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DERIVATION OF SEISMIC RESPONSE SPECTRA FROM THE COMBINATION OF FUZZY LOGIC THEORY AND A NON-LINEAR NUMERICAL MODEL

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SUMMARY

In this paper, we present a methodology that combines the fuzzy set theory and a non-linear numerical model to determine seismic response spectra of soil columns. Using the numerical model, a database of response spectra is constituted. It corresponds to different sets of the fuzzy parameters (thickness of layers, plasticity index, shear wave velocity, etc.). A fuzzy relation is then established between the input and the output, considering the model as a black box. Using the fuzzy inference concept, new response spectra can then be immediately derived for a new set of fuzzy parameters, without any further calculations with the numerical model.

INTRODUCTION

Numerical techniques (e.g. FEM, BEM) are more and more used for the resolution of complex engineering problems. The development of powerful computing tools has widely contributed to this expansion. However, the accuracy of their output is penalised by the data-inherent uncertainties. Traditionally, probabilistic methods are used to handle uncertainties in engineering problems. Nevertheless, there are two sources of uncertainties: statistical variability of parameters and, the fuzziness of information about parameters.

The Fuzzy Set Theory, which was first introduced by Lotfi A. Zadeh in 1965, is based on the idea that the human knowledge cannot be simultaneously wide and precise. It can be considered as a means for quantifying the ambiguity and including it in a machine-understandable logic. The most innovating aspect is the fuzzy reasoning in which both variables and the relations between them are considered as fuzzy. In civil engineering, the pioneer works were presented by [Blockley, 1979], [Brown, 1979] and [Yao, 1980]. In civil engineering, many analyses were carried out using this method and interesting results were presented by [Valliappan and Pham, 1993], [Chuang, 1995] and [Elton *et al.*, 1995]. However, most of the applications were limited to the resolution of problems in which uncertain parameters intervene in relatively simple equations. For more complex formulas, the interval arithmetic leads to an extremely large, even infinite, band in which the solution varies. This is due to the process of minimisation and maximisation in each operation on the fuzzy parameters. Moreover, in non-linear numerical models, the solution is obtained after successive operations and thus the fuzzy output depends on the accuracy of the discretization schemes in time and space domains.

In the proposed paper, we present a methodology that combines the fuzzy set theory and a non-linear numerical model to determine seismic response spectra. First, using the numerical model, a database of response spectra is constituted. It corresponds to different sets of the fuzzy parameters and serves to construct their membership functions. A fuzzy relation is then established between the input and the output, considering the model as a black box. Using the fuzzy inference concept, new response spectra can then be immediately derived for a new set of fuzzy parameters, without any further calculations with the numerical model.

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BASIC PRINCIPLES OF FUZZY LOGIC

The fuzzy set theory can be considered as a complement to the theory of probabilities. Indeed, there are two sources of uncertainties: randomness and fuzziness. The latter represents the vagueness, imprecision or the ambiguity of the information. These two aspects are fundamentally different: a random or probabilistic phenomenon becomes certain after the realisation of an event whereas the ambiguity in the definition of a concept persists and is time-insensitive. The need to consider imprecise or fuzzy data has led L.A. Zadeh to introduce the theory of possibilities. His idea was to replace classical -or crisp- sets, for which the membership is a binary notion (i.e. yes/no or 0/1), by fuzzy sets, for which the transition from membership to non-membership is smooth and not abrupt, i.e. sets whose elements have a membership degree between 0 and 1.

Fuzzy set theory

Fuzzy sets and membership function

A fuzzy set A of a universe X is characterised by a membership function μ_A which affects to each element *x* of *X* a real number of the interval [0, 1], $\mu_A : X \longrightarrow [0,1]$. It is usually represented as:

$$A = \{\mu_A(x)/x\}, x \in X \qquad (1)$$

$\mu_A(x)$ is the membership degree of *x* to *A*: the closest is this value to unity, the highest is the membership of *x* to *A*. The symbol "/" is used as a separator between the element *x* and its membership degree $\mu_A(x)$. If *A* is a crisp set, this function can take only one of the two values 0 or 1 whether *x* belongs to *A* or not.

The construction of the membership function of a parameter requires statistical data or the opinion of an expert who gives an estimation of the interval where the parameter can vary as well as its most likely value.

Operations on fuzzy sets

The most common operations on fuzzy sets are given in equations 2 to 9. The operations which are usually applied to classical sets, such as the union, the intersection and the complement, are extended to fuzzy sets:

- algebraic sum: $\mu_{A\oplus B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x)$ (2)
- algebraic product: $\mu_{A\otimes B}(x) = \mu_A(x).\mu_B(x)$ (3)
- bounded sum: $\mu_{A+B}(x) = (\mu_A(x) + \mu_B(x)) \wedge 1$ (4)
- bounded difference: $\mu_{A-B}(x) = (\mu_A(x) - \mu_B(x)) \vee 0$ (5)
- union: $\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x)$ (6)
- intersection: $\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x)$ (7)
- complement: $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$ (8)
- cross product: $\mu_{AxB}(x,y) = \mu_A(x) \wedge \mu_B(y)$ (9)

where $\wedge$ and $\vee$ represent the minimum and the maximum: $a \wedge b = \min(a,b)$ and $a \vee b = \max(a,b)$.

α–cuts and the extension principle

The extension principle permits to apply the classical mathematical functions (multiplication, inverse, power, exponential...) to fuzzy sets. It is therefore possible, for simple mathematical equations, to calculate analytically fuzzy quantities. However, in many cases, the formulas are not explicit and are heavy to use. One of the possibilities to bypass this difficulty is the method of α–cuts. The α–cut of a fuzzy set *A* is the set of all the elements whose membership degree is greater or equal to α :

$$A_\alpha = \{x, \mu_A(x) \geq \alpha\}, \qquad \alpha \in (0,1] \qquad (10)$$

A fuzzy set can be entirely reconstructed from its α–cuts, i.e. its membership function can be reconstituted from the bounds of the intervals obtained by α–cuts. It is then possible to use the α–cuts for the definition of a fuzzy set instead of using its membership function. This feature is very useful when arithmetic operations on fuzzy sets are involved. Indeed, for each level α, classical interval calculus can be used to determine the corresponding α–cut of the result. The final fuzzy result is then reconstituted from the successive intervals.

Operations on fuzzy numbers

Direct operations on membership functions of fuzzy numbers are feasible while the complexity of the analytical functions is limited and the number of fuzzy variables is reduced. When those conditions are not satisfied, which is the most common case, it is more convenient and efficient to use the α–cut method. The following equations give the most usual operations on α–cuts of fuzzy numbers:

- fuzzy addition: $A_\alpha(+)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (+) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right]$ (11)
- fuzzy subtraction: $A_\alpha(-)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (-) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} - b_2^{(\alpha)}, a_2^{(\alpha)} - b_1^{(\alpha)}\right]$ (12)
- fuzzy multiplication: $A_\alpha(\times)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (\times) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} \times b_1^{(\alpha)}, a_2^{(\alpha)} \times b_2^{(\alpha)}\right]$ (13)
- fuzzy division: $A_\alpha(\div)B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (\div) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] = \left[a_1^{(\alpha)} \div b_2^{(\alpha)}, a_2^{(\alpha)} \div b_1^{(\alpha)}\right]$ (14)

Fuzzy reasoning

Most of the applications of fuzzy logic are limited to fuzzy arithmetic, which consist in operations on fuzzy numbers and involve interval calculus combined with the concept of α–cuts. This is suitable when the output can be easily determined using the fuzzy input and a mathematical model. However, in most cases, the numerical models are too sophisticated and their use requires a good mastery of the theory. Moreover, after an analysis with a set of parameters, the engineer may sometimes need to know what would be the output if he chooses another value of an input parameter. To avoid heavy repetitive calculations with the numerical model, one fast alternative is the use of *fuzzy reasoning*, which is based on the concepts of *fuzzy relations* and *fuzzy inference*.

Fuzzy relations

A relation *R* between a fuzzy set *X* and a fuzzy set *Y* is a fuzzy set of the cross product *X* x *Y* whose membership function is defined as $\mu_R : X \text{x} Y \longrightarrow [0,1]$. Using the notation of fuzzy sets, *R* can be expressed as:

$$R = \int_{X \text{x} Y} \mu_R(x,y)/(x,y); \quad x \in X, y \in Y. \tag{15}$$

If *R* and *S* are fuzzy relations of *X* x *Y* and *Y* x *Z* respectively, the max–min composition *R* o *S* of the two relations is a fuzzy relation of *X* x *Z* whose membership function is defined as:

$$\mu_{R\text{o}S}(x,z) = \bigvee_y \left(\mu_R(x,y) \wedge \mu_S(y,z)\right). \tag{16}$$

Fuzzy inference

Expressions like "*x* is large" or "the flow is low" are very common in fuzzy logic. They are called *fuzzy propositions*, and constitute the basic element of fuzzy reasoning. Their general form is: "*x* is *A*".

The expression: "IF x is A THEN y is *B*", where *A* and *B* are two fuzzy sets, is called *fuzzy implication*. This implication, $R = A \rightarrow B$, can be considered as a binary relation, whose membership function is $\mu_{A\rightarrow B}$. There are several formulas giving $\mu_{A\rightarrow B}(x,y)$ in terms of $\mu_A(x)$ and $\mu_B(y)$; the most famous are those proposed by Zadeh and Mamdani:

- *Ra* (Zadeh): $a \rightarrow b = 1 \wedge (1 - a + b)$ (17)

- *Rc* (Mamdani): $a \rightarrow b = a \wedge b$ (18)

Fuzzy reasoning consists in deducting a fuzzy set *B'* which corresponds to a new fuzzy set *A'* assuming the same relation *R* that links the fuzzy sets *A* and *B*:

rule	IF x is A THEN y is B
fact	x is A'
conclusion	y is B'

The conclusion *B'* can be obtained from the composition of the fuzzy set *A'* and the fuzzy condition $A \rightarrow B$:

$$B' = A' \text{o} R = A' \text{o} (A \rightarrow B) \tag{19}$$

The fuzzy inference rules can be generalised to more complex situations in which multiple conditions are involved. The following form is very common:

rule	IF x is A AND y is B THEN z is C
fact	x is A' AND y is B'
conclusion	z is C'

The conclusion *C'* can be obtained in the same way as with elementary rules by using a composition of the fuzzy set *A'* x *B'* and the fuzzy condition $A \text{x} B \rightarrow C$:

$$C' = (A' \text{x} B') \text{o} R(A, B; C) = (A' \text{x} B') \text{o} (A \text{x} B \rightarrow C) \tag{20}$$

This approach is still valid when the number of elementary conditions is greater than two, and when there is a set of rules, which can be elementary or complex. Generally, when several conditions or rules are involved, one has to simplify the expressions so as to reduce them to their most elementary form.

Defuzzification

In general, the output of a fuzzy logic analysis is a fuzzy set expressed as a distribution of possibilities [Dubois and Prade, 1980]. It is therefore necessary to "defuzzify" this fuzzy result to extract one or several ready-to-use crisp results. Among the methods of defuzzification, the moment's method is one of the most famous. It consists in replacing the fuzzy result by the centre of gravity of the area under its membership curve. The abscissa of the obtained point is the defuzzified result, which can be used for subsequent stages of the analysis; the ordinate may be seen as the degree of confidence on this result.

This method has the advantage of furnishing a sole result, which can constitute the final expected solution or an intermediate datum of the analysis (e.g. the value of an input parameter of a numerical model). However, its drawback is the loss of information while replacing the fuzzy set by a point.

FUZZY RESPONSE SPECTRA

Problem description

The determination of the seismic response spectrum of a soil profile follows several steps: description of the soil profile, numerical simulation of the soil response using an input accelerogram and finally, use of the surface acceleration as the input acceleration to calculate the response spectrum of a single degree of freedom oscillator.

In this section, we present a methodology that combines the fuzzy set theory and a non-linear numerical model to estimate, in a fast way, seismic response spectra. The numerical model permits, for a given soil profile and an input accelerogram, the evaluation of the surface acceleration and then the corresponding response spectrum, assuming a cyclic elasto-plastic behaviour for the soil layers. A complete description of this model can be found in [Mellal, 1997]. First, using the numerical model, a database of response spectra is constituted. It corresponds to different sets of the fuzzy parameters (thickness of layers, plasticity index, shear wave velocity, etc.) and serves to construct their membership functions. A fuzzy relation is then established between the input and the output, considering the model as a black box. Using the fuzzy reasoning concept, new response spectra can then be immediately derived for new sets of fuzzy parameters, without any calculation with the numerical model.

Input data

The simulations with the numerical model take into account several factors such as the soil layers thickness, the shear wave velocities, the soil type and the seismic signal duration. Table 1 lists the values of these parameters. The soil type is taken into account by the means of the plasticity index PI to which correspond G/D-γ curves, which give the shear modulus and the damping ratio as a function of the cyclic shear strain.

Nine deterministic simulations were carried out (combinations h = 20, 50, 80 m and PI = 0, 30, 100) using the numerical model. Figure 1 shows the normalised response spectra that correspond to the computed surface accelerations at the top of soil profiles. These spectra constitute the database for the subsequent analyses.

Table 1: soil profiles characteristics

accelerogram:	duration = 20 s, PGA = 0.2 g
bedrock:	ρ = 2100 kg/m3, *Vs* = 800 m/s, *Vp* = 1500 m/s
soil profiles:	ρ = 1500 kg/m3, *Vs* = 100 m/s, *h* = 20, 50, 80 m, PI = 0, 30, 100

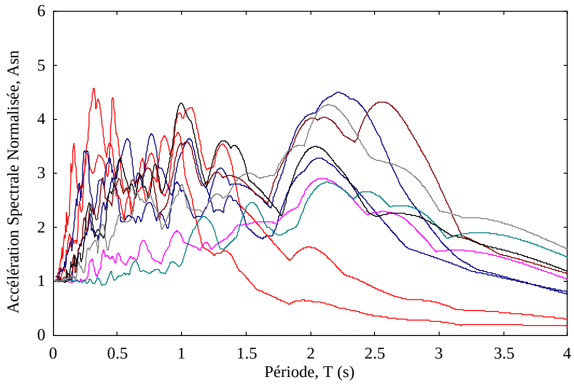

Figure 1: Normalised response spectra obtained by numerical simulations

Membership function of the fuzzy parameters

In this section, we consider the thickness and the plasticity index of the soil profile as fuzzy parameters. All the other parameters are constant and their values are those given in the previous section. Figure 2 gives the membership function of the thickness h and the plasticity index PI, considered as fuzzy parameters. The intervals of variation of h and PI are respectively [20,80] and [0,100] with the highest membership degree (α = 1) attributed to h = 50 m and PI = 30.

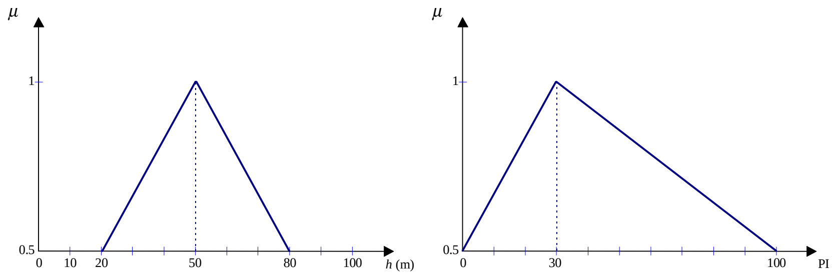

Figure 2: Membership function of *h* and PI

Calculus of the fuzzy response spectrum

In figure 3 are presented the response spectra corresponding to the α–cuts $\alpha = 0.5$ and $\alpha = 1$ of the fuzzy parameters h and PI. For $\alpha = 0.5$, $h \in [20,80]$ and $\mathrm{PI} \in [0,100]$; the envelope curve was obtained from the results of nine deterministic simulations which correspond to the different combinations of h and PI (h = 20, 50, 80 m and PI = 0, 30, 100). For $\alpha = 1$, only one spectrum is calculated: the one corresponding h = 50 m and PI = 30. To be more rigorous, for each α–cut, the resulting envelope spectrum should contain all the spectra corresponding to the combination of the intervals of variation of the fuzzy parameter h and PI. A larger number of simulations is thus recommended for a better accuracy of the resulting envelope spectra.

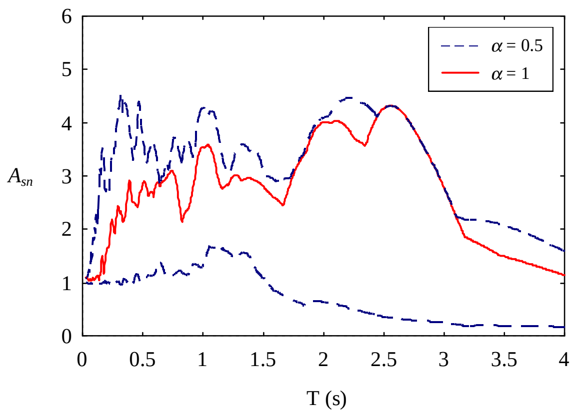

Figure 3: Fuzzy response spectrum

For each period T, the corresponding membership function can be established from its α–cuts. Figure 4 gives the membership function of the fuzzy spectrum for the period T = 1.5 s.

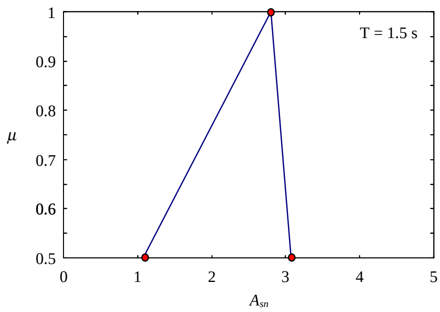

Figure 4: Membership function of the fuzzy spectrum (T = 1.5 s)

Derivation of new response spectra using fuzzy reasoning

In this section, fuzzy reasoning is used to determine the fuzzy response spectra that corresponds to two new fuzzy sets $\underset{\sim}{h}'$ and $\underset{\sim}{\mathrm{PI}}'$ whose membership functions are represented on figure 5. The fuzzy inference rules permit to express the new fuzzy spectra in terms of the new fuzzy parameters and a relation between the initial input and output (equation 21). Using Zadeh's relation *Ra* (equation 17) and the max–min composition, $\underset{\sim}{A}'_{sn}$ can be expressed as the union of the elementary conclusions which correspond to each input parameter (equation 22):

$$\underset{\sim}{A}'_{sn} = (\underset{\sim}{h}' \wedge \underset{\sim}{\mathrm{PI}}') \circ R(\underset{\sim}{h}, \underset{\sim}{\mathrm{PI}}; \underset{\sim}{A}_{sn}). \tag{21}$$

$$\underset{\sim}{A}'_{sn} = \left[\underset{\sim}{h}' \circ R_a(\underset{\sim}{h}; \underset{\sim}{A}_{sn}) \right] \cup \left[\underset{\sim}{\mathrm{PI}}' \circ R_a(\underset{\sim}{\mathrm{PI}}; \underset{\sim}{A}_{sn}) \right] = \underset{\sim}{A}'^{h}_{sn} \cup \underset{\sim}{A}'^{\mathrm{PI}}_{sn} \tag{22}$$

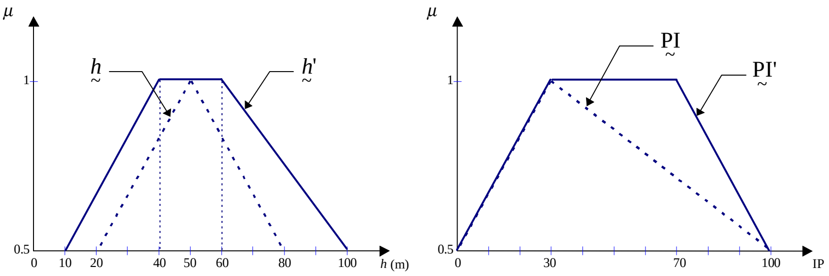

Figure 5: Membership function of *h'* and PI'

The elementary relations between the input, $\underset{\sim}{h}$ and $\underset{\sim}{\mathrm{PI}}$, and the output $\underset{\sim}{A}_{sn}$, also called elementary fuzzy matrices, are presented hereafter. The results are shown for the period T = 1.5 s, the process is obviously the same for any other value.

$R^h = h$

	A_{sn} 1.08	2.80	3.07
10	1	1	1
20	1	1	1
40	0.667	1	0.667
50	0.5	1	0.5
60	0.667	1	0.667
80	1	1	1
100	1	1	1

and

$R^{\mathrm{PI}} = \mathrm{PI}$

	A_{sn} 1.08	2.80	3.07
0	1	1	1
30	0.5	1	0.5
70	0.786	1	0.786
100	1	1	1

(23)

The elementary conclusions are obtained by combining the membership functions of the fuzzy sets $\underset{\sim}{h}'$ and $\underset{\sim}{\mathrm{PI}}'$ with the elementary fuzzy matrices R^h and R^{PI} using the max–min composition. The resulting fuzzy spectral acceleration $\underset{\sim}{A}'_{sn}$ which corresponds to $\underset{\sim}{h}'$ and $\underset{\sim}{\mathrm{PI}}'$ is finally obtained from equation 22 by calculating for each element, the maximum of the membership degrees on the elementary fuzzy sets $\underset{\sim}{A}'^{h}_{sn}$ and $\underset{\sim}{A}'^{\mathrm{PI}}_{sn}$. Figure 6 shows the membership function of the obtained spectral acceleration; the membership function of the initial spectral acceleration corresponding to $\underset{\sim}{h}$ and $\underset{\sim}{\mathrm{PI}}$ is represented as well. The former is obtained by fuzzy reasoning whereas the latter is obtained by simulation with the numerical model.

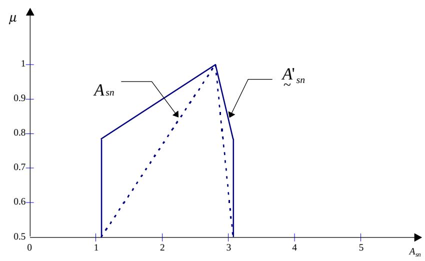

Figure 6: Membership function of the fuzzy spectrum obtained by fuzzy inference (T = 1.5 s)

Response spectra for non-fuzzy parameters - Defuzzification

The membership functions of the input parameters and the fuzzy results can be used to have a quick estimate of the response spectra corresponding to a particular set of parameters with crisp values, e.g. h=70 and PI=50. First, the membership degrees of *h* and PI are determined from their respective membership functions, in this example $\mu_{h=70} = 0.667$ and $\mu_{PI=50} = 0.857$. The corresponding fuzzy response spectra is then obtained by excluding from the initial response spectra all the values which are greater than the minimum of the membership degrees of *h* and PI (figure 7). This result is then defuzzified by calculating the centre of gravity of the area under the membership function curve of the resulting spectrum (figure 8). The abscissa of this point, $A_{sn} = 2.19$, is the estimated spectral acceleration, and the ordinate, $\alpha = 0.578$, is the "degree of confidence" on this value.

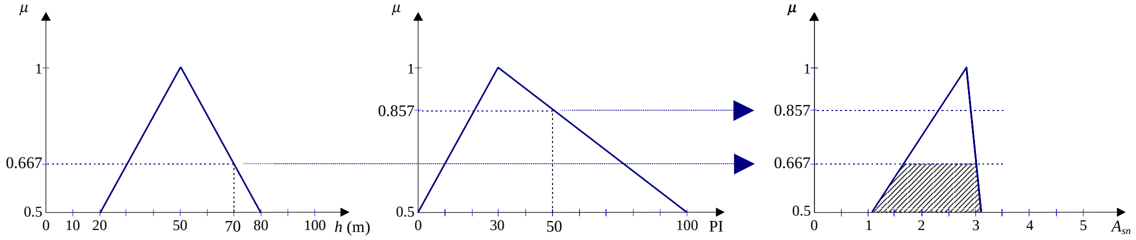

Figure 7: Deduction of a fuzzy spectrum for non-fuzzy parameters

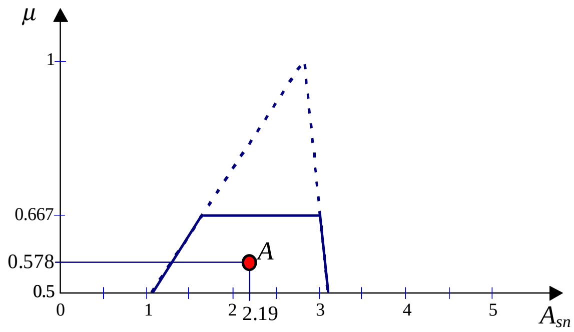

Figure 8: Defuzzification of the fuzzy spectrum

CONCLUSION

The combination of fuzzy set theory and a non-linear numerical model has permitted to evaluate seismic response spectra using the concept of fuzzy reasoning. This approach is interesting in situations in which fast estimation of response spectra is needed or when there is a lack of information concerning one or several parameters. In addition to the importance of the database, the accuracy of such analyses depends directly on the fuzzy inference rule and it is therefore essential to select the most suitable among the available methods. The proposed methodology can be easily adapted to other problems for which the complexity of the resolution model prevents the application of fuzzy arithmetic directly on the parameters.

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