A TWO-LEVEL OPTIMIZATION METHOD FOR SEISMIC DESIGN OF ELASTIC THREE-DIMENSIONAL FRAMES

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SUMMARY

A two-level algorithm is presented for optimum design of a 3D-frame under constraints on seismic and elastic static responses. The optimal stiffnesses and corresponding horizontal seismic loads are first found from a shear model considering the responses evaluated by a response spectrum approach. In the lower level optimization problem, the cross-sectional areas of beams of plane frames are optimized so that the stiffness of each story is equal to the specified value calculated from the obtained stiffness of the shear model, and the response stresses are within the specified limits. In this process, the cross-sectional areas of columns are fixed because a column belongs to two plane frames in mutually perpendicular directions. In the upper level problem, sensitivity analysis of the optimal objective values of the lower level problems is carried out to modify the cross-sectional areas of columns. This way, the lower level problems can be solved independently, and two problems are successively solved to obtain the optimal design of a 3-D frame. The efficiency of the algorithm is discussed in an example of a 27-story 3D-frame.

INTRODUCTION

There has been variety of methods presented for seismic design and optimum design of plane frames [Arora 1997, Nakamura *et al*. 1993]. Those methods, however, cannot be directly applied to a large-scale 3D-frame due to limitations on computational capacity and approximation introduced to reduce the computational cost. Although the recent dramatic progress in the field of computer science enabled us to carry out analysis and optimization of moderately large frames, it is critical to develop more efficient algorithms for the designers to use those methods in design practice.

Multilevel optimization method may be successfully applied for optimizing large-scale frames [Sobieski *et al*. 1985, Grierson and Chiu 1984, Friedman and Fuchs 1987]. In the standard formulation of multilevel optimization problem, interaction between the upper and lower problems are modeled by using the parametric programming techniques [Fiacco 1983, Guddat *et al*. 1990] which may also be used in conjunction with the substructure method for optimizing large structures [Svensson 1987]. These methods are combined to be called multidisciplinary optimization method [Sobieski and Haftka 1997] where the process of optimizing a large structural system is divided into several hierarchical and non-hierarchical subsystems. Note that the subsystem can represent a subprocess as well as a physical substructure. To the authors' knowledge, however, there is no efficient multilevel optimization algorithm specially developed for large 3-D building frames.

In this paper, a two-level algorithm that is suitable for parallel computing is presented for optimization of large 3-D building frames. The 3D-frame is considered as an assemblage of plane frames, and is optimized by using an optimization algorithm based on a parametric programming approach [Ohsaki 1997]. The efficiensy of the algorithm is demonstrated in the example of a 27-story three-dimensional frame.

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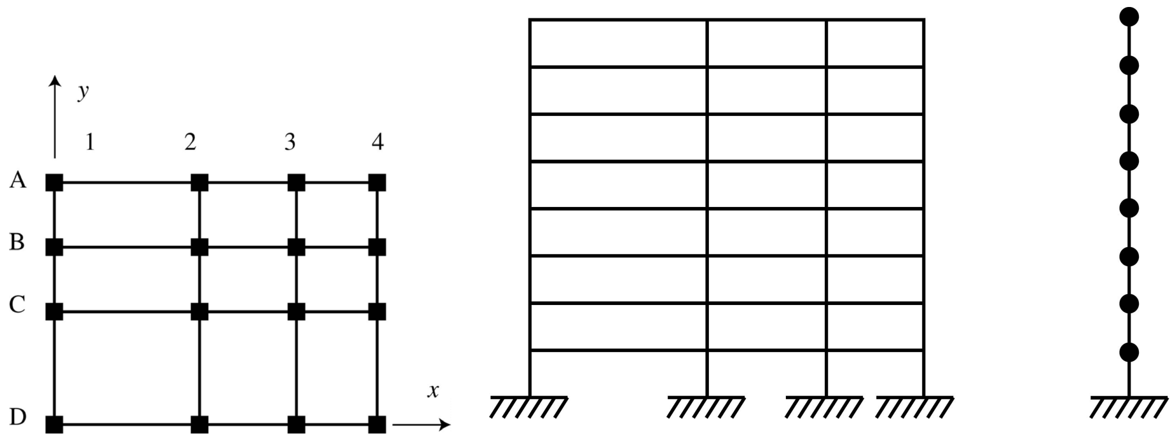

(a) Plan view (b) Elevation (c) Shear model

Fig. 1: A three-dimensional frame model.

DEFINITION OF SEISMIC LOAD AND STORY STIFFNESS

Consider a 3-D frame as shown in Fig. 1, which is regular but not necessarily symmetric. To reduce the computational cost for analysis and design of a large 3-D frame, the frame is divided into plane frames assuming that the interaction among the frames in the same direction is negligibly small. The effect of the torsional moment from the beams that are perpendicular to the plane frame is also neglected. In the process of evaluating the horizontal earthquake loads and the optimal interstory stiffnesses, the 3-D frame is further simplified into a shear model as shown in Fig. 1(c).

The story stiffness of the plane frame is to be found so that the mean-maximum interstory drift is equal to the specified value under a set of earthquakes that are compatible with the given design response spectrum. Let $\mathbf{D}^* = \{D_i^*\}$ denote the vector of interstory stiffness of the shear model. The stiffness matrix $\mathbf{K}^s(\mathbf{D}^*)$ and the mass matrix $\mathbf{M}^s(\mathbf{D}^*)$ are the functions of $\mathbf{D}^*$. The r th eigenvalue and eigenvector are denoted by $\Omega_r(\mathbf{D}^*)$ and $\mathbf{\Phi}_r(\mathbf{D}^*)$, respectively, which are defined by

$$\mathbf{K}^s(\mathbf{D}^*)\mathbf{\Phi}_r(\mathbf{D}^*) = \Omega_r(\mathbf{D}^*)\mathbf{M}^s(\mathbf{D}^*)\mathbf{\Phi}_r(\mathbf{D}^*) \tag{1}$$

The interstory drift of the i th story corresponding to $\mathbf{\Phi}_r(\mathbf{D}^*)$ is denoted by $\Psi_{ri}(\mathbf{D}^*)$. Then the mean-maximum interstory drift $\delta_i(\mathbf{D}^*)$ for the set of earthquake excitations that are compatible with the specified displacement response spectrum $S_D(\Omega_r)$ is evaluated by the Square-Root-of-Sum-of-Squares (SRSS) method as

$$\delta_i(\mathbf{D}^*) = \sqrt{\sum_{r=1}^{p} \left[S_D(\Omega_r)\beta_r(\mathbf{D}^*)\Psi_{ri}(\mathbf{D}^*)\right]^2} \tag{2}$$

where $\beta_r(\mathbf{D}^*)$ is the participation factor for the r th mode, and p is the number of modes to be used for evaluation of the responses.

Let $\bar{\delta}_i$ denote the specified interstory drift. The story stiffness $\mathbf{D}^* = \{D_i^*\}$ is found so that

$$\delta_i(\mathbf{D}^*) = \bar{\delta}_i \tag{3}$$

is satisfied. Note that (3) is solved by using a Newton-Raphson type algorithm, an unconstrained optimization technique, or a fully-stressed design algorithm [Haftka *et al.* 1990]. Since the numbers of design variables D_i^* and the constraints (3) are same, the values of D_i^* are uniquely determined by (3). It is desired that D_i^* should be recalculated after optimization of cross-sectional areas of beams and columns, because the mass matrix depends on the cross-sectional areas. In the following, the argument $\mathbf{D}^*$ is not written explicitly for simpler presentation of the formulations.

The story shear force Q_i^* and the horizontal load P_i^* are defined as follows in terms of the interstory drift and the story stiffness:

$$Q_i^* = D_i^* \overline{\delta}_i \tag{4}$$

$$P_i^* = Q_i^* - Q_{i+1}^* \tag{5}$$

OPTIMUM DESIGN OF PLANE FRAMES FOR SPECIFIED STORY STIFFNESS

A 3-D frame is considered as an assemblage of plane frames, and is optimized by using a two-level optimization algorithm. Consider a set of plane frames in one of two principal directions; i.e. x - or y -direction in Fig. 1(a). The story stiffness of the 3-D frame is shared by the plane frames so that the torsional deformation does not occur even if the locations of the columns are not symmetric.

Let S and S^j denote the area of each floor and the area covered by the j th plane frame. The mass is assumed to be uniformly distributed at each floor. Then the interstory stiffness $\overline{D}_i^j$ of the i th story to be specified for the j th plane frame, the horizontal load, and interstory shear of a plane frame are given as

$$\overline{D}_i^j = \frac{S^j}{S} D_i^*, \quad P_i^j = \frac{S^j}{S} P_i^*, \quad Q_i^j = \frac{S^j}{S} Q_i^* \tag{6}$$

In this case, the interstory drifts of the plane frames in each direction are same without any interaction through the shear force of the floor. In the following, the values corresponding to the j th frame is indicated by the superscript j . Let A_i^j and $I_i^j\left(A_i^j\right)$ denote the cross-sectional area and the moment of inertia of the i th member of the j th plane frame. Note that $I_i^j\left(A_i^j\right)$ as well as the section modulus $Z_i^j\left(A_i^j\right)$ is a function of A_i^j . Two load vectors $\mathbf{P}^{Vj}$ and $\mathbf{P}^{Hj} = \left\{P_i^j\right\}$ corresponding to the self-weight and the horizontal loads defined by (6), respectively, are considered for evaluating the responses.

The frame considered here does not necessarily have an axis of symmetry. Therefore, the responses for horizontal loads in the two opposite directions should be evaluated. The stress σ_j^i of the i th member of the j th plane frame is defined as the maximum value of the absolute values of the stresses at the two faces of two ends of the member for the loads $\mathbf{P}^{Vj} + \mathbf{P}^{Hj}$ and $\mathbf{P}^{Vj} - \mathbf{P}^{Hj}$. For the beams, the moment $\pm \frac{1}{12} q\left(L_i^j\right)^2$ due to the distributed load is added at each end, where q is the vertical load per unit length and L_i^j is the length of the i th member. The effect of axial deformation is considered in evaluating the stresses of the columns. Note that the axial force does not exist in the beams because of the assumption of rigid floor.

The interstory drift of the frame which is same for $\mathbf{P}^{Vj} + \mathbf{P}^{Hj}$ and $\mathbf{P}^{Vj} - \mathbf{P}^{Hj}$ is denoted by δ_i^j . Then the story stiffness D_i^j of the frame is calculated from

$$D_i^j = \frac{Q_i^j}{\delta_i^j} \tag{7}$$

Let $\overline{\sigma}_i^j$ denote the upper bound for σ_i^j . The lower bound for A_i^j is denoted by $\overline{A}_i^j$. Then the problem for minimizing the total mass W^j of the j th plane frame is stated as

Minimize $$W^j = \sum_{i=1}^{m^j} \rho L_i^j A_i^j \tag{8}$$

Subject to $$\sigma_i^j \le \overline{\sigma}_i^j \tag{9}$$

$$D_i^j \ge \overline{D}_i^j \tag{10}$$

$$A_i^j \ge \overline{A}_i^j \tag{11}$$

where m^j is the number of members of the j th plane frame, and ρ is the mass density of the beams and columns [Ohsaki 1997]. The constraint (10) is equivalent to a constraint on the interstory drift if the relation (7) is used for the specified value of Q_i^j.

In the examples, the optimal solutions are found by using the method of modified feasible directions which utilizes sensitivity information. The details of design sensitivity analysis are not explained here, because it is rather straightforward for static elastic responses.

OPTIMUM DESIGN OF 3-D FRAMES

Optimum design of a 3-D frame is found by successively optimizing the plane frames. A serious difficulty arises, however, from the fact that the columns belong to two plane frames in different directions, whereas the beams are included in only one frame. Therefore, the cross-sectional properties of the columns cannot be modified independently in the process of optimizing a plane frame.

In the proposed two level optimization algorithm, the upper and lower problems are solved successively to find the optimum design of a 3-D frame. The cross-sectional areas of the columns are fixed in the lower-level problem for optimizing the beams of the plane frames. After optimization of all the plane frames is completed, the cross-sectional areas of the columns are modified in the upper level problem based on a parametric programming approach. Then the beams of plane frames are optimized again in the lower level problem for the updated values of cross-sectional areas of the columns.

Suppose the member numbers of a plane frame are assigned so that the members $1,2,\cdots,m_b^j$ are beams and $m_b^j+1, m_b^j+2,\cdots,m^j$ are columns, where m_b^j is the number of beams. Let λ_i^j, η_i^j and μ_i^j denote Lagrangian multipliers. The design variables for optimization of j th plane frame are the cross-sectional areas of beams which are denoted by $A_1^j, A_2^j, \cdots A_{m_b^j}^j$. Then the Lagrangian Π^j is written as

$$\Pi^j = \sum_{i=1}^{m^j} \rho L_i^j A_i^j + \sum_{i=1}^{m^j} \lambda_i^j (\sigma_i^j - \overline{\sigma}_i^j) + \sum_{i=1}^{m^j} \eta_i^j (\overline{A}_i^j - A_i^j) + \sum_{i=1}^{n^f} \mu_i^j (\overline{D}_i^j - D_i^j) \tag{12}$$

where n^f is the number of stories. The multipliers are calculated at each step of optimization if a primal-dual method is used. Even if the multipliers are not available, those are easily calculated after the solution has converged.

The member numbers are assigned also for the 3-D frame so that the members $1,2,\cdots,m_b$ are beams and $m_b+1, m_b+2,\cdots,m$ are columns, where m is the total number of members and m_b is the number of beams. In the following, the values for the total 3-D frame is indicated without superscript j. The objective function is the total structural volume that is defined by

$$W = \sum_{i=1}^{m} \rho L_i A_i \tag{13}$$

Let A_k^C denote the cross-sectional area of the k th column. The sensitivity coefficients of C with respect to A_k^C are calculated from [Fiacco 1983, Sobieski *et al.* 1985]

$$\frac{\partial W}{\partial A_k^C} = \rho L_k + \sum_{j=1}^{n^s} \sum_{i=1}^{m^j} \lambda_i^j \frac{\partial \sigma_i^j}{\partial A_k^C} - \sum_{j=1}^{n^s} \sum_{i=1}^{n^f} \mu_i^j \frac{\partial D_i^j}{\partial A_k^C}, \quad (k = 1,2,\cdots,m-m_b) \tag{14}$$

where n^s is the number of plane frames. The cross-sectional area of the columns are modified based on the steepest decent algorithm.

The optimization algorithm is summarized as:

1. [Definition of optimal story stiffness and horizontal seismic loads:]

   Find optimal story stiffness D_i^* and corresponding seismic load P_i to define $\overline{D}_i^j$ and Q_i^j to be specified for optimizing the plane frames.

2. [Initialize A_i^C:]

   Optimize each plane frame by considering cross-sectional areas of all the members as independent design variables. Then the value of A_i^C is defined, e.g., as the mean value or the maximum value of A_i^C in the plane frames in two different directions. This process can be parallely carried out, but can be omitted if a default initial value is given for the cross-sectional areas of columns.

3. [Lower level problem:]
   Find optimal cross-sectional areas of beams by optimizing plane frames for fixed cross-sectional areas of columns, and calculate Lagrangian multipliers for the constraints. Note that the optimization of each plane frame can be carried out independently and parallely, if possible, also for this lower level problem.
4. [Upper level problem:]
   Calculate from (14) the sensitivity coefficients of the objective function with respect to A_i^C and modify A_i^C based on the steepest decent method. A move limit ΔA is given for preventing divergence, and ΔA is reduced to $\beta\Delta A$ $(0 < \beta < 1)$, if the objective value has increased, to enable a convergence to the optimal value. There might be no feasible solution, however, if A_i^C is reduced too drastically; e.g. the constraints of story stiffness cannot be satisfied by optimizing the cross-sectional areas of beams if all the A_i^C ' in a story have too small values. For such a case, A_i^C is modified as follows depending the constraints that are violated:

   Interstory drift: $A_i^C = \gamma_1 A_i^C \quad (\gamma_1 > 1)$ (15)

   Stress of beam: $A_i^C = \gamma_2 A_i^C \quad (\gamma_2 < 1)$ (16)

   Stress of column: $A_i^C = \gamma_3 A_i^C \quad (\gamma_3 > 1)$ (17)

   where $\gamma_1, \gamma_2, \gamma_3$ are the parameters to be specified. Note that the cross-sectional area of the columns should be increased if the stress constraints of beams are violated.
5. Go to 2 if the solution is not converged.

EXAMPLES

Optimum designs have been found for a 27-story 3-D frame with a plan view as shown in Fig. 1(a). The span lengths (m) in x- and y-directions are $(W_{12}, W_{23}, W_{34}) = (12,10,10)$ and $(W_{\mathrm{AB}}, W_{\mathrm{BC}}, W_{\mathrm{CD}}) = (8,8,10)$, respectively., where W_{ij} is the distance between the lines i and j defined in Fig. 1(a). The elastic modulus is $205.8\ \mathrm{GPa}$ and the mass at each floor is $800\ \mathrm{kg/m}^2$. The mass density ρ is $7.86\times10^{-3}\ \mathrm{kg/cm}^3$. The values of I_i^j and Z_i^j are defined as the functions of A_i^j as

Column: $I_i^j = 1.2\left(A_i^j\right)^2, \quad Z_i^j = 1.0\left(A_i^j\right)^{1.5}$ (18)

Beam: $I_i^j = 8.0\left(A_i^j\right)^2, \quad Z_i^j = 2.0\left(A_i^j\right)^{1.5}$ (19)

The lower bound for the cross-sectional area is $50\ \mathrm{cm}^2$.

The response spectrum is given as the minimum value of the following functions of the eigenvalues [Newmark and Hall 1982]:

$$
\begin{aligned}
S_{D1}(\Omega_r) &= C_A / \Omega_r \\
S_{D2}(\Omega_r) &= 16.2 C_A A_A \Omega_r^{-1.36} \\
S_{D3}(\Omega_r) &= C_A A_A / \Omega_r \\
S_{D4}(\Omega_r) &= C_V A_V \sqrt{\Omega_r} \\
S_{D5}(\Omega_r) &= C_D A_D
\end{aligned}
\tag{20}
$$

where the parameters for acceleration, velocity and displacements are

$$C_A = 201.0\ \mathrm{cm/s}^2, \quad C_A = 25.0\ \mathrm{cm/s}, \quad C_D = 18.75\ \mathrm{cm} \tag{21}$$

and the parameters A_A, A_V, A_D are defined by the damping factor h_r of the r th mode as

$$
\begin{aligned}
A_A &= 3.21 - 0.68 \log(100 h_r) \\
A_V &= 2.31 - 0.41 \log(100 h_r) \\
A_D &= 1.82 - 0.27 \log(100 h_r)
\end{aligned}
\tag{22}
$$

Six modes are considered for evaluating the seismic responses of the shear model. The damping factors are proportional to the frequency, and $h_1 = 0.02$.

The columns of the 3-D frame and the beams of each plane frame are divided, respectively, into 15 and 5 groups

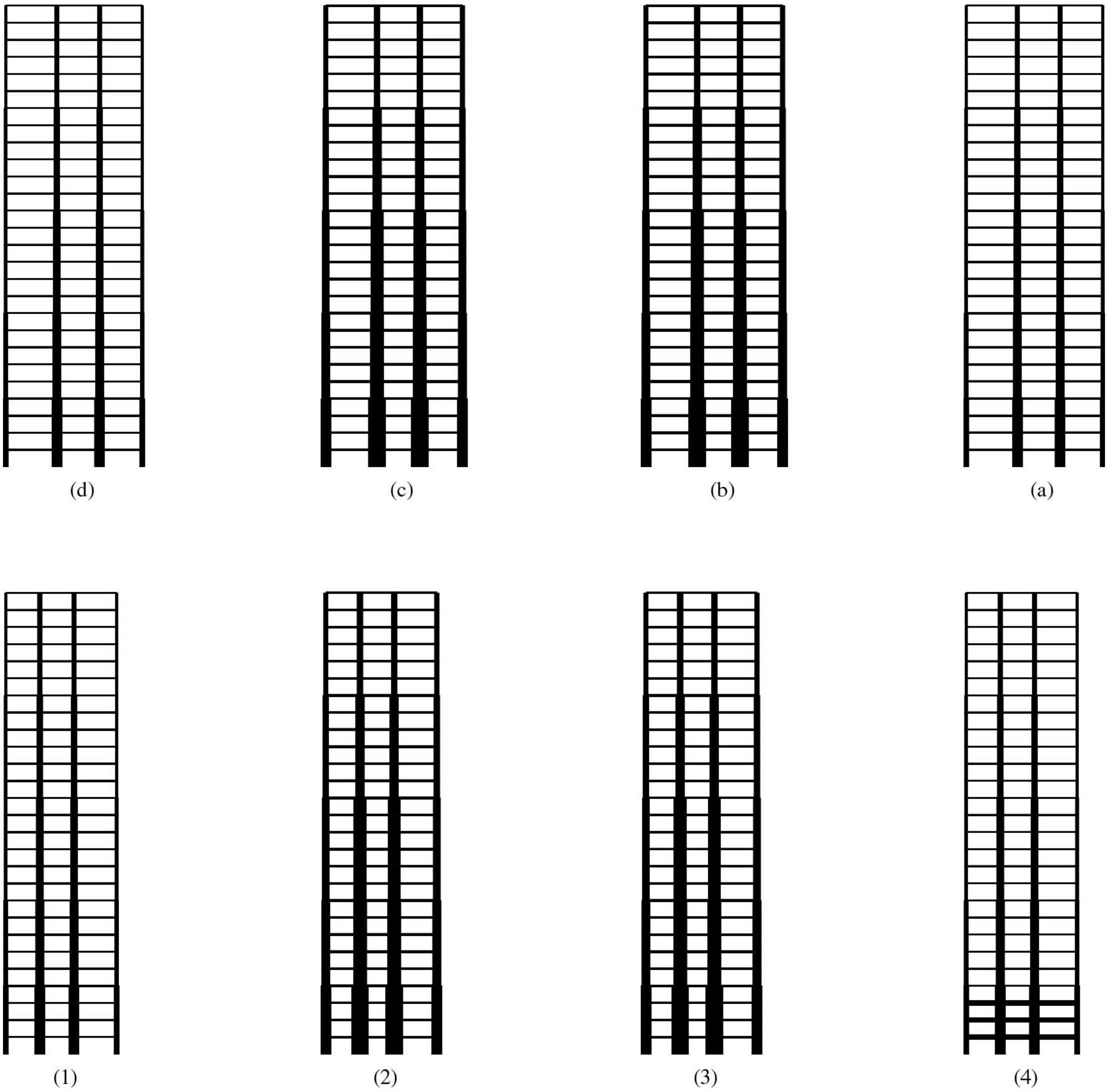

Fig. 2: Optimization results.

with same cross-sectional areas. The package DOT [VR&D 1995] has been used for optimization, and the method of modified feasible directions is used. Optimization has been carried out on Sun Ultra2 (UltraSPARC $300\ \mathrm{MHz} \times 2$). The upper bounds for the maximum interstory drift and the stresses are 2.0 cm and 323.4 MPa, respectively. The parameters for modification of the cross-sectional areas of columns are $\gamma_1 = 1.25$, $\gamma_2 = 0.95$, $\gamma_3 = 1.25$, $\beta = 0.8$ and the initial value of ΔA is $10.0\mathrm{cm}^2$

The cross-sectional area of a column at the first stage is chosen as the mean value of the optimal cross-sectional areas of the columns of the plane frames in two directions. The two level procedure has converged at the fourth step of the upper level problem. The optimal solution at the fourth step is as shown in Fig. 2, where (a)-(d) and (1)-(4) in the figure are as defined in Fig. 1(a), and the width of each member is proportional to the cross-sectional area. Note that the total mass of the beams and columns is 896.36 ton and the mass per $1\,\mathrm{m}^2$ of the floor is 50.387 kg .

The maximum and minimum values of the cross-sectional areas for this case are $1009.36\,\mathrm{cm}^2$ and $79.571\,\mathrm{cm}^2$, respectively. It may be observed from Fig. 2 that the columns in the core have large cross-sectional areas compared with the exterior columns. CPU time for this example is 770.65 sec which is moderately small.

CONCLUSIONS AND DISCUSSIONS

A two-level algorithm has been presented for optimizing a 3D-frame under constraints on seismic responses as well as the static responses under self-weight. First, the design earthquake loads and the optimal interstory stiffness are found from a shear model considering the earthquake responses evaluated by a response spectrum approach. In the lower level, the cross-sectional areas of beams are optimized so that the stiffness of each story is equal to the specified value, and the response stresses are within the limits. In the upper level, the sensitivity analysis of the optimal objective values of the lower level problems is carried out to modify the cross-sectional areas of columns.

It has been shown in the example of a 27-story 3-D building frame that the solution converges in several iterative steps of the upper level problem, and the computational time for obtaining the optimal solution is small enough to be applied in design practice. Note that the lower level problems can be solved parallely, and the CPU time will be drastically reduced in the distributed computing environment.

The story masses should be updated if more accurate solutions are needed. The stresses should be evaluated at the face of the facees of the connections to incorporate more realistic situation. The effect of interaction among the plane frames in the same and different directions should be evaluated for the optimal solution by carrying out a response analysis of the total 3-D frame.

REFERENCES

Arora, J.S. (ed.) (1997). *Guide to Structural Optimization*, ASCE.

Fiacco, A.V. (1983). *Introduction to Sensitivity and Stability Analysis in Nonlinear Programming*, Academic Press.

Friedman, Z. and Fuchs, M.B. (1987). ``Multilevel optimal design of thin-walled continuous beams, *Comp. & Struct.*, **25**, 405-414.

Grierson, D.E. and Chiu, T.C.W. (1984). ``Optimal synthesis of frameworks under multilevel performance constraints,'' *Comp. & Struct.*, **18**(5), 889-898.

Guddat, J. Vasquez, F.G. and Jongen, H.T. (1990). *Parametric Optimization*: *Singularities, Pathfollowing and Jumps*, John Wiley & Sons.

Haftka, R.T. Gördal, Z. and Kamat, M.P. (1990). *Elements of Structural Optimization*, Kluwer Academic Publishers.

Nakamura, T. Tsuji, M. and Takewaki, I. (1993). ``Design of steel frames for specified seismic member ductility via inverse eigenmode formulation,'' *Comput. & Struct.*, **47**, 1017-1030.

Newmark, N.M. and Hall, W.J. (1982). *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, CA.

Ohsaki, M. (1997). ``Optimization of building structural systems using parametric multidisciplinary optimization method, *J. Struct. Engng.*, AIJ, **43B**, 79-88. (in Japanese)

Sobieszczanski-Sobieski and Haftka, R.T (1997). ``Multidisciplinary aerospace design optimization: survey of recent developements,'' *Structural Optimization*, **14**, 1-23.

Sobieszczanski-Sobieski, J. James, B. and Dovi, A. (1985). ``Structural optimization by multilevel decomposition,'' *AIAA J.*, **23**, 1775-1782.

Svensson, B. (1987). ``A substructuring approach to optimum structural design, *Comp. & Struct.*, **25**, 251-258.

VR&D, *DOT User's Manual*, Ver. 4.20. Vanderplaats Research & Development Inc., Colorado Spring.