SEISMIC HAZARD AND DESIGN BY USING ENERGY FLUX

Erdal SAFAK[1] And Steve HARMSEN[2]

SUMMARY

Energy flux provides a dynamic measure of seismic energy, and can be used to characterize the intensity of ground shaking, as well as the response of structures. Energy flux is defined as the amount of energy transmitted per unit time through a cross-section of a soil or a structural medium. It is equal to kinetic energy multiplied by the propagation velocity of seismic waves. For ground motions, the peak and the sum over time of energy flux provide two simple measures of shaking intensity. By definition, energy flux accounts for site amplification. The examples show that energy flux gives a good detail of the shaking intensity in deep sedimentary basins. Energy flux can be used directly as input to structures. For multi-story buildings subjected to vertically propagating plane seismic waves, the flow of energy in the building can be formulated in terms of upgoing and downgoing energy fluxes in each story. The formulation results in energy flux, as well as energy demand and dissipation, time histories at each story of the building. Since it is in discrete-time domain, the formulation is applicable to both linear and nonlinear structures. An example for the methodology is presented by using a 10-story building founded on a two-layer soil medium over bedrock.

INTRODUCTION

Seismic energy is one of the key parameters characterizing the intensity of ground shaking and the magnitude of structural damage. The concept of using energy for seismic hazard assessment and seismic design was first suggested by Housner (1956) more than forty years ago. Recently, new studies on developing energy-based methods for seismic design have appeared in the literature (e.g., Uang and Bertero, 1988; Sucuoglu and Nurtuğ, 1995). Although the concept is far from being adopted by practicing engineers, the energy-based methods will likely be the foundation of future seismic design procedures. Most of the previous studies on the subject have used a static measure of seismic energy, because they were based on the total energy at the end of the earthquake. In this paper, we introduce a dynamic measure of energy, the energy flux, to characterize seismic hazard and structural response.

Energy flux is defined as the amount of energy transmitted per unit time through a cross-section of a medium, and is expressed by the following equation:

$$E(t) = \frac{1}{2}\rho \; A \; v^2(t) \; V$$

where ρ and A are the mass density and the cross-sectional area, $v(t)$ is the velocity response, and V is the propagation velocity of seismic waves. Note that $E(t)$ is equal to kinetic energy multiplied by the propagation velocity.

SEISMIC HAZARD

[1] *Research Structural Engineer, U.S. Geological Survey, Golden, CO, USA*
[2] *Research Physical Scientist, U.S. Geological Survey, Golden, CO, USA*

We characterize seismic hazard by considering the energy flux for a unit area (i.e., $A=1.0$) of the soil layer next to the surface. We define two simple measures of energy flux, the peak value, E_{max}, and the sum over time, E_{sum},. which are

$$E_{\max}(t) = \frac{1}{2}\rho\ V\ [v^2(t)]_{\max} \qquad \text{and} \qquad E_{sum} = \frac{1}{2}\rho\ V \int_{t=0}^{T} v^2(t)\, dt$$

where T is the duration of the earthquake.

In order to use E_{max} and E_{sum} for seismic hazard assessment, we need attenuation equations that show the variation of E_{max} and E_{sum} with magnitude and distance. At present, these equations are not available. However, we can approximate them if we know the attenuation equations and the PDF (probability density functions) of peak velocities. The attenuation equations for peak velocities, including their standard deviations, are available in the literature. For the PDF, we can assume that they are lognormal. For each magnitude and distance, the mean value and the standard deviation of the corresponding attenuation function specify the two parameters of the lognormal distribution. Once we know the PDF of peak velocities, we can easily calculate the PDF of peak squared-velocities, and consequently the PDF of peak energy flux at each magnitude and distance. It can be shown that if the PDF of peak velocities (for given magnitude and distance) is lognormal with parameters a and b, the PDF of peak squared-velocities is also lognormal with parameters $2a$ and $2b$ (Safak, 1999).

To approximate the attenuation equations for E_{sum}, we first note that by definition E_{sum} is related to the mean-square value of $v(t)$. The theory of extreme values of random variables provides closed form expressions for the relationship between the mean-square and the peak values of random variables (e.g., Vanmarcke, 1983). By using these expressions, we can approximate the distribution of the mean-square value of a random variable in terms of the distribution of its peak value. This leads to the distribution function for E_{sum}, and consequently the attenuation functions for the mean value and the standard deviation of E_{sum}.

One of the advantages of using energy flux to characterize ground motions is that energy flux automatically accounts for site amplification, because its definition includes the propagation velocity of seismic waves. It can be shown that for a soil layer over bedrock subjected to vertically-propagating plane shear waves, the soil to rock ratio of peak energy flux is equal to the maximum site amplification (Safak, 1999b).

To give an example of ground motion characterization by energy flux, we present in Figure 1 the distribution of total energy flux in Santa Clara Valley, Northern California, for a hypothetical $M=6.2$ earthquake on the Monte Vista fault. The earthquake and the resulting ground motions were simulated by using a three-dimensional finite-difference model of the valley and the fault rupture (Harmsen et al., 1999). The fault dips about 65 degrees southwest, and the rupture propagates up from the southeast corner to the northwest corner of the fault plane as shown schematically in the figure. The areas with large values of energy flux generally correspond to the deepest layers of the sediments in the basin. For comparison, we present in Figure 2 the 3-second PSRV (pseudo-spectral response velocities) for the same simulation. The PSRV also show increased values for the deep sedimentary regions, but fails to convey the detail and the resolution captured by the energy flux. In particular, a large area over the Cupertino basin exhibits very distinct high-energy flux regions, but fairly uniform PSRV values.

STRUCTURAL RESPONSE

Energy flux can be used directly as input to a structure, which makes it possible to study the dynamics of energy flow in the structure, including the time variations of energy demand and dissipation. Such a detailed knowledge on energy provides important new tools for seismic design and safety evaluation.

For multi-story buildings on layered soil media and subjected to vertically propagating plane shear waves, we can combine the building and the soil layers into a single layered system by assuming that the building is an extension of the soil media and each story of the building represents another layer. This model allows us to propagate the seismic energy starting from the bedrock and to account for the energy losses due to soil-structure interaction. Figure 1 shows a schematic view of such a layered model. The input energy flux, $E_0(t)$, enters the system at the bedrock-soil interface, and propagates upward through the soil and building layers. The energy flow in the system is composed of two components, the upgoing energy flux and the downgoing energy flux, as shown in Fig. 3a. The upgoing and downgoing energies are partially reflected and partially transmitted to the next layer as they cross layer interfaces. Reflections and transmissions are characterized by the reflection and transmission coefficients, α and β, as shown schematically in Fig. 3b. To determine α and β, we first calculate the reflection and transmission coefficients for velocities by solving the two equations depicting the equality of displacements and the equilibrium of shear forces at the interface. We then determine α and β by inserting the velocity reflection and transmission coefficients in Eq. 1. It can be shown that the reflection and transmission coefficients for energy flux are independent of the direction of energy flow. In other words, α and β at an interface are identical for upgoing and downgoing energy fluxes. This is in contrast to the reflection and transmission coefficients for velocities, which are different for upgoing and downgoing waves. The details of the calculations as well as the final expressions for α and β can be found in Safak (1999a, 1999b). For interfaces with concentrated mass, such as the soil-foundation interface or the building floors, α and β are frequency dependent, whereas for interfaces with no concentrated mass, such as soil layers, α and β constant. It can be shown that a concentrated mass at an interface acts as a low-pass filter; it transmits the low frequency components to the next layer while blocking the high frequency components (Safak, 1999a). Note that due to the principle of conservation of energy $\alpha+\beta=1$; that is, the sum of reflected and transmitted energies is equal to the input energy.

To formulate the flow of energy in the layers, we select two energy flux variables in each layer: $U(t)$, the upgoing energy flux at the top of the layer, and $D(t)$, the downgoing energy flux at the bottom of the layer. Fig. 3c shows three consecutive layers (layers $j-1$, j, and $j+1$) with their energy flux variables. The upgoing energy flux $U_j(t)$ is equal to the transmitted portion of the upgoing energy flux $U_{j-1}(t)$ from the top of layer $j-1$, plus the reflected portion of the downgoing energy flux $D_j(t)$ from the bottom of layer j. Similarly, the downgoing energy flux $D_j(t)$ is equal to the transmitted portion of the downgoing energy flux $D_{j+1}(t)$ from the bottom of layer $j+1$, plus the reflected portion of the upgoing energy flux $U_j(t)$ from the top of layer j. When upgoing and downgoing energies travel across a layer, their amplitudes are reduced due to damping in the layer. Damping is characterized by an exponential function of the following form: $A_j(f)=\exp(-\pi f\, \tau_j / Q_j)$, where τ_j and $1/Q_j$ are the wave travel time and the damping coefficient, respectively, of layer j, and f denotes the frequency in Hz. The reduction in the energy flux amplitudes is equal to $A^2_j(f)$. By using appropriate damping coefficients, $A_j(f)$ can be shown to represent various forms of damping, such as viscous and structural damping. More on damping is given in Safak (1999a). Based on the foregoing arguments, we derive the damped energy flux equations for layer j as follows:

$$U_j(t) = A_j^2(f)\cdot[\alpha_{j-1}(f)\cdot D_j(t-\tau_j) + \beta_{j-1}(f)\cdot U_{j-1}(t-\tau_j)]$$
$$D_j(t) = A_j^2(f)\cdot[\alpha_j(f)\cdot U_j(t-\tau_j) + \beta_j(f)\cdot D_{j+1}(t-\tau_j)]$$

For given $E_0(t)$, these equations can be solved recursively starting from the bedrock and continuing upward. The net energy flux at the top and the bottom of layer j are

$$E_{top,j}(t) = U_j(t) + D_j(t-\tau_j)$$
$$E_{bot,j}(t) = U_j(t-\tau_j) + D_j(t)$$

The energy flux entering layer j is $U_j(t-\tau_j)$ from the bottom, and $D_j(t-\tau_j)$ from the top. The energy flux exiting layer j is $D_j(t)$ from the bottom, and $U_j(t)$ from the top. The energy dissipated per second, $E_{dis,j}(t)$, by layer j is the difference of the entering and exiting energy fluxes, which is

$$E_{dis,j}(t) = U_j(t-\tau_j) + D_j(t-\tau_j) - U_j(t) - D_j(t)$$

The dissipated total energy, $E_{tot,j}$, in layer j is

$$E_{tot,j} = \int_0^T E_{dis,j}(t)\, dt$$

$E_{tot,j}$ is a key parameter for design and safety analysis. Every story in a building should be able to dissipate this energy without failure. Energy dissipation is accomplished through elastic and inelastic displacements, damping, and other energy dissipation devices (e.g., friction and fluid dampers).

As an example, we subjected a 10-story building founded on a two-layer soil medium to one of the rock-site records from the Northridge earthquake. The detail on the characteristics of the building and the input can be found in Safak (1999a). Figure 4 shows the upgoing energy fluxes at the top and the bottom of the soil and building layers, and the dissipated energy in each layer.

CONCLUSIONS

Energy flux represents the amount of seismic energy transmitted per unit time through a cross-section of a medium, and is equal to kinetic energy multiplied by the propagation velocity of seismic waves. Energy flux provides a simple means to study the dynamics of seismic energy flow in the ground and structures. The peak or the integral of energy flux can be used to characterize ground motions. By definition, energy flux automatically accounts for site amplification. Using ground energy flux as input, we can investigate the time varying characteristics of energy flow, energy demand, and energy absorption throughout a structure. Such a detailed knowledge of energy provides new tools to advance the methodologies used for seismic design and safety assessment. For multi-story buildings founded on layered soil media and subjected to vertically propagating seismic waves, we combine the soil layers and the building into a single layered system and develop energy flux equations that account for soil-structure interaction. The methodology presented is applicable to both linear and nonlinear structures.

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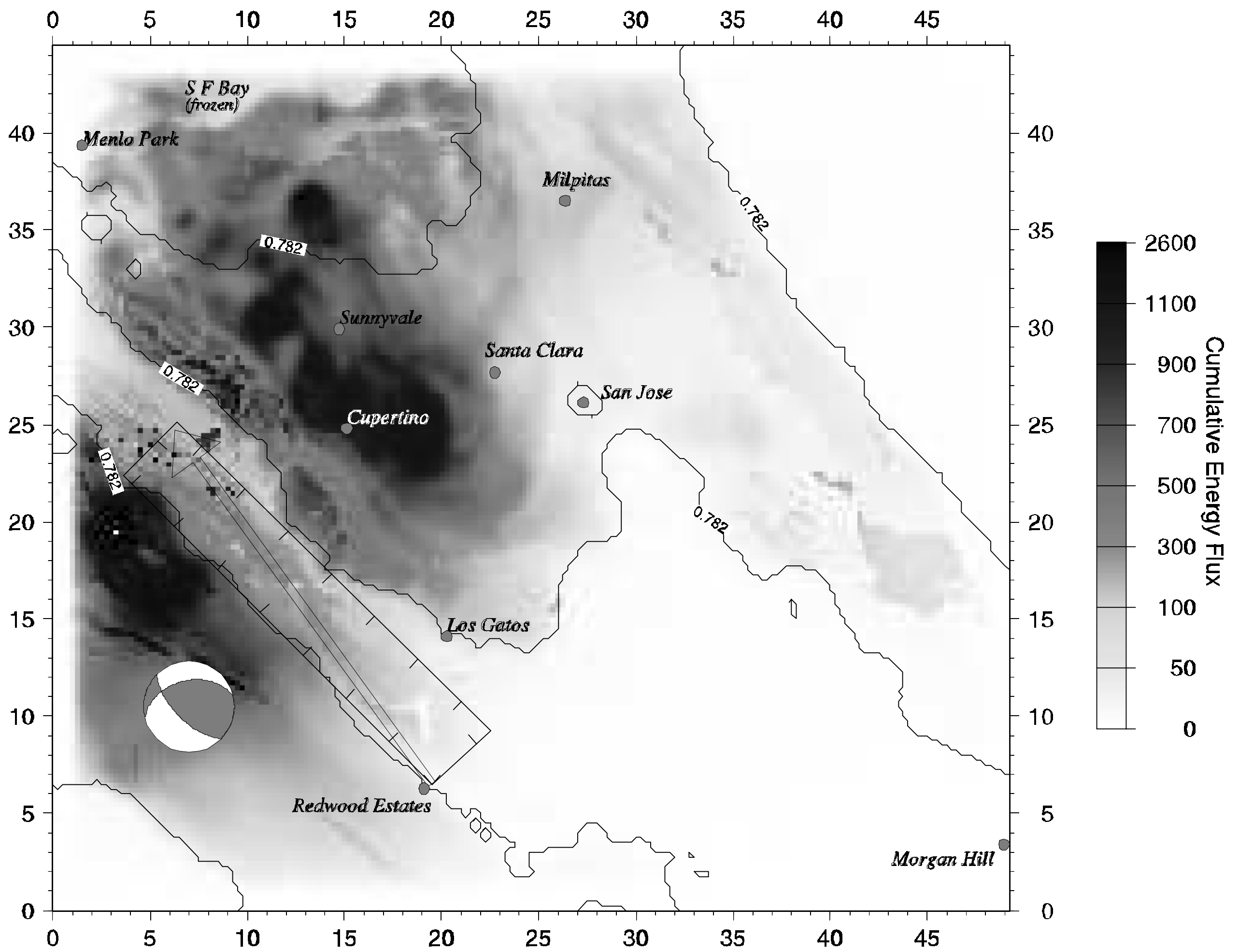

FIGURE 1 - Distribution of total energy flux in Santa Clara Valley, calculated from a 3D finite-difference model, for a magnitude 6.2 earthquake on the Monte Vista fault.

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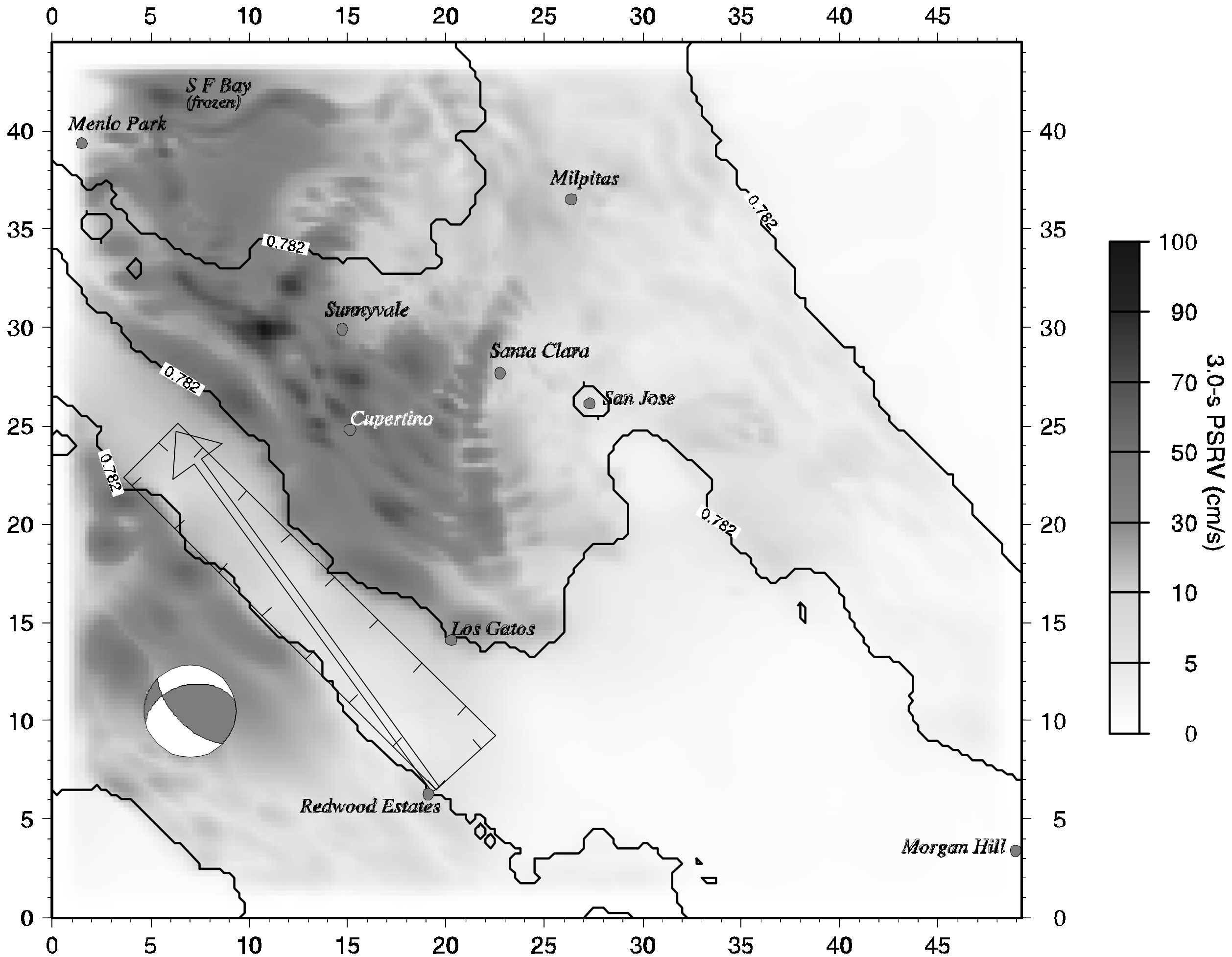

FIGURE 2 - Distribution of pseudo-spectral response velocity at 3.0 seconds in Santa Clara Valley, calculated from a 3D finite-difference model, for a magnitude 6.2 earthquake on the Monte Vista fault.

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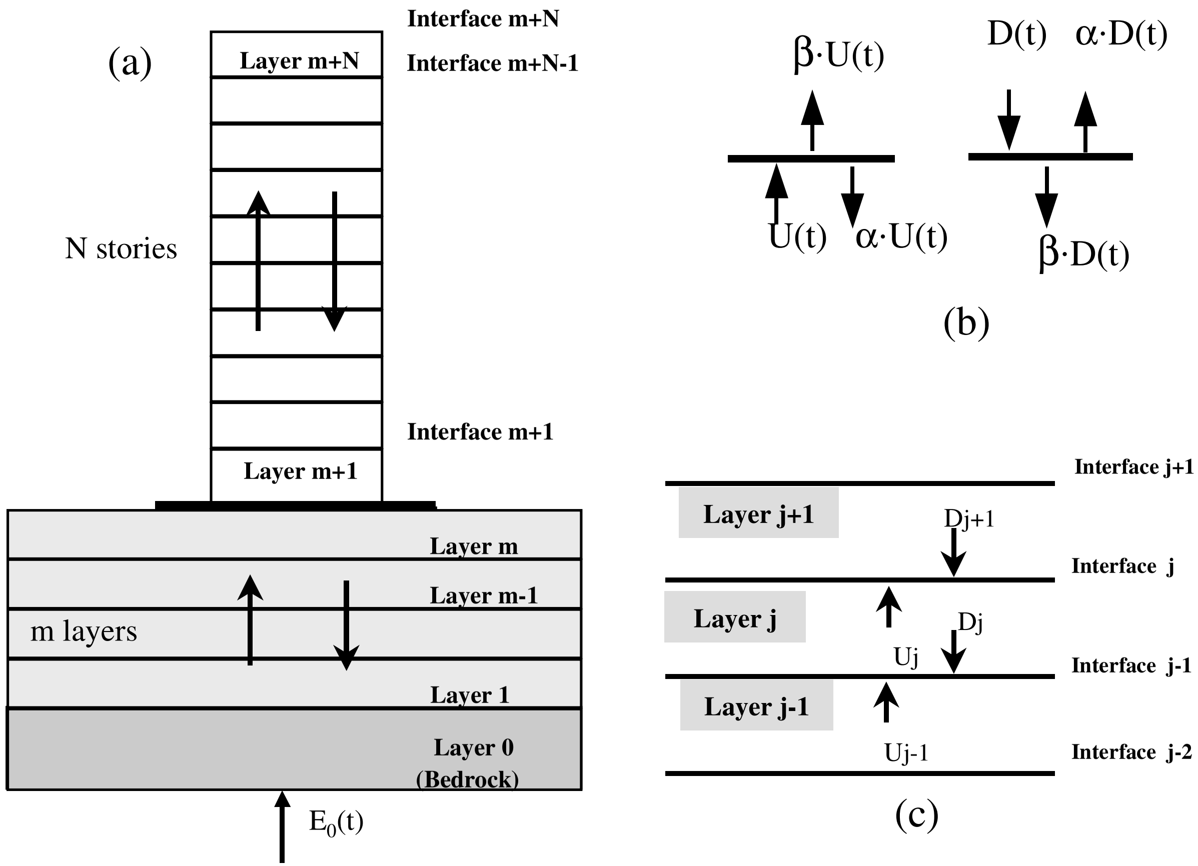

FIGURE 3 - Bedrock-soil-building system: (**a**) layers, interfaces, and upgoing and downgoing waves, (**b**) reflection and transmission of upgoing and downgoing waves, and (**c**) three consecutive layers with upgoing and downgoing waves.

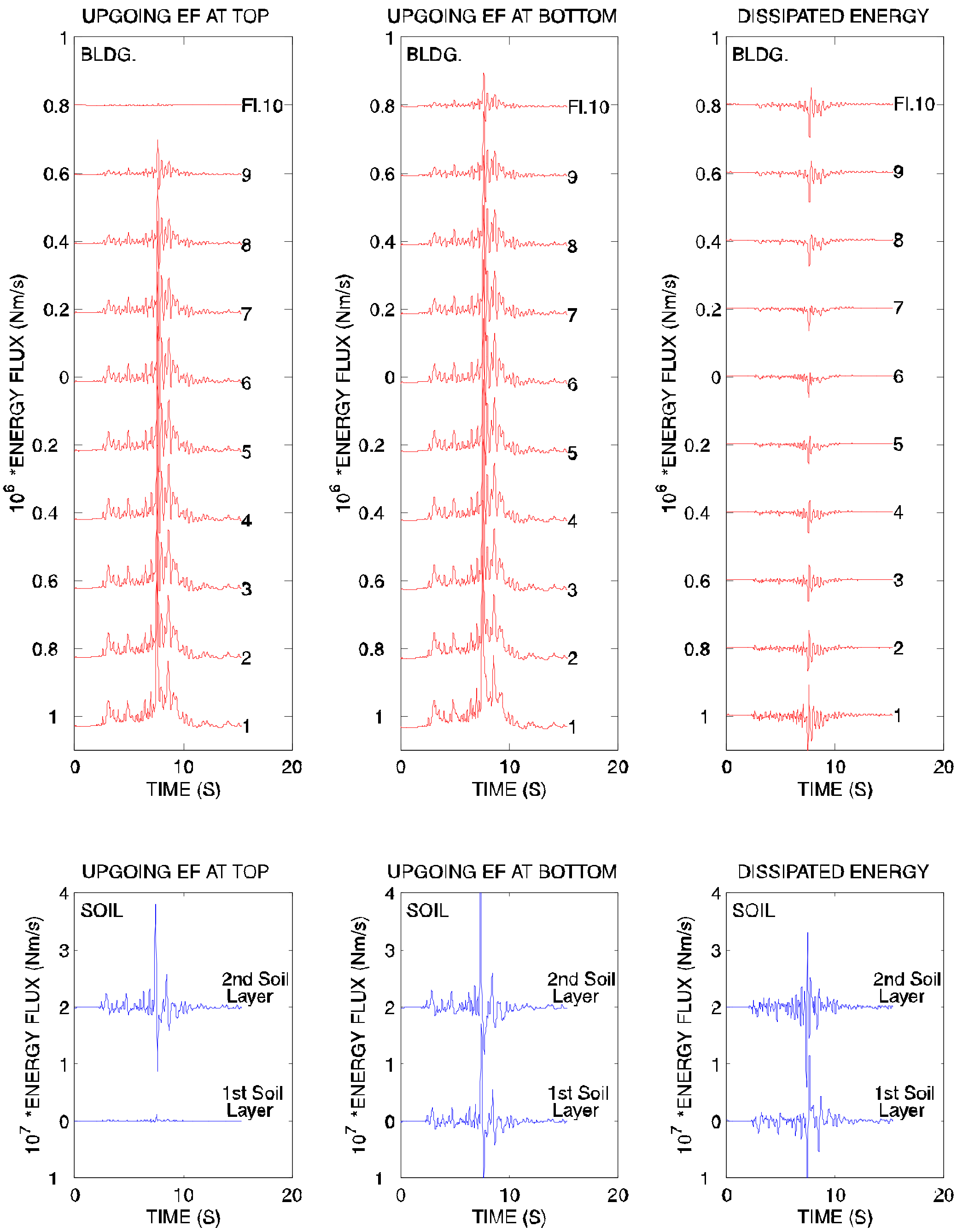

FIGURE 4 - Upgoing energy fluxes at the top and the bottom of layers, and the dissipated energy in a 10-story building founded on 2-layer soil media and subjected to a recorded ground motions from the Northridge earthquake.

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Abstract

Earthquake safety of structures depends largely upon the structure's ability to absorb the seismic energy that is transmitted through the foundation. Development of energy-based methods for seismic hazard and structural response has been a popular research topic in recent years. Most of the studies on this subject, however, are based on the total energy transmitted from the ground to the structure at the end of the earthquake, and do not account for the dynamic nature of the problem. A better understanding of seismic energy and its effect on structural response can be obtained by studying the flow of energy in the structure during an earthquake.

Energy flow can best be described by energy flux, which is the amount of energy transmitted per unit time through a cross section of a soil or a structural medium. For seismic waves, the energy flux is defined by the following equation:

$$\text{Energy flux} = E(t) = \frac{1}{2}\rho\, A\, v^2(t)\, V$$

where ρ is the mass density, A is the cross-sectional area, $v(t)$ is the velocity response, and V is the propagation velocity of seismic waves. Since the definition incorporates propagation velocity, the energy flux automatically accounts for site effects. The seismic energy transmitted per unit time into the structure through the foundation can be calculated in terms of the energy flux in the ground and the energy reflection coefficient at the soil-foundation interface. For buildings founded on layered soil media and subjected to vertically-incident plane shear waves, energy flux can be formulated in the discrete-time domain by modeling the building as an extension of the layered soil medium, and considering each story as another layer in the wave propagation path. The formulation results in a pair of simple finite-difference equations for each layer, which can be solved recursively starting from the bedrock.

Energy flux provides a convenient tool to investigate the dynamics of seismic energy and its propagation in structures. Seismic hazard, including site effects, can be assessed by using various measures of energy flux, such as the peak or the sum over time. Both linear and nonlinear structures can be analyzed using energy flux. Numerical examples show the advantages of using energy flux to characterize ground motions and structural response.

[1] *Research Structural Engineer, U.S. Geological Survey, Golden, CO, USA*
[2] *Research Physical Scientist, U.S. Geological Survey, Golden, CO, USA*

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