RESPONSE OF BASE ISOLATED STRUCTURE IN CHAOTIC DYNAMIC SYSTEM UNDER EARTHQUAKE MOTION WITH LARGE AMPLITUDE

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SUMMARY

This paper describes nonlinear response of base isolated structure subjected to ground motion with large amplitude from the viewpoint of chaotic dynamics. Since the isolators sustaining the superstructure show hardening characteristics in hysteretic curves under these circumstances, the dynamic system should be expressed mathematically by a nonlinear equation with polynomial terms on restoring force. It is called Duffing's equation when it is proportional to the cube of displacement and exciting ground motion is sinusoidal wave. Authors extend theoretically it to the equation on multi-mass system installed isolators at the base and solve it numerically by means of Runge-Kutta's method. Here ground motions are sinusoidal wave, El Centro 1940 NS, the 1995 Hyogo-Ken-Nanbu Earthquake and Miyagi-Ken Oki earthquake of 1978 modified so that their maximum velocities may reach 150 kine. Analytical results on one-mass system are compared with those acquired by solving equivalent linear equation, where the initial natural periods of the system are set 3.0 seconds and dampings are set 5%. The difference between them is remarkable when subjected to El Centro 1940 NS. The nonlinear earthquake responses are also affected by initial displacement and small difference between natural periods. Finally they conclude the nonperiodic motions exist essentially in nonlinear response of the base isolated structure and viscous damping is effective to make it stable and predictable. The idea presented here may leads to effective control of the structure and show us a new interpretation of irregularity and complexity of actual earthquake damages.

INTRODUCTION

Over the last two decades, chaos theory has been studied in the many fields of science and nowadays complexity of vibration problems in chaotic dynamic system is gradually revealed. Ueda described chaotic phenomena in the dynamic system governed by Duffing's equation in the late 1970's[6] and innovative research works on subharmonic resonance and chaotic motion of offshore structures were summarized and published by Thompson and Stewart[5]. These phenomena existing essentially in nonlinear system have never been discussed in the past dynamic analyses in the field of earthquake engineering. However recent development of response control devices made it possible to provide structures with any hysteretic characteristics and viscous dampings and some of them may necessitate changing the past idea in order to face real nonlinearity due to the effect of the new structural elements produced artificially. Under these situations authors reviewed the past experiments and modelings on seismic isolator and found that a vibratory system with hardening characteristics caused by large deflection is idealized by Duffing's equation.

This paper presents a numerical method solving directly the vibratory equation governing nonlinear behaviors of base isolated structure under seismic motion with extremely large amplitude which causes hardening in hysteretic curves of isolators, showing analytical results and a new interpretation of complexity of the nonlinear phenomena.

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NONLINEAR EQUATIONS ON BASE ISOLATED STRUCTURE

The nonlinear vibratory equation of one-mass system which denotes a structure sustained by isolators with hardening characteristics can be written as

$$m\ddot{y} + c\dot{y} + k_1 y + k_2 y^3 = -m\ddot{a}_0 = F \quad (1)$$

where y, a_0, m and c are relative displacement, exciting ground motion, mass and damping respectively and k_1 and k_2 are elastic and nonlinear spring constants. They are set by idealizing experimental hysteretic curves by Tada, Sakai, Takayama, Shimizu and Ando[3, 4] as shown in Figure1. Here the Eq.(1) is called Duffing's equation when the exciting force F is sinusoidal wave.

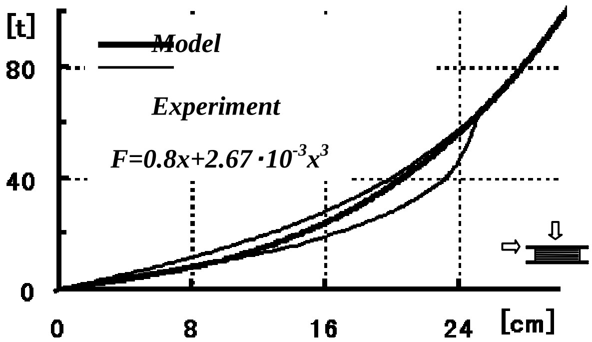


Figure 1: Idealized nonlinear hysteretic curve.

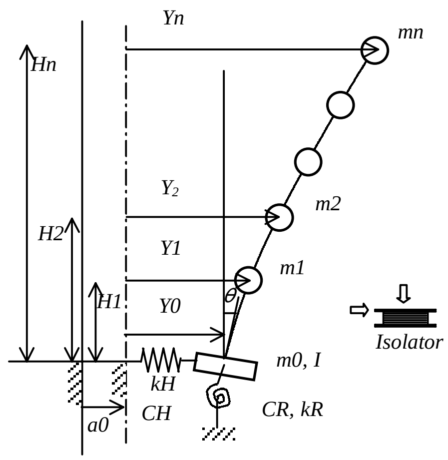


Figure 2: Multi-mass system with isolators.

Extending the above idea to a vibration problem of the multi-storied structure installed seismic isolators on the base as shown in Figure 2, the vibratory equation can be written as

$$[m]\{\ddot{y}\} + [c]\{\dot{y}\} + [k]\{y\} + \{\theta^3\} = -[m]\{\ddot{a}_0\} \quad (2)$$

where

$$\{y\} = \{\theta, y_0, y_1, \cdots, y_n\}^T \quad (3)$$

and

$$\{\theta^3\} = \{k_{R2}\theta^3, k_{H2}y_0^3, 0, 0 \cdots, 0\}^T \quad (4)$$

Here $[m]$,$[c]$ and $[k]$ denote mass, damping and stiffness matrices and $\{y\}$ and $\{a_0\}$ do displacement and ground motion vectors. The constants k_{R2} and k_{H2} are springs for rocking and sway motions related to nonlinear terms of the equation. How to obtain the values from experiments on isolators is described afterward. The details of the above matrices and vectors are

$$[m] = \begin{bmatrix} I & & & \\ & m_0 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix} \quad (5)$$

$$[k]=\begin{bmatrix} k_{R1}+\sum_i\sum_j k_{ij}H_iH_j & \sum_i\sum_j k_{ij}H_i & -\sum_i k_{i1}H_i & \cdots & -\sum_i k_{in}H_i \\ \sum_i\sum_j k_{ij}H_i & k_{H1}+\sum_i\sum_j k_{ij} & -\sum_i k_{i1} & \cdots & -\sum_i k_{in} \\ -\sum_j k_{1j}H_j & -\sum_j k_{1j} & k_{11} & \cdots & k_{1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\sum_j k_{nj}H_j & -\sum_j k_{nj} & k_{n1} & \cdots & k_{nn} \end{bmatrix} \quad (6)$$

$$[c]=\gamma[k] \quad (7)$$

and

$$\gamma = 2 \cdot h/\omega \quad (8)$$

.

Eqs.(1) and (2) are arranged as

$$\ddot{x} = -c/m \cdot \dot{x} - \left(k_1 x + k_2 x^3\right)/m - \ddot{a}_0 \quad (9)$$

and

$$\{\ddot{y}\} = -[m]^{-1}[c]\{\dot{y}\} - [m]^{-1}[k]\{y\} - [m]^{-1}\{\theta^3\} - \{\ddot{a}_0\} \quad (10)$$

,

which are solved numerically by means of Runge-Kutta's method.

ACCURACY OF NUMERICAL ANALYSIS

Since the mathematical solution of nonlinear equations is normally unknown, numerical approaches are inevitable for pursuing the complex behaviors governed by them. Therefore the accuracy of analysis, especially how to distinguish numerical errors from chaotic phenomena becomes very significant because the nonperiodic motion in phase space somewhat looks like them. Here the numerical errors in Runge-Kutta's method are discussed by means of changing time increment using a well-known free vibration problem of one-mass system. Figure 3 shows orbits of free vibration of the one-mass system with no damping given initial displacement of 10 centimeters. The calculation was repeated 10^8 times at maximum. Since it has to continue moving on a ellipsoidal orbit everlastingly, time increment more than 0.001 second shown in Figure 3 (d) are recommended.

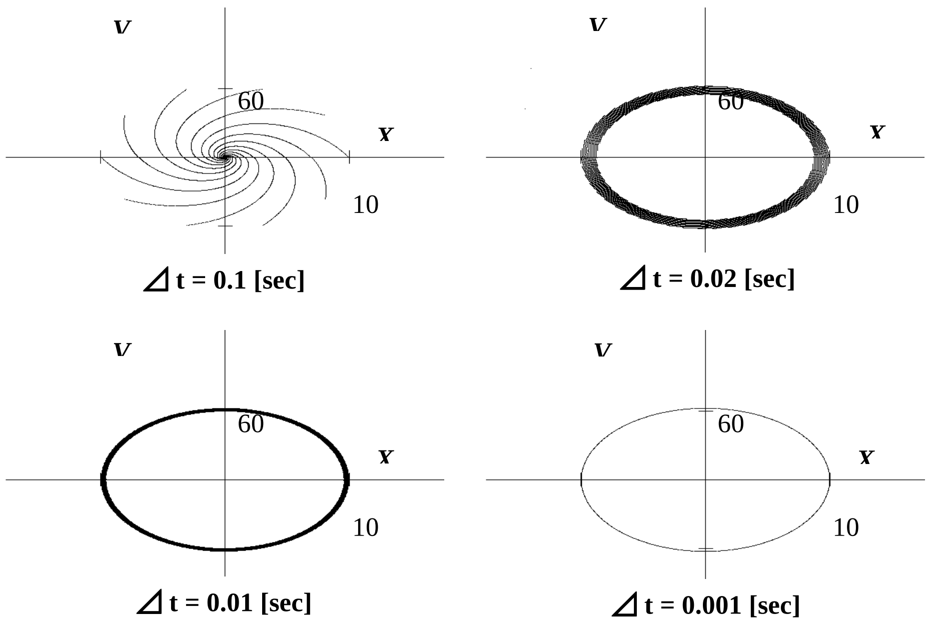


Figure 3: Phase projection of orbit of one-mass system free vibrating.

Authors also discussed the issue, comparing numerical results with those acquired from Newmark's β-method (β=1/4) using a five story building model [1]. The time increment less than 0.005 second was recommended.

NONLINEAR RESPONSE OF ONE-MASS SYSTEM WITH HARDENING HYSTERESIS

Next analytical results acquired chaotic dynamically are compared with those based on linear equation where its stiffness is replaced considering the status of response displacement. Here exciting ground motions are sinusoidal wave, El Centro 1940 NS, the 1995 Hyogo-Ken-Nanbu Earthquake and Miyagi-Ken Oki Earthquake of 1978 modified so that their maximum velocities may reach 150 kine. The time increment for computing is 0.001 second. Figure 4 shows comparison of nonlinear hysteretic curve proportional to the cube of displacement with one of linear approximation combining spring constants in eight small domains. The difference between them is obviously small as far as it can be seen as a static load-displacement relationship. However it is quite large and significant in dynamics since it determine the essential characteristics on the solution of differential equations mathematically .

Figure 5 shows comparison of nonlinear response with one by the linear approximation where the stiffness is replaced every 10 centimeters in isolator's displacement. The initial natural period of the system is set 3.0 seconds and damping is set 5%. Resonant sinusoidal waves with the periods of 1.5 and 3.0 seconds are imposed as shown in Figure 5 (a) and (b). The difference between them are small in time history respectively.

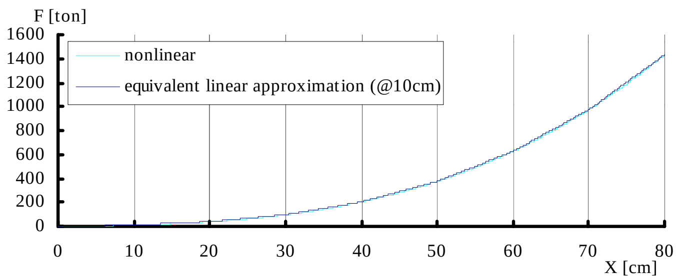


Figure 4: Nonlinear load-displacement relationship due to hardening and linear approximation.

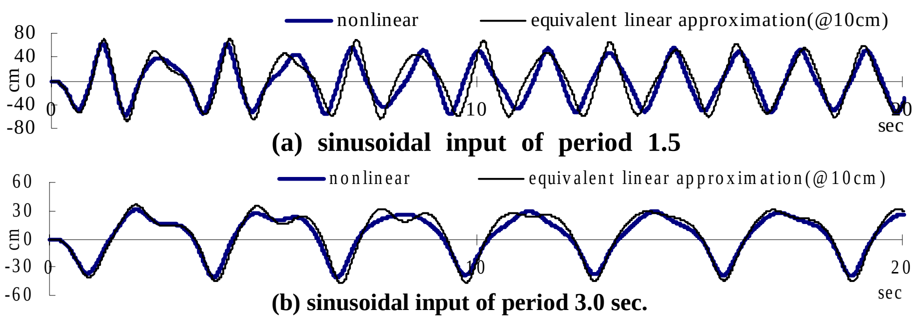
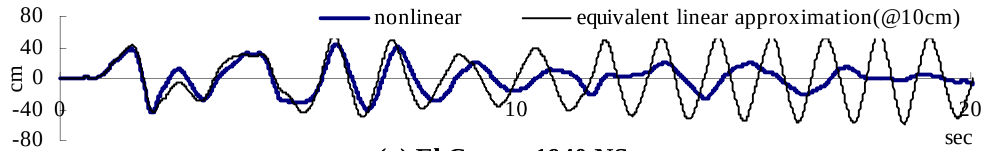


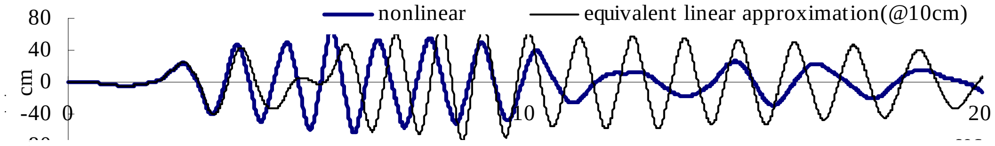
Figure 5: Comparison of nonlinear response with linear approximation in time history (sinusoidal wave).

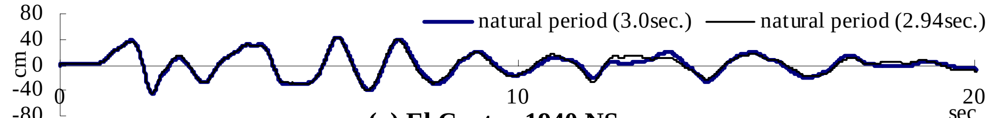
Figure 6 shows the comparison of the system with the initial natural period of 3.0 seconds when subjected to El Centro 1940 NS (a) and the 1995 Hyogo-Ken-Nanbu Earthquake EW at JR Takatori Station (b) [2]. Generally difference between responses is remarkable and seems to become larger and larger with the passing of the time. However this trend cannot always be recognized and they are sometimes in good accordance with each other. In facts, the difference like that was not seen in the case of the 1995 Hyogo-Ken-Nanbu Earthquake NS component. Thus nonlinear phenomena in chaotic dynamic system are complex and hard to predict by means of superposing results from sinusoidal excitation and it seems difficult to cope with it using linear approximation even if a number of spring constants are adopted.

Next influence of small difference in the natural periods to the response is examined using the same model with the initial natural period of 3.0 seconds and 5% damping. Figure 7 shows the comparison of responses in time history, where the difference between the periods is 2%. It is quite small when subjected to El Centro NS shown in Figure 7 (a). However it becomes remarkable in the later phase in the case of Hyogo-Ken-Nanbu Earthquake EW (b) and Miyagi-ken Oki Earthquake of 1978 NS (c). Similarly Figure 8 shows influence of initial displacement of 5 centimeters to the responses, which can be though a problem related to accuracy of construction and structural damage due to earthquake. The trend of analytical results is the almost same with the above one shown in Figure 7.

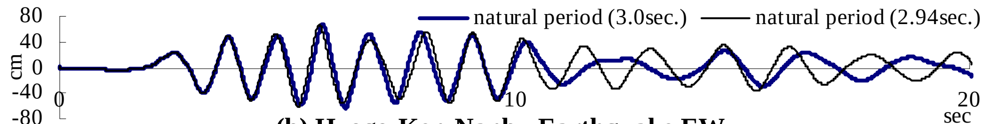


(a) El Centro 1940 NS

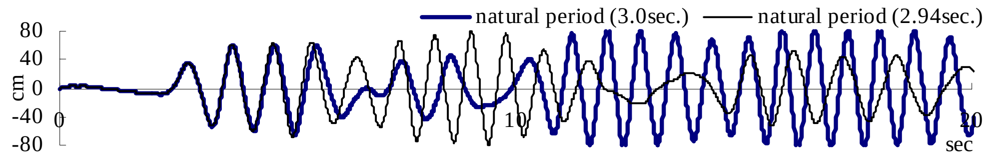




(a) El Centro 1940 NS

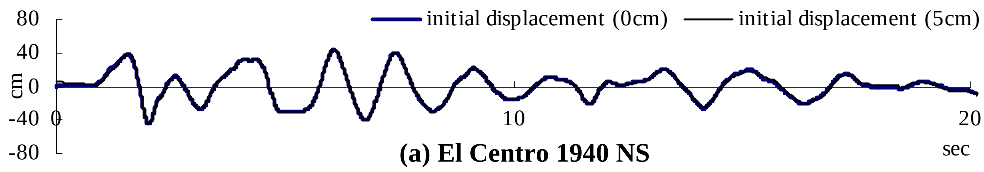


(b) Hyogo-Ken-Nanbu Earthquake EW

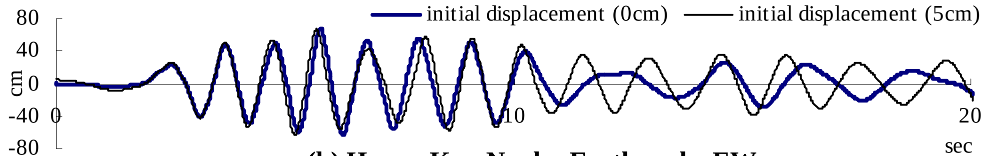


(c) Miyagi-Ken Oki Earthquake of 1978 NS

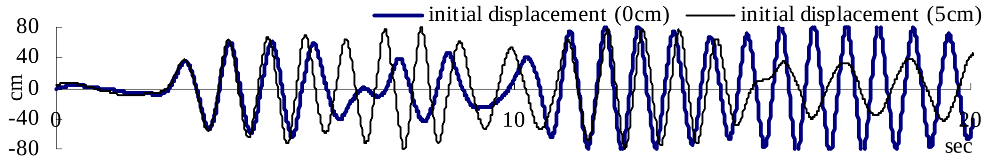
Figure 7: Influence of small difference between natural periods to nonlinear response.



(a) El Centro 1940 NS



(b) Hyogo-Ken-Nanbu Earthquake EW



(c) Miyagi-Ken Oki Earthquake of 1978 NS

Figure 8: Influence of initial displacement to nonlinear response.

NONPERIODIC BEHAVIORS IN UNDAMPED SYSTEM

It is the most significant issue that we have some difficulty in predicting nonlinear response of full-scale buildings because the system respond sharply to initial conditions and the modeling always requires some approximations. Therefore it is necessary to make it stable for them and frequency components of earthquakes.

Here nonperiodic responses of one-mass system with no damping to sinusoidal motions are discussed in order to reveal the essential characteristics of it. Figure 9 shows Poincare sections of responses of the system with the initial period of 3.0 seconds. The periods of exciting motions are 2.5, 3.0, 3.5 and 4.0 seconds. Obviously attractors are different from each other and those shown in Figure 9 (a), (c) and (d) seem chaotic, which suggest nonperiodic motions in phase space governed by certain rule. Figure 10 shows phase projections of responses of the system with 5% damping to the waves of period 3.0 and 3.5 seconds. The point representing response is attracted into the orbit and its Poincare section should be a point attractor, which shows periodic and stable vibration. Viscous damping plays an important role in making it stable and predictable.

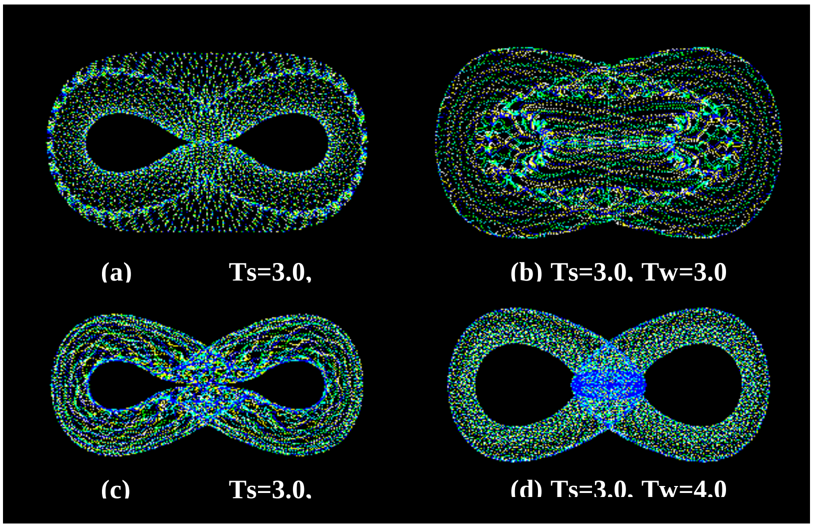
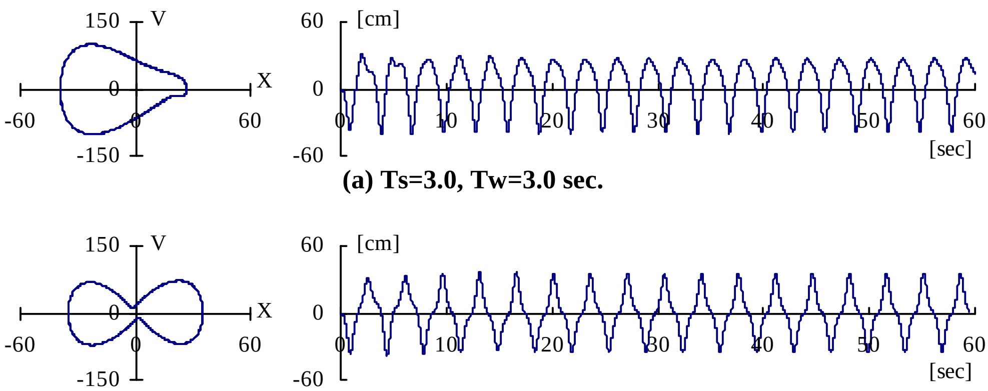


Figure 9: Poincare section of nonlinear responses in phase space to sinusoidal excitation.



(a) Ts=3.0, Tw=3.0 sec.

(b) Ts=3.0, Tw=3.5 sec.

Figure 10: Phase projection and time history of nonlinear responses of damped one-mass system.

ANALYTICAL RESULTS ON A FIVE STORY BUILDING IN CHAOTIC DYNAMIC SYSTEM

Here nonlinear response of a five story and base isolated building shown in Figure 11 is described. Table 1 shows mass, height of story and damping. Size of column section is 60 x 60 centimeters and one of beam section is 30 x 60 centimeters. Restoring forces of an isolator in lateral and vertical directions are expressed as

$$F_H = 0.8x + 2.67 \times 10^{-3} x^3$$
$$F_V = 333y + 7259y^3 \qquad (11)$$

(12)

based on the experiments by Tada, Sakai, Takayama, Shimizu and Ando[3, 4]. Assuming rigid base, moment M and shear force F causing rocking and sway motions of the building with isolators are given as

$$M(\theta) = \left(\Sigma_i k_{V1} \times l_i^2\right)\theta + \left(\Sigma_i k_{V2} \times l_i^4\right)\theta^3 = k_{R1}\theta + k_{R2}\theta^3 \tag{13}$$

$$F(x) = \left(\Sigma_i k_{H1}\right)x + \left(\Sigma_i k_{H2}\right)x^3 = k_{H1}x + k_{H2}x^3 \tag{14}$$

where ik_{V1}, ik_{H1}, ik_{V2} and ik_{H2} denote linear and nonlinear spring constants of i-th isolator and li is distance from it to the center of rocking motion. Thus k_{R1}, k_{R2}, k_{H1}, k_{H2} in Eq.(4) and (6) in chapter 2 are obtained.

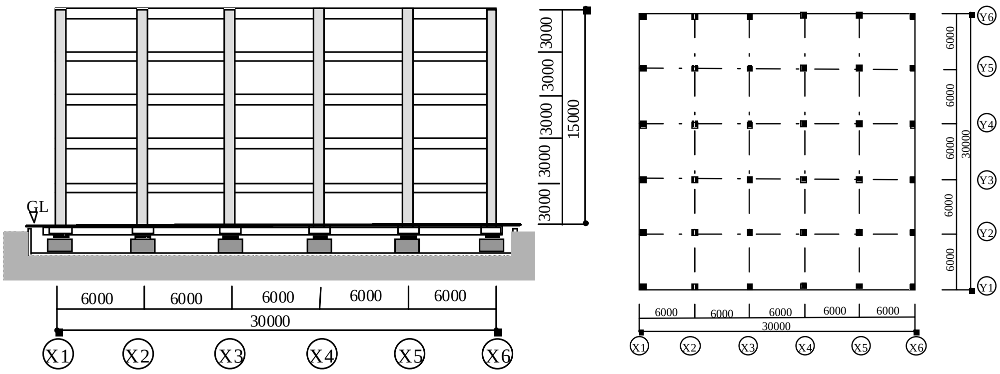
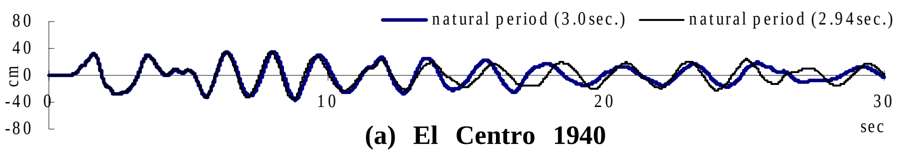


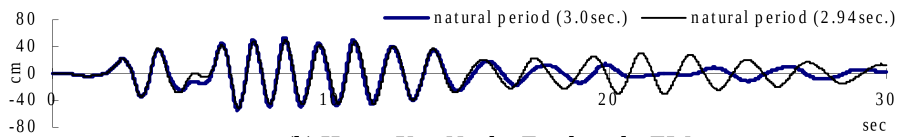
Figure 11: Analytical model of a five story building sustained by seismic isolators.

Table 1: Analytical parameters of a five story building

stoy	mass	hight	damping
5	0.92 ton*sec^2/cm	3.00 m	0.05
4	0.92 ton*sec^2/cm	3.00 m	0.05
3	0.92 ton*sec^2/cm	3.00 m	0.05
2	0.92 ton*sec^2/cm	3.00 m	0.05
1	0.92 ton*sec^2/cm	3.00 m	0.05
sway	0.92 ton*sec^2/cm	3.00 m	0.05
rocking	1.38*10^6 ton*cm^2	3.00 m	0.05
column - 600mm x 600mm, beam - 300mm x 600mm			



(a) El Centro 1940



(b) Hyogo-Ken-Nanbu Earthquake EW

Figure 12: Influence of small difference between natural periods to nonlinear response of a five story building.

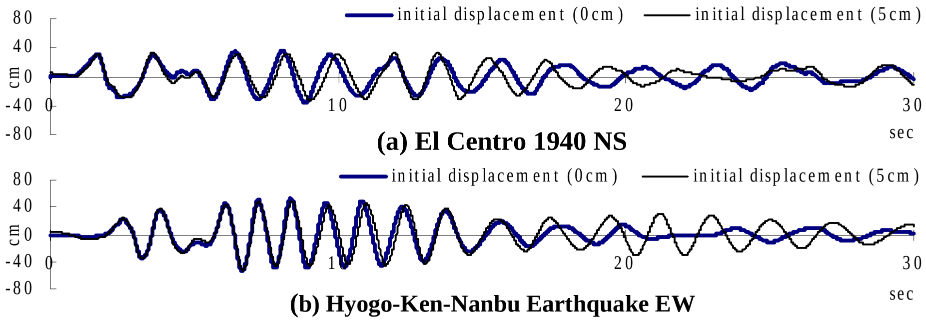


Figure 13: Influence of initial displacement to nonlinear response of a five story building.

Here influence of small difference between the first natural periods to response is examined using the above structure designed so that the period and damping may be adjusted to 3.0 seconds and 5%. Figure 12 shows the comparison of responses in time history, where the difference between the periods is 2%. It is comparatively small when subjected to El Centro NS shown in Figure 13 (a). However it becomes rather remarkable in the later phase after 18 seconds in the case of Hyogo-Ken-Nanbu Earthquake EW (b). Similarly Figure 13 shows influence of initial displacement to the response and the trend of the figure is the same as described above.

CONCLUDING REMARKS

1. Nonlinear earthquake responses in chaotic dynamic system are complex and hard to predict by means of superposing results from sinusoidal excitations. It seems difficult to estimate all of them using linear approximation even if a number of linear spring constants are prepared.
2. Some numerical results show influence of small difference between initial natural periods to nonlinear response becomes larger and larger with the passing of the time. Influence of initial displacement to it, which can be thought a problem related to accuracy of construction and earthquake damage, also shows the same trend. The similar results can be recognized in an example of a five story building.
3. Nonperiodic behaviors exist essentially in nonlinear response of the system with hardening hysteresis and some of them are chaotic. Viscous damping is effective to make them stable and predictable.

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