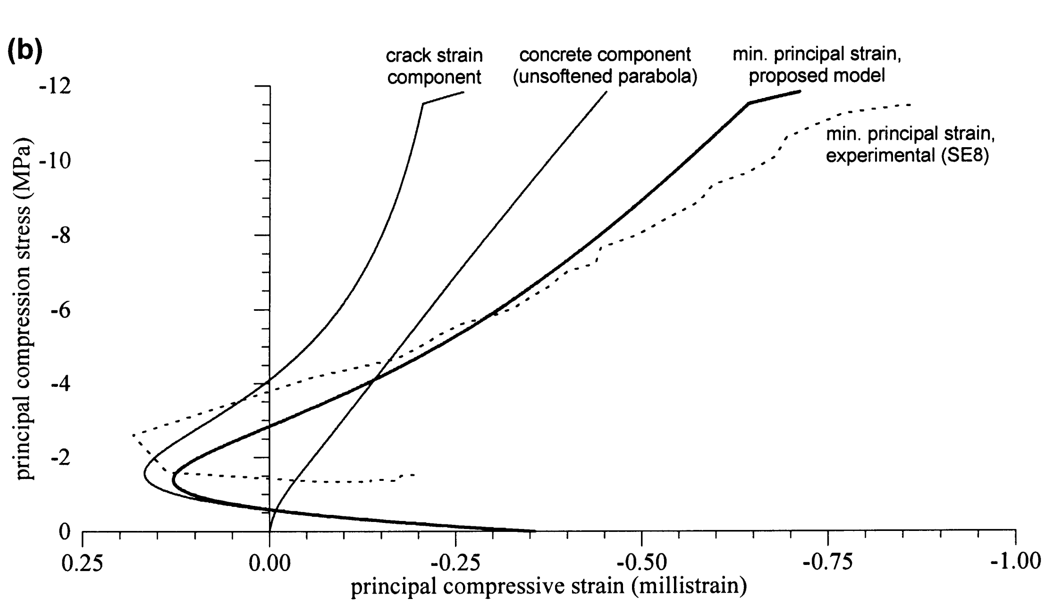


Fig. 8 - Concrete response: (a) monotonic; (b) cyclic