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TRI-AXIAL NON-LINEAR RESTORING FORCE MODEL OF R/C STRUCTURE USING THE THEORY OF PLASTICITY

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SUMMARY

This paper discusses, firstly, the analogy between the non-linear behaviors of a model structure and the theory of plasticity. One simple model structure that consists of a horizontal rigid plate and two vertical springs is provided. The horizontal rigid plate is supported by two springs. Each spring has the same load-deformation relation of tri-linear type. In the theory of plasticity, the concept of plastic potential and Prager's kinematic hardening rule are applied. Two-dimensional non-linear behaviors of the model structure can be perfectly described by the theory of plasticity. At the same time, a method is proposed to correct error of formulation in the analysis by the theory of plasticity. Then, the criterion of loading, neutral loading and unloading, as well as judgement of states are narrated. After that, the discussion about the adaptation of the theory of plasticity in accordance with a model structure of square section is carried on. The material of the section doesn't transfer tensile stress, and is rigid perfectly-plastic in the compressive strain region. The relation of two-dimensional forces and two-dimensional deformations is examined. Some part of restoring force-deformation relation of this model structure is far different from the result by the theory of perfect plasticity. Based on the above, a tri-axial non-linear restoring force model using the theory of plasticity is presented, lastly. Prager's kinematic hardening rule is used. The object of the model is a reinforced concrete structure. The model has two yield surfaces. One is translating and the other is fixed. The model has elastic, elastic-perfectly plastic, elastic-hardening, and, elastic-hardening-perfectly plastic states.

INTRODUCTION

During an earthquake, structures are attacked by three-dimensional ground motion. Three-dimensional earthquake response analysis of structures is necessary for aseismic design. To analyze the response of a structure to a strong earthquake, tri-axial non-linear restoring force characteristics of the structure should be formulated. Some studies on the formulation of the non-linear restoring force characteristics have been presented [Takizawa and Aoyama, 1976; Takiguchi and Ogura, 1998; Takiguchi, Ogura and Mu, 1998]. If the restoring force characteristics of the structure are macroscopically formulated using the theory of plasticity, three-dimensional earthquake response analysis can be carried out conveniently. About the tri-axial restoring force characteristics, it is not easy to formulate by only arranging the experimental results. It is important to examine in detail the formulation method, which has an essential meaning. This paper discusses, firstly, the analogy between the non-linear behaviors of a model structure and the theory of plasticity. One simple two-spring model structure is provided. Restoring force characteristic of each spring is tri-linear type. In the theory of plasticity, the concept of plastic potential and Prager's kinematic hardening rule are applied. Then, the criterion of loading, neutral loading and unloading, as well as judgement of states are narrated. After that, the discussion about the adaptation of the theory of plasticity in accordance with a model structure of square section is carried on. The theory of perfect plasticity is used. The relation of two-dimensional forces and two-dimensional deformations is examined. Finally, a tri-axial non-linear restoring force model using the theory of plasticity is proposed. The object of the tri-axial non-linear restoring force model is a reinforced concrete structure.

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TWO-SPRING MODEL STRUCTURE

Model structure

One simple structural model that consists of a rigid plate and two springs is supposed, which is shown in Figure 1. The origin of coordinate axes (x, y, z) is at the center point O of the rigid plate. Each of the distance between the spring and the center point O in y direction is expressed by L_x. The rigid plate is subjected to the bending moment around x-axis (M_x) and the force in the direction of z-axis (N_z). Rotational deformation around x-axis and translational deformation along z-axis of the center point of the rigid plate is expressed by ϕ_x, δ_z respectively. Each spring of this structure model has the same load (P)-deformation (δ) relation, which is tri-linear type shown in Figure 2. P is load, δ is deformation of the end point of the spring. The second slope ratio of the P-δ relation is expressed by γ, and the third of that is equal to zero.

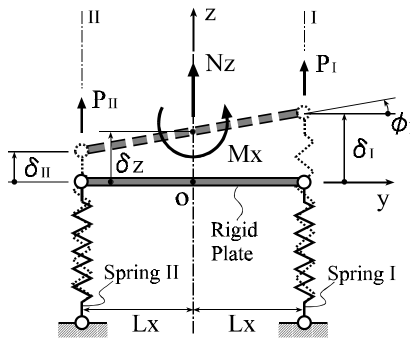


Figure 1: Model Structure

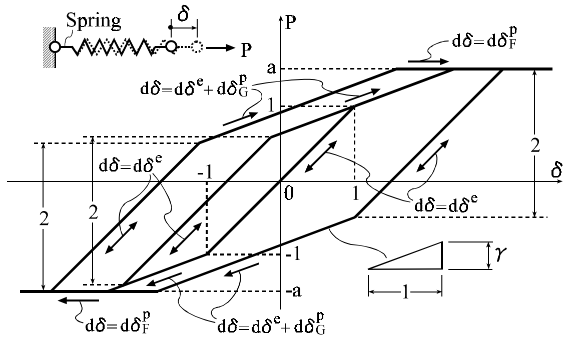


Figure 2: $P-\delta$ Relation of Springs

In order to adapt the theory of plasticity to the case of the different uniaxial restoring force characteristics in every axis, the transformation matrix [A] is used, which is defined as $[A]=\begin{bmatrix} 1/L_x & 0 \\ 0 & 1 \end{bmatrix}$. Therefore, force vector $\{M\}^T=\lfloor M_x \quad N_z \rfloor$ and deformation vector $\{\phi\}^T=\lfloor \phi_x \quad \delta_z \rfloor$ can be transformed as forms shown in Eqs. (1) and (2). (In this paper, the symbols are defined as: $\{\ \}$ is for row vector, $\lfloor\ \rfloor$ is for line vector, and $[\ ]$ is for matrix.)

$$\{\overline{M}\}=[A]\cdot\{M\} \tag{1}$$

$$\{\overline{\phi}\}=[A]^{-1}\cdot\{\phi\} \tag{2}$$

, where $\{\overline{M}\}$ and $\{\overline{\phi}\}$ is force vector, deformation vector respectively in accordance with the force space $(\overline{M}_x-\overline{N}_z)$.

Assumption of yield surface and hardening rule

Two yield surfaces are assumed in the force space $(\overline{M}_x-\overline{N}_z)$, which are shown in Figure 3. One is fixed yield surface expressed as $F(\overline{M})=0$ ($F(\overline{M})=\left|\overline{M}_x\right|+\left|\overline{N}_z\right|-2\cdot a=0$), and the other is translating yield surface expressed as $G(\overline{M}-\alpha)=0$ ($G(\overline{M}-\alpha)=\left|\overline{M}_x-\alpha_x\right|+\left|\overline{N}_z-\alpha_z\right|-2=0$). $\{\alpha\}$ is the translating index of translating yield surface, and is defined as $\{\alpha\}^T=\lfloor \alpha_x \quad \alpha_z \rfloor$. Prager's kinematic hardening rule [Prager, 1956] is employed in establishing the conditions for subsequent yield from a plastic state.

Restoring force characteristics based on the theory of plasticity

Generally, if fixed yield surface and translating yield surface were assumed, four states could be presented, which are elastic, elastic-hardening, elastic-perfectly plastic, and elastic-hardening-perfectly plastic states. The basic derivation of this part has been narrated in complete detail [Takiguchi and Ogura, 1998; Takiguchi, Ogura and Mu, 1998].

At the elastic state, total incremental deformations vector $\{d\bar{\phi}\}$ ($\{d\bar{\phi}\}^T = \lfloor d\bar{\phi}_x \quad d\bar{\delta}_z \rfloor$) is assumed as $\{d\bar{\phi}\} = \{d\bar{\phi}^e\}$. $\{d\bar{\phi}^e\}$ is the elastic incremental deformation vector defined as $\{d\bar{\phi}^e\}^T = \lfloor d\bar{\phi}_x^e \quad d\bar{\delta}_z^e \rfloor$.

At the elastic-hardening state, $\{d\bar{\phi}\}$ is assumed as $\{d\bar{\phi}\} = \{d\bar{\phi}^e\} + \{d\bar{\phi}_G^p\}$. $\{d\bar{\phi}_G^p\}$ is the incremental deformation vector associated with work-hardening plasticity, and is defined as $\{d\bar{\phi}_G^p\}^T = \lfloor d\bar{\phi}_{x\,G}^p \quad d\bar{\delta}_{z\,G}^p \rfloor$. Following to Prager's kinematic hardening rule, incremental translation $\{d\alpha\}$ of translating yield surface ($G(\bar{M} - \alpha) = 0$) is defined as $\{d\alpha\} = c \cdot \{d\bar{\phi}_G^p\}$. c is the work-hardening constant, and is defined as $c = \dfrac{2 \cdot \gamma}{1 - \gamma}$.

At the elastic-perfectly plastic state, $\{d\bar{\phi}\}$ is assumed as $\{d\bar{\phi}\} = \{d\bar{\phi}^e\} + \{d\bar{\phi}_F^p\}$. $\{d\bar{\phi}_F^p\}$ is the incremental deformation vector associated with perfect plasticity, and is defined as $\{d\bar{\phi}_F^p\}^T = \lfloor d\bar{\phi}_{x\,F}^p \quad d\bar{\delta}_{z\,F}^p \rfloor$.

At the elastic-hardening-perfectly plastic state, $\{d\bar{\phi}\}$ is assumed as $\{d\bar{\phi}\} = \{d\bar{\phi}^e\} + \{d\bar{\phi}_G^p\} + \{d\bar{\phi}_F^p\}$.

When the force is on the smooth yield surface, according to the flow rule, the direction of plastic deformation increment vector is normal to the yield surface at the current force point. When the force is at a singular point, the direction of plastic deformation increment vector is not uniquely. With reference to the previous researches [Koiter, 1953; Yin, 1986] of this problem, an assumption about this case is given as below.

Plastic deformation increment in the case that the force is at a singular point

At the singular point, yield surfaces $F_1 = 0, \cdots, F_i = 0, \cdots, F_n = 0$ adjoin.

$$\{d\bar{\phi}_F^p\} = d\lambda_1 \cdot \left\{\frac{\partial F_1}{\partial \bar{M}}\right\} + \cdots + d\lambda_i \cdot \left\{\frac{\partial F_i}{\partial \bar{M}}\right\} + \cdots + d\lambda_n \cdot \left\{\frac{\partial F_n}{\partial \bar{M}}\right\}, \quad d\lambda_1, \cdots, d\lambda_i, \cdots, d\lambda_n \geq 0$$

The angle between $\{d\bar{\phi}_F^p\}$ and $\{d\bar{\phi}^e\} + \{d\bar{\phi}_F^p\}$ becomes minimum, where $\{d\bar{\phi}_F^p\}$ is plastic deformation increment vector which associates with yield surfaces $F_1 = 0, \cdots, F_i = 0, \cdots, F_n = 0$ and $\{d\bar{\phi}^e\}$ is elastic deformation increment vector.

The relation between force increment vector $\{d\bar{M}\}$ and deformation increment vector $\{d\bar{\phi}\}$ can be generally expressed as $\{d\bar{M}\} = [\bar{K}] \cdot \{d\bar{\phi}\}$, in which $[\bar{K}]$ is stiffness matrix which connects $\{d\bar{M}\}$ and $\{d\bar{\phi}\}$ completely defined in past researches [Takiguchi and Ogura, 1998; Takiguchi, Ogura and Mu, 1998]. Using Eqs. (1) and (2), the relation between $\{dM\}$ and $\{d\phi\}$ is expressed as

$$\{dM\} = [A]^{-1} \cdot [\bar{K}] \cdot [A]^{-1} \cdot \{d\phi\} \tag{3}$$

, where $\{dM\}$ and $\{d\phi\}$ is force increment vector, deformation increment vector respectively in accordance with the force space (M - N).

The analytical results about the non-linear behaviors of the model structure between from the $P - \delta$ relation of the springs and from the theory of plasticity are compared. To say in other words, the analogy between non-linear behaviors of the model structure and the theory of plasticity is investigated. It is assumed that the distance between the spring and the center point O in y direction $L_x = 3/2$, the stiffness of the springs after yielding $\gamma = 1/3$, and $a = 3/2$. The model structure is analyzed under the condition of the monotonic incremental deformations ϕ_x and δ_z, which satisfies $\phi_x = 4 \cdot \delta_z / 3$. The result is shown in Figure 4. Input data is (ϕ_x, δ_z), and response is (M_x, N_z). The response of (M_x, N_z) by using the $P-\delta$ relation of the springs is coincident

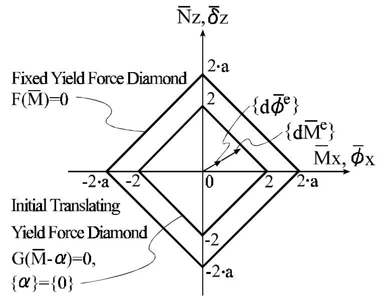


Figure 3: Yield Surfaces

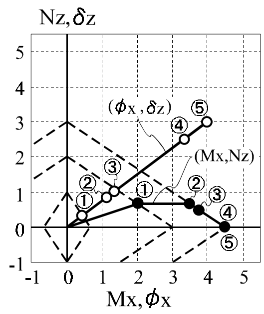
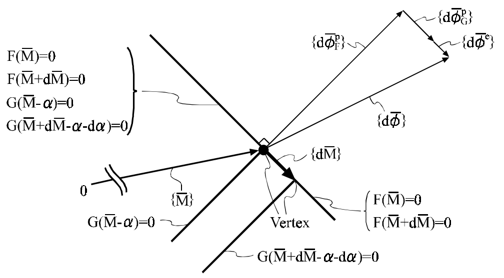
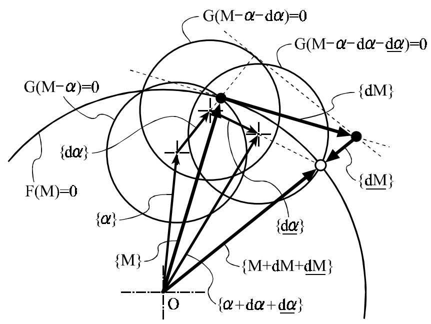


Figure 4: Response of (M_x, N_z) to (ϕ_x, δ_z)





with the response by using the theory of plasticity.

Figure 5: Incremental Force and Incremental Deformation at a Stage between and

Figure 6: Error of Formulation

In the force space, two yield surfaces were assumed. One is translating yield surface ($G(\overline{M}-\alpha)=0$), the other is fixed yield surface ($F(\overline{M})=0$) which represents the perfect plasticity without hardening. This original assumption requires that once the force point reaches the fixed yield surface, the force increment vector ($\{d\overline{M}\}$) should be tangent to the fixed yield surface and the force point should stay on the fixed yield surface during following deformation increment ($\{d\overline{\phi}\}$). Examine the response of force increment to the deformation increment in the numerical example above, those tally with the original assumption. A partial result of stage between and is indicated in Figure 5. However, this result based on the numerical analysis has no universality, because the yield surfaces assumed above are particular, which are composed of straight lines. In addition, if use loading criteria and judgement of states which will be seen later in Section 3., this result can not be obtained. As an general situation, the translating yield surface and the fixed yield surface are assumed as two circular surfaces shown in Figure 6, which are expressed as $G(M-\alpha)=0$ and $F(M)=0$. In Figure 6, current force point corresponding to force vector ($\{M\}$) reaches an intersection of two yield surfaces. Based on the analysis by flow rule, loading criteria and judgement of states, once a deformation increment is given now, a force increment vector ($\{d\overline{M}\}$) is brought about, and subsequent force point reaches an intersection of a tangent to fixed yield surface and a tangent to subsequent translating yield surface ($G(M-\alpha-d\alpha)=0$). Since the position of subsequent force point that is out of two yield surfaces is against the original assumption, it is sure that an error of formulation exists in the analysis. A method is proposed to correct the error of formulation. The intersection of fixed yield surface and the line from origin point O to subsequent force point is referred as new subsequent force point, which is a white round point shown in Figure 6. At this time, force vector is expressed as $\{M+dM+\underline{dM}\}$. $\{\underline{dM}\}$ is the error force increment vector. The corresponding translating yield surface is expressed as $G(M-\alpha-d\alpha-\underline{d\alpha})=0$. $\{\underline{d\alpha}\}$ is the error incremental translation of translating yield surface.

LOADING CRITERIA AND JUDGEMENT OF STATES

A yield surface in the force space defines the boundary between the elastic region and the plastic region. If a force point lies inside the surface, corresponding state is an elastic state and only the elastic behaviors is expected. On the other hand, the state of force point lying on the yield surface is referred as a plastic state. Either the elastic or the elastic-plastic behavior occurs. For the case that a yield surface is associated with work-hardening plasticity, if the force point tends to move out of the yield surface, a loading process occurs and elastic-plastic deformation is observed. The configuration of the yield surface changes so that the force point always stays on the yield surface. If the force point moves along the yield surface, a neutral loading process occurs. The associated deformation is elastic. And if the force point tends to move into the yield surface, an unloading process occurs. Only elastic deformation occurs and the yield surface remains unchanged. The criterion of loading, neural loading and unloading are given as below with reference to the previous researches [Yin and Qu, 1981; Yin and Qu, 1982; Chen, Yamaguchi and Zhang, 1991].

Loading criteria

In the case that the force is on the smooth yield surface $F = 0$.
$f > 0$: Loading / $f = 0$: Neutral Loading / $f < 0$: Unloading

$$f = \left\{ \frac{\partial F}{\partial \overline{M}} \right\}^T \cdot [\overline{K}] \cdot \{d\overline{\phi}\}$$

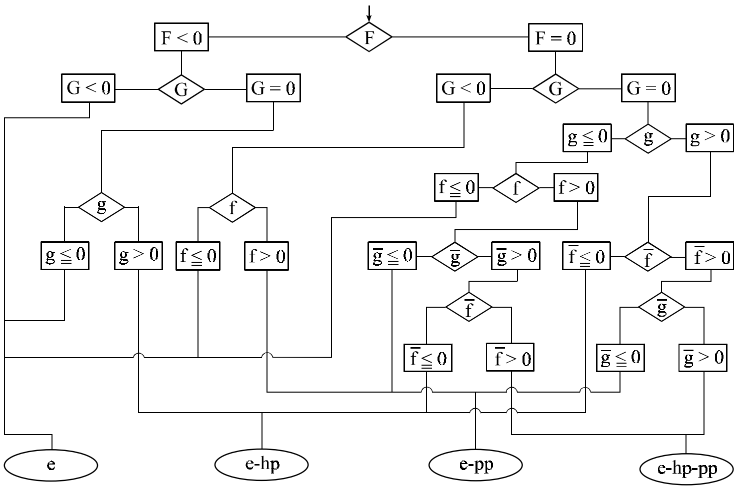
In the case that the force is at a singular point.
At the singular point, yield surfaces $F_1 = 0, \cdots, F_i = 0, \cdots, F_n = 0$ adjoin.
Any $f_i > 0$: Loading / Any $f_i = 0$ and All $f_j < 0$ $(j \neq i)$: Neutral Loading / All $f_i < 0$: Unloading

$$f_i = \left\{ \frac{\partial F_i}{\partial \overline{M}} \right\}^T \cdot [\overline{K}] \cdot \{d\overline{\phi}\}$$

$[\overline{K}]$ is stiffness matrix which connects the force increment vector $\{d\overline{M}\}$ and the total deformation increment vector $\{d\overline{\phi}\}$ ($\{d\overline{M}\} = [\overline{K}] \cdot \{d\overline{\phi}\}$) by assuming that plastic deformation increment vector associated with the yield surface $F = 0$ or with the yield surface $F_1 = 0, \cdots, F_i = 0, \cdots, F_n = 0$ is zero ($\{d\overline{\phi}_F^p\} = \{0\}$).

In the practical analysis, it is important how to judge the states. Two yield surfaces are assumed as mentioned in Section 2., one is translating yield surface ($G(\overline{M} - \alpha) = 0$), the other is fixed yield surface ($F(\overline{M}) = 0$). Four states are presented, which are elastic, elastic-hardening, elastic-perfectly plastic, and elastic-hardening-perfectly plastic states, and are expressed as State e, State e-hp, State e-pp, and State e-hp-pp respectively, here. Factors

$$g = \left\{ \frac{\partial G}{\partial \overline{M}} \right\}^T \cdot [E] \cdot \{d\overline{\phi}\}, \quad f = \left\{ \frac{\partial F}{\partial \overline{M}} \right\}^T \cdot [E] \cdot \{d\overline{\phi}\}, \quad \overline{g} = \left\{ \frac{\partial G}{\partial \overline{M}} \right\}^T \cdot [\overline{K}_G^p] \cdot \{d\overline{\phi}\}, \quad \overline{f} = \left\{ \frac{\partial F}{\partial \overline{M}} \right\}^T \cdot [\overline{K}_F^p] \cdot \{d\overline{\phi}\}$$

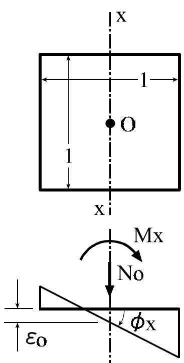


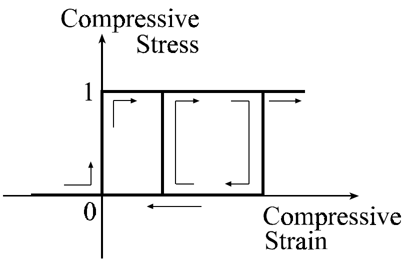
g , f , $\bar{g}$ and $\bar{f}$ are proposed for judgement of states. The judgement process is shown in Figure 7.

Figure 7: Flow Diagram of States Judgement

MODEL STRUCTURE OF SQUARE SECTION

To investigate the adaptation of the theory of plasticity further, another model structure is supposed, which is a square section shown in Figure 8. The square section is subjected to the bending moment (M_x) around x-x axis and the force (N_o) on the center point O in the direction normal to the section. Curvature around x-x axis and dimensional strain of the center point O is expressed by ϕ_x , ε_o respectively. The characteristics of material are given in Figure 9. The material of the section doesn't transfer tensile stress, and is rigid perfectly-plastic in the compressive strain region. The $M_x - N_o$ interaction diagram is shown in Figure 10.





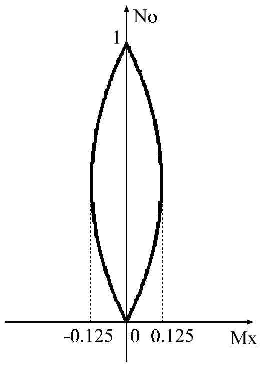


Figure 8: Model Structure of Square Section **Figure 9: Stress-Strain Relationship** **Figure 10: $M_x - N_o$ Interaction**

The condition of $N_o = \text{const.} = 0.25$ is set up on the square section. The responses of M_x and ε_o to cyclic input data ϕ_x by using the characteristics of material are shown in Figure 11. The monotonic deformation and the cyclic deformation can be simply defined. In Figure 11, the monotonic deformation history is from point ① to point ③ , and the cyclic deformation history is from point ③ to point ⑦. On the other hand, when the $M_x - N_o$ interaction curve shown in Figure 10 is regarded as a yield surface, the theory of perfect plasticity is used. The state of force lying on the yield surface is referred to as a perfectly plastic state. At this time, the flow rule requires that the plastic strain increment vector be always outwards normal to the yield surface. Based on the above, a result by comparing the response based on the characteristics of material with the response based on the theory of perfect plasticity is obtained, what is the consistency as to the monotonic deformation history and the great difference as to the cyclic deformation history. Because the material characteristics of this section model could be regarded as simplified stress-strain relationship of concrete, this result is important for the analysis about multi-dimensional restoring force behaviors of composite structure.

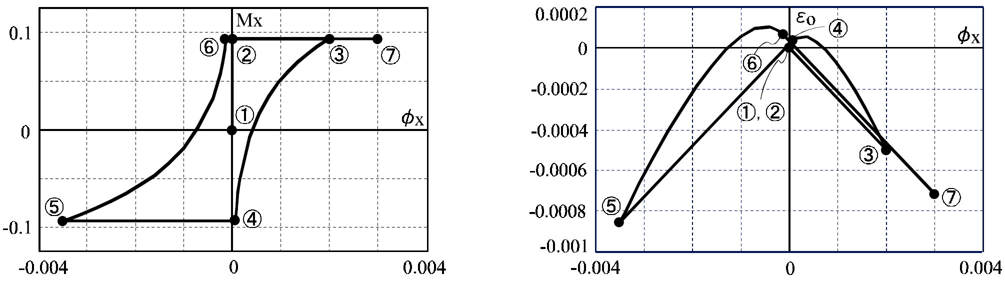


Figure 11: Responses of M_x and ε_o to ϕ_x History under Condition of $N_o = const. = 0.25$ by Section Analysis

TRI-AXIAL RESTORING FORCE MODEL OF R/C STRUCTURE

A tri-axial non-linear restoring force model using the theory of plasticity is presented here. The object of the model is a reinforced concrete structure. The model has two yield surfaces in the force space ($Q_x - Q_y - Q_z$). One is translating yield surface ($G(Q-\alpha)=0$), the other is fixed yield surface ($F(Q)=0$), which are shown in Figure 12. At the same time, the model consists of four states, which are elastic, elastic-hardening, elastic-perfectly plastic, and elastic-hardening-perfectly plastic states. Prager's kinematic hardening rule is applied.

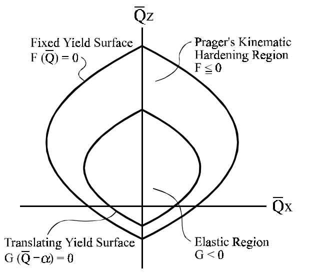


Figure 12: Non-linear Restoring Force Model

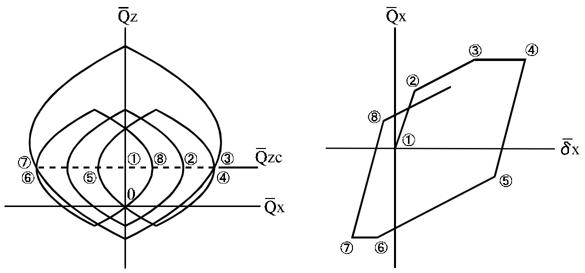


Figure 13: $\overline{Q}_x - \overline{\delta}_x$ Relation under the Conditions of $\overline{Q}_y = 0$ and $\overline{Q}_z = \overline{Q}_{zc}$

The tri-axial force vector $\{Q\}^T = \lfloor Q_x \quad Q_y \quad Q_z \rfloor$ and deformation vector $\{\delta\}^T = \lfloor \delta_x \quad \delta_y \quad \delta_z \rfloor$ could be transformed as forms shown in Eqs. (4) and (5) by using transformation matrix [A] defined as $[A] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{k_x^e / k_y^e} & 0 \\ 0 & 0 & \sqrt{k_x^e / k_z^e} \end{bmatrix}$. k_x^e, k_y^e and k_z^e are elastic stiffness with respect to x, y and z axis.

$$\{\overline{Q}\} = [A] \cdot \{Q\} \tag{4}$$

$$\{\overline{\delta}\} = [A]^{-1} \cdot \{\delta\} \tag{5}$$

The relations between incremental force vector $\{d\overline{Q}\}$ ($\{d\overline{Q}\} = \lfloor d\overline{Q}_x \quad d\overline{Q}_y \quad d\overline{Q}_z \rfloor$) and the total incremental deformation vector $\{d\overline{\delta}\}$ ($\{d\overline{\delta}\}^T = \lfloor d\overline{\delta}_x \quad d\overline{\delta}_y \quad d\overline{\delta}_z \rfloor$) are described by Eqs. (6), (7), (8) and (9) in accordance with four states, respectively.

At the elastic state, $\{d\overline{\delta}\}$ is defined as $\{d\overline{\delta}\} = \{d\overline{\delta}^e\}$. $\{d\overline{\delta}^e\}$ is the elastic incremental deformation vector defined as $\{d\overline{\delta}^e\}^T = \lfloor d\overline{\delta}_x^e \quad d\overline{\delta}_y^e \quad d\overline{\delta}_z^e \rfloor$. Relation between $\{d\overline{Q}\}$ and $\{d\overline{\delta}\}$ is given as

$$\{d\overline{Q}\} = [E] \cdot \{d\overline{\delta}^e\} \tag{6}$$

, where [E] is the elastic stiffness matrix defined as $[E] = \begin{bmatrix} k_x^e & 0 & 0 \\ 0 & k_x^e & 0 \\ 0 & 0 & k_x^e \end{bmatrix}$.

At the elastic-hardening state, $\{d\bar{\delta}\}$ is defined as $\{d\bar{\delta}\} = \{d\bar{\delta}^e\} + \{d\bar{\delta}^p_G\}$. $\{d\bar{\delta}^p_G\}$ is the incremental deformation vector associated with work-hardening plasticity, and is defined as $\{d\bar{\delta}^p_G\}^T = \lfloor d\bar{\delta}^p_{x\,G} \quad d\bar{\delta}^p_{y\,G} \quad d\bar{\delta}^p_{z\,G} \rfloor$. Relation between $\{d\bar{Q}\}$ and $\{d\bar{\delta}\}$ is given as

$$\{d\bar{Q}\} = [\bar{K}^p_G] \cdot \{d\bar{\delta}\} \tag{7}$$

At the elastic-perfectly plastic state, $\{d\bar{\delta}\}$ is defined as $\{d\bar{\delta}\} = \{d\bar{\delta}^e\} + \{d\bar{\delta}^p_F\}$. $\{d\bar{\delta}^p_F\}$ is the incremental deformation vector associated with perfect plasticity, and is defined as $\{d\bar{\delta}^p_F\}^T = \lfloor d\bar{\delta}^p_{x\,F} \quad d\bar{\delta}^p_{y\,F} \quad d\bar{\delta}^p_{z\,F} \rfloor$. Relation between $\{d\bar{Q}\}$ and $\{d\bar{\delta}\}$ is given as

$$\{d\bar{Q}\} = [\bar{K}^p_F] \cdot \{d\bar{\delta}\} \tag{8}$$

At the elastic-hardening-perfectly plastic state, $\{d\bar{\delta}\}$ is defined as $\{d\bar{\delta}\} = \{d\bar{\delta}^e\} + \{d\bar{\delta}^p_G\} + \{d\bar{\delta}^p_F\}$. Relation between $\{d\bar{Q}\}$ and $\{d\bar{\delta}\}$ is given as

$$\{d\bar{Q}\} = [\bar{K}^p_{G\,F}] \cdot \{d\bar{\delta}\} \tag{9}$$

$[\bar{K}^p_G]$, $[\bar{K}^p_F]$ and $[\bar{K}^p_{G\,F}]$ in the equations above are stiffness matrices defined in the past research [Takiguchi and Ogura, 1998]. The relation of $\bar{Q}_x - \bar{\delta}_x$ under the conditions of $\bar{Q}_y = 0$ and $\bar{Q}_z = \bar{Q}_{zc}$ is shown in Figure 13. The relations between $\{d\bar{Q}\}$ and $\{d\bar{\delta}\}$ in accordance with respective state could be generally expressed by Eq. (10). Using Eqs. (4) and (5), the relation between $\{dQ\}$ and $\{d\delta\}$ can be obtained by Eq. (11).

$$\{d\bar{Q}\} = [\bar{K}] \cdot \{d\bar{\delta}\} \tag{10}$$

$$\{dQ\} = [A]^{-1} \cdot [\bar{K}] \cdot [A]^{-1} \cdot \{d\delta\} \tag{11}$$

, where $\{dQ\}$ and $\{d\delta\}$ is force increment vector, deformation increment vector respectively in accordance with the force space ($Q_x - Q_y - Q_z$).

CONCLUSIONS

1. Two-dimensional non-linear behaviors of two-spring model structure can be described perfectly by the theory of plasticity. Each of spring has the same load-deformation relation of tri-linear type. In the theory of plasticity, the concept of plastic potential and Prager's kinematic hardening rule are applied. In addition, the loading criteria and the judgement of states are proposed.
2. Two-dimensional non-linear restoring force behaviors of square section model structure are far different from the result by the theory of perfect plasticity as to the cyclic deformation history. The material of the section doesn't transfer tensile stress, and is rigid perfectly-plastic in the compressive strain region.
3. A tri-axial non-linear restoring force model using the theory of plasticity is presented. The object of the model is a R/C structure. The model has two yield surfaces. One is translating and the other is fixed. The model has elastic, elastic-hardening, elastic-perfectly plastic, and elastic-hardening-perfectly plastic states.

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