0572

FUNDAMENTAL ASPECTS OF SITE RESPONSE DETERMINED FROM INVERSION OF VERTICAL ARRAY DATA

Laurie G BAISE[1] And Steven D GLASER[2]

SUMMARY

We analyse site response using parametric autoregressive moving-average (ARMA) models to quantify the effect of individual soil intervals on the through passing wave. This method allows us to relate effects of site response from different sites. The formulation is statistically robust and allows for estimation of uncertainty of the model parameter estimates. To analyse site response spatial consistency across a site or similar sites, the site response model from one site is tested for applicability at a similar soil interval at a site of common geologic character by quantifying the error of prediction.

In this paper, we examine the earthquake ground motion data sets for vertical arrays at four deep soil sites: Port Island, Rokko Island, and Chiba, Japan; and Lotung, Taiwan. We use this analysis to examine the variability of site response at geologically similar sites. At the Chiba site, multiple vertical arrays have been installed over a broad area, so the site response is examined and compared at horizontal separations of 15 m, 143 m and 247 m. At 15-m separation, the site response is virtually identical; as the separation increases, however, the response diverges. A single predictive model can be used for all boreholes, with error increasing with horizontal distance. A similar comparison is made between the Port Island and the Rokko Island vertical arrays, which are separated by approximately 5 km and are both man-made islands in the same sedimentary basin. According to our analyses, the site response estimated using the vertical array recordings at the two islands is similar. The Port Island site response amplifies the ground motion slightly more than observed at Rokko Island, but the Port Island filter could be used to adequately predict the Rokko Island site response. The Chiba and Lotung site responses were compared and found to be dissimilar although the geologic profile showed consistencies.

INTRODUCTION

Site response is an issue in engineering seismology that has traditionally been examined using spectral ratios comparing soil sites to nearby rock sites. With access to vertical array data and improved inversion methods, we now have the opportunity to examine site response on more detailed levels. By analysing site response at a variety of locations with vertical arrays, we can begin to deconstruct the physics of site response and decipher which characteristics of a site are the most influential in determining response during an earthquake. Are the material properties (shear modulus and damping coefficient) of primary importance? or depth to bedrock? or the soil profile (geometry of reflecting layers)? or the soil age, grain size, and mineralogy? How much does site response vary horizontally? Do geologically similar sites have a similar site response?

In this paper, we examine four vertical arrays installed at deep soil sites with extensive earthquake ground motion data sets: Lotung, Taiwan; Port Island and Rokko Island, Japan; and Chiba, Japan. Each of the sites is located in a deep sedimentary basin, which provides the basis for comparison. Port Island and Rokko Island are located about 5 km apart in the Osaka Bay near Kobe and recorded six of the same earthquakes, aftershocks of the January 17, 1995 Kobe earthquake. Therefore, this data set provides an excellent opportunity to test the consistency of site response of geologically similar sites. The Chiba Seismometer array has 15 installed vertical arrays with the largest horizontal separation of 300 m and the shortest of 5 m. For this analysis, we use four

[1] *Graduate Research Assistant, Dept. of Civil and Environmental Engineering, University of California, Berkeley, CA 94720*
[2] *Assistant Professor, Dept. of Civil and Environmental Engineering, University of California, Berkeley, CA 94720*

vertical arrays at the Chiba site which each recorded the same set of earthquakes, comparing the site response at the central borehole C0, with that recorded at 15 m, 143 m, and 247 m horizontal separation. The Lotung site has two vertical arrays with a horizontal separation of 46 m with one vertical array near a building to test for soil structure interaction. The Lotung and Chiba sites are geologically similar and therefore, the site response is compared at the two sites. We use system identification (SI) to determine parametric site response filter models, and the parametric model of choice is an auto-regressive moving average (ARMA) model.

METHODOLOGY

System Identification

The goal of SI is to model a system in a manner that provides needed mechanical information about that system. The most common techniques have evolved from electrical and mechanical engineering, and involve solving the inverse problem for the system transfer function. A simple system model is a ratio of weighted polynomials. The weights are the parameters relating system input and output. Such a model, referred to as an ARMA model, is based on discrete time series analysis:

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \ldots + b_0 x_t + b_1 x_{t-1} + \ldots \qquad (1)$$

where *yj* is the actual output data sequence, *xj* is the input sequence, and *t* is the time step counter. The theory and assumptions behind the method, with emphasis on geo-problems, are reviewed in a paper and report by the author [Glaser, 1995, 1998].

The ARMA model represents the time history of earthquake ground motion very well, and has been successfully applied to relevant seismological and structural problems [e.g. Popescu and Demetriu, 1990; Safak, 1988; Polhemus and Cakmak, 1981]. Several researchers [Robinson and Treital, 1978; Kanasewich, 1981; Hubral et al., 1980] have shown that the horizontally layered earth system containing a source at the surface and a receiver at depth is an autoregressive (AR) process. By the theorem of reciprocity, the system with a source at depth and a receiver on the surface can therefore be considered an AR process [Aki and Richards, 1980]. By identifying the physical phenomena of earthquake ground motion recorded at the surface of a horizontally layered media as an autoregressive process, we have narrowed the field of potential models to use in our identification. The MA parameters capture convolutional aspects of the modelled system and provide phase information. Wold’s decomposition shows that the combined ARMA model results in the most parsimonious system filter.

Choosing the Data Window

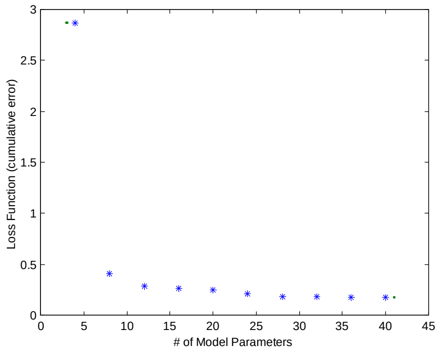
As each of the study sites is located in a sedimentary basin, surface waves are expected to develop at the edges of the basin and propagate horizontally across the basin; therefore, the assumption of one-dimensional vertically propagating waves is not totally accurate. To account for this, the data window used for model estimation is shortened to include the direct Shear wave. The Shear wave is assumed to propagate vertically due to the large impedance contrast between bedrock and soil.

A common issue in site response analysis encountered when using vertical array data is the inclusion of the downgoing wave in the borehole recording. For instance at the Chiba site, the downgoing S-wave surface reflection is evident in the 20 m recording at 0.15 s delay. One way of handling this is to limit the window for estimating the model to only include upgoing input waves, but that would be a very short window (0.15 s, or 30 points), leading to poor resolution of the model. Since we are solving an inverse problem, and the recorded time series report all motions endured by the soil, we utilise a black box method, which does not require a physical model for forward estimation. Rather, a complex-valued rational polynomial model is determined which most accurately maps a given input motion to a given output motion. The best-fit model captures the “essence” of the intervening system without the need to solve for a forward physical model. When the physical behaviour is no longer ARMA, the model can no longer identify site response and the error will be large.

Choice of Model Order, and Validation

The lengths of the AR and MA processes (model order) are free variables which must be explicitly chosen so that the model best represents the physical process in question, and the statistical appropriateness of the chosen model order is verified by several accepted methods [Bohlin, 1987; Priestley, 1992]. The initial assumption made for these analyses was that the soil systems acting on the various strong motions recorded were basically linear. The linear SI algorithm used is a mathematically rigorous least-square optimisation process to determine the rational polynomial coefficients (weights) [Ljung, 1987] and results in model parameters that are constant with time. An important property of the ARMA model is invoked when choosing the model order – the associated 2*n*-2*n* ARMA difference equation is the difference equation of the integral of the equation of motion of a *n*-degree-of-freedom lumped mass oscillator [Beck, 1978; Ghanem et al., 1991]. Therefore, we use 2*n*-2*n* ARMA models so that we can use our understanding of lumped mass models to interpret the site response models. As with all inverse problems, the model solution is nonunique. A unique model is found by regularising the problem through acceptance of some least-square criteria, limiting possibilities to order 2*n*-2*n*, acceptance of the concept that a simpler model is better, and other selection procedures described.

Each input/output pair of strong motion records was initially submitted to an overall algorithm that calculates the loss function (normalised sum of squared prediction errors) versus model order increased in 2*n*-2*n* steps for a suite of pre-selected model orders. A typical loss function plot is shown in Fig. 1, for the Port Island Event 5 EW, 0-32 m interval. This plot shows that the waveforms only carry information from the first 1-3 modes (4-12 parameters) - there was virtually no improvement in estimation quality (smaller error) for more parameters. Examination of the pole and zero plot of the estimated complex roots insured that excessive, overlapping parameters were not included [e.g. Astrom and Soderstrom, 1974].



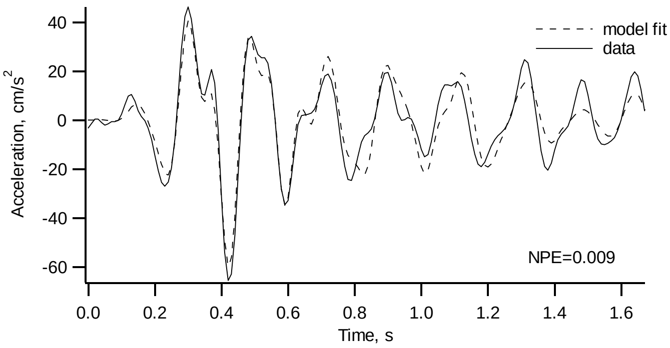


Fig. 1: Loss Function for 0-32 m system. **Fig. 2: Actual versus 3-DOF model output at Port Island.**

In most cases, the fits were excellent for a small number of parameters. An example is given in Fig. 2 which shows the model and actual values of output for Port Island, Event 5, 0-32 m, EW component for a 3-DOF model (12 parameters). For this calculation, the rms error function varied from 0.38 for the 3-DOF model to 0.26 for a 10-DOF model, indicating that very little information was left to be extracted from the data by the more complicated models. Experience has shown that any estimate above the second or third mode is tenuous at best, although numerically we are only limited by computational power as to how many modes we want to calculate. In general, the increased model order may sharpen the peaks or add small secondary peaks but does not seem to effect the location of the primary peaks. Given that the data does not have an infinite signal-to-noise ratio, i.e. there is noise present from many sources including quantization, there is only a limited amount of information that can be taken from the data [Shannon, 1949]. Additional modes will try to fit the noise rather than the system itself.

A final test for the chosen model was insuring 99% confidence in both the whiteness of the residual autocorrelation function and the cross-correlation function of the input signal and output residuals [Bohlin, 1987]. The residuals of the model are the difference between the predicted value and the actual value at each time step ($y_t - \hat{y}_t$) where y_t is actual output at time *t*, and $\hat{y}_t$ is the prediction of output at time *t* made at time *t-1*. The residual analysis plot for the example model, shown in Fig. 3, demonstrates that no additional statistically significant information was left in the data beyond the 3-DOF system; therefore, a 3-DOF model was the choice to represent this interval and event.

The SI process described above identifies the weights that map the input time series to the output time series. Given the extreme congruence between actual and calculated interval outputs (e.g. signal entering the soil layer at 32 m and recorded at the surface), the technique is obviously effective. In Fig. 2, a linear mapping with just 12 parameters in the model of the EW acceleration motions passing through the 32 m to surface layer is close to a perfect fit, and strongly suggests that the calculated parameters are carrying some information about how the soil layer affected the through-passing waveform.

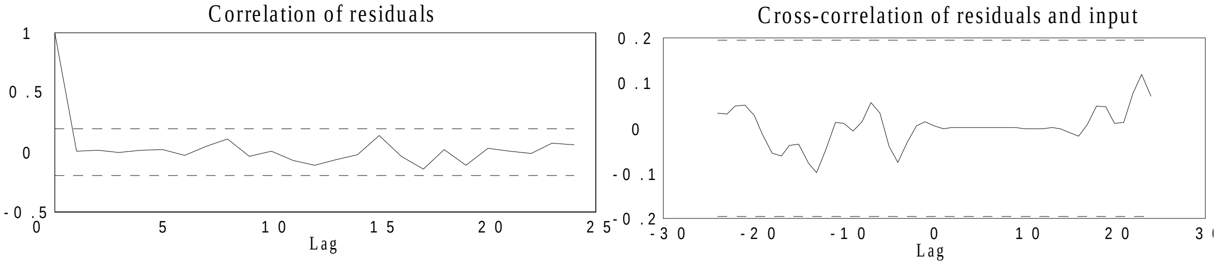


Fig. 3: Residual Analysis plot for 3-DOF model, Port Island, 0-32 m, EW.

Assessing Error and Predictive Capability

Once a filter model is estimated using least square optimisation, the goal is to use the model to predict soil behaviour for other earthquakes or at similar sites using different input motions. To use the site response filter-models as a predictive tool, actual recorded input waveforms are fed into the model to produce output waveforms, which can then be compared to actual ground motions recorded at the site to assess the predictive power of the model.

The accuracy of the predicted ground motion is quantified by the mean squared error (MSE) normalised to the maximum amplitude of the ground motion (peak ground acceleration, PGA). The MSE is an overall measure of the error magnitude for a model prediction and is directly related to the loss function used in the least square estimation algorithm. Effects of event duration are removed by calculating the error only over the significant portion of shaking, i.e. > 20% PGA. By normalising the error by the PGA and over the duration of significant ground shaking, we are able to directly compare the goodness of fit of the model predictions of different events. Because earthquake ground motion time series are non-stationary, the mean value of the squared error term will most likely not be stationary. Therefore, it is necessary to specify the portion of the earthquake that will be analysed for error with the assumption of a constant mean. By analysing the window of the earthquake containing all the peaks with values greater than or equal to 20% PGA for that earthquake and normalising the records by the PGA of the output, the resulting normalised prediction error (NPE) provides a useful statistic with which the goodness of fit of a model to a input/output data set can be assessed and compared with other cases.

PORT AND ROKKO ISLANDS, JAPAN

Site Description

Port Island and Rokko Island are reclaimed islands located in Osaka Bay, south of Kobe City, Japan. The fills for both islands consisted of residual granite soils derived from the Rokko Mountains [Iwasaki and Tai, 1996]. The fill was fluvially placed and the grain size varies over the two sites depending on the degree of weathering of the original material. The northern portion of Port Island where the vertical array is located was constructed primarily of the residual granite soils, while the Rokko Island fill consisted of residual granite soils mixed with material from the Kobe Group, a layered Miocene sediment, leading to higher fines content [UCB, 1995]. The underlying natural deposits are soft alluvial clay, approximately 2 to 3 m thick, over stiffer Pleistocene terrace deposits which continue for approximately 2 km to the bedrock. A more detailed description of these sites can be found in UCB [1995].

The geologic profile at the vertical array at Port Island consists of 19 m of loose sandy fill over 8 m of native Holocene alluvial clay materials, underlain by Pleistocene terrace deposits to a depth of approximately 2 km. In August 1991, a four-level 3-D accelerometer downhole array was installed at the north-west corner of the island, with accelerometers placed at the surface, 16 m, 32 m, and 83 m depth [Iwasaki et al., 1996]. A similar vertical array was installed at Rokko Island. For the following analysis, three site response soil intervals are compared at each site: Interval A (16 m depth to surface); Interval B (32 m depth to surface); and Interval C (83 m depth to surface).

Analysis

Ground motion recordings were made simultaneously at Rokko Island and Port Island for five different aftershocks to the January 17, 1995 Kobe earthquake. For the aftershock (Event 5, JMA=4.2) occurring on February 2, 1995, site response models were estimated at both sites and then compared to test the consistency of site response at the two geologically similar sites. Initially, site response models for only the filled portion of the site profile (16 m depth to the surface) were estimated and compared. For this interval, the Port Island site response filter model slightly underestimates the Rokko Island site response with a NPE of 0.053, as shown in Fig. 4. The second interval analysed (32 m depth to the surface) which spans native soils as well as fill had similar results. On the other hand, interval B at Port Island is overestimated using the Rokko Island filter model for that interval with a NPE of 0.214, as shown in Fig. 5.

On comparison in Fig. 6, the interval A frequency functions for the site response models from the two islands have alternate resonant peaks. The Port Island site response filter model has a strong peak at 6 Hz and a less dominant peak at 12 Hz, while the Rokko Island site response model has a strong peak at 13 Hz and a weaker peak at 5 Hz. Overall, the Port Island site response model has a higher degree of amplification than the Rokko Island model but the models are of similar shape explaining why the predictions are not too erroneous. As seen in Fig. 7, the frequency functions for both interval A and B at Rokko Island have peaks at the same frequencies. The B interval (32 m to surface) has a higher degree of amplification than the shallower interval A (16-m to surface). Similar results were seen at Port Island.

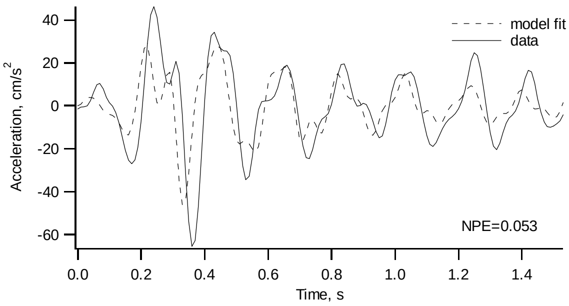


Fig. 4: Interval A at Rokko Island with Port Island filter Island model.

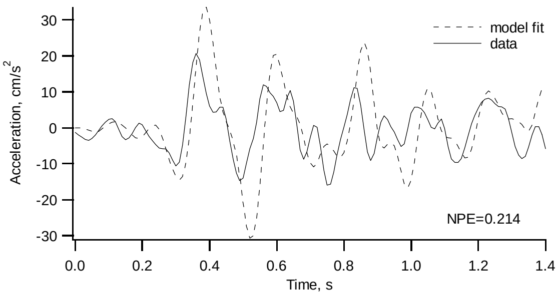


Fig. 5: Interval B at Port Island with Rokko filter model.

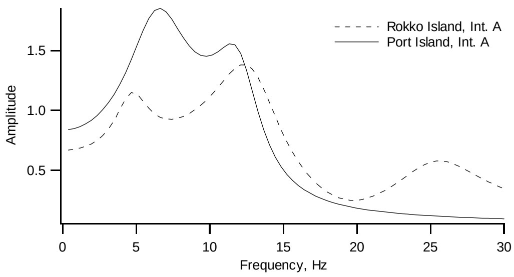


Fig. 6: Frequency functions at Port Island and Rokko for Island for interval A (16 m to surface).

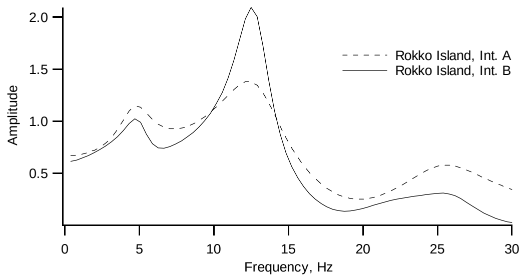


Fig. 7: Frequency functions at Rokko Island intervals A and B.

CHIBA, JAPAN

Site Description

The Chiba Seismometer Array was installed at the Chiba Experiment Station of the Institute of Industrial Science at the University of Tokyo in 1982 approximately 30 km east of Tokyo. The geologic profile at the site consists of approximately 3 to 5 meters of loam underlain by 2 to 4 meters of sandy clay which is subsequently underlain by a stiffer dilivium sand layer [Katayama et al, 1990]. The sand layer's stiffness increases with depth and is interspersed with clayey layers. The 15 borehole logs at the site indicate flat-lying pervasive layers, consistent over the site. The water table was reported at 5 m depth [Katayama et al, 1990].

The Chiba seismographic array is comprised of 44 three-component piezoelectric accelerometers densely placed on the ground surface and at depth in boreholes. The instruments were located geometrically to provide spatial coverage of the site. Borehole C0 has 5 instruments at depths of 1, 5, 10, 20 and 40 meters. P1, P6, and P8 have instruments at 1, 10, and 20 meters and are used in this study along with the C0 instruments. The largest ground motion recording was approximately 0.3g as a result of the December 17, 1987 Chibaken-Toho-Oki earthquake. The database consists of 27 recorded events with PGA ranging from less than 0.01g to 0.3g with most events between 0.02 and 0.08g.

Analysis

At Chiba, site response filter models were determined for two intervals at the most central borehole, C0: 10 m to the surface, and 20 m to the surface. These site response filter models were used to predict the site response at boreholes, P1, P6, and P8 with horizontal separation of 15 m, 143 m and 247 m respectively. As expected the NPE increased when the model was applied to input data at boreholes with increasing horizontal separation from C0. Results from the deeper interval are presented in Fig. 8 and demonstrate that even at 247 m distance, the site response filter model predicts the surface ground motion given the 20 m input motion, with NPE=0.05.

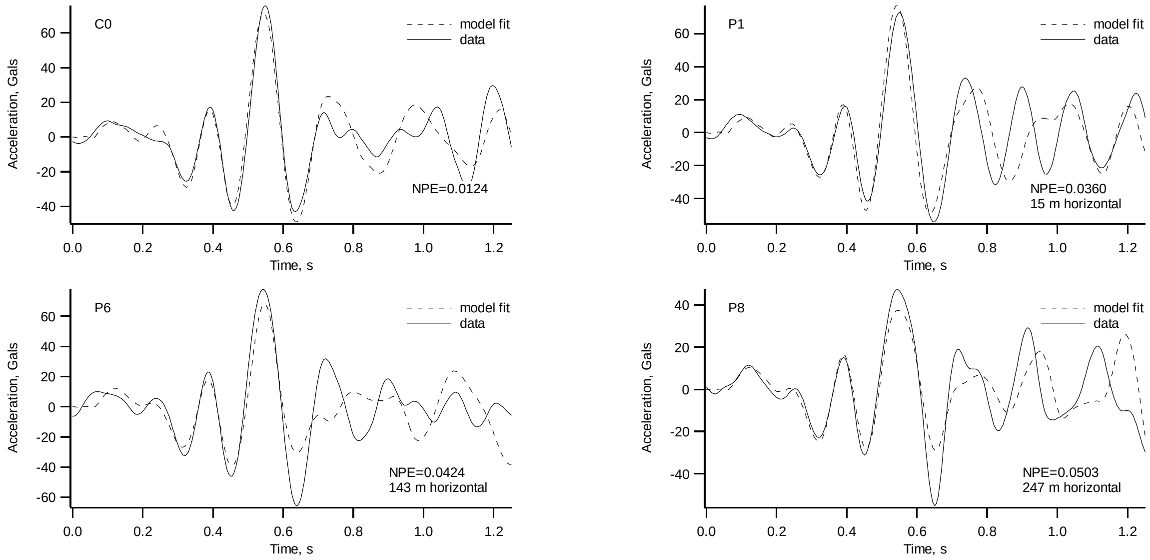


Fig. 8: Site response model derived for the C0 borehole 20 m to surface interval used to predict the surface ground motions at borehole C0, P1, P6, and P8.

LOTUNG, TAIWAN

Site Description

The geology of the Lotung site is summarised by Wen and Yeh [1984] and Tang [1987]. The area consists of a recent alluvium layer 40 to 50 m thick overlying a Pleistocene formation that varies from 150 to 500 m in thickness. Underlying the Pleistocene material is a Miocene basement rock. A simplified soil profile consists of 30-35 m of silty sand and sandy silt with some gravel, above clayey silt and silty clay. The site has been extensively investigated [Anderson, 1993; Anderson and Tang, 1989] with five independent testing programs.

Analysis

Because the Lotung site has a similar geology to the Chiba site (alluvial clays and sands), we compared the site response at the two sites. The Chiba 10-m to surface site response model was used to simulate the site response at Lotung between 11 m and the surface, a similar interval, during Event 7. The resulting predicted waveform is greatly inaccurate and overpredicted at the surface, as shown in Fig. 9, indicating that the Chiba model of this soil interval is inappropriate for the similar depth interval at the Lotung site.

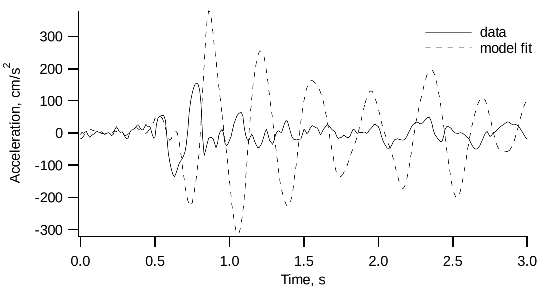


Fig. 9: Lotung surface Event 7 ground motion predicted with Chiba filter.

DISCUSSION AND CONCLUSIONS

We have demonstrated spatial consistency of site response at three deep soil sites. We have shown that sites with a similar geologic setting, such as Port and Rokko Islands or the Chiba site, will result in a similar site response. In other words, minor material and structural variations that would be expected across a site do not seem to strongly affect the overall site response. The site response at Port and Rokko Islands is consistent even though the sites are approximately 5 km apart. At Chiba, a horizontal separation of 247 m results in a similar site response. However, although geologically similar sites, the site response at Chiba was not similar enough in character to be used for predictive purposes at the Lotung site. Port and Rokko Island are located in a single sedimentary basin with the same geologic units while Chiba and Lotung are thousands of miles apart in different countries and geologic settings. So although the geologic units are of similar material (alluvial clays and sands), the overall geologic history of the sedimentary basin is not the same.

The more thorough comparison of site response at Port and Rokko Islands indicated that the two sites have very similar responses. The site response models have similar shaped frequency functions with two resonant peaks (5-6 Hz and 12-13 Hz). When comparing the deeper interval (32-m to the surface) with 16 m to the surface, the model amplification increased as a greater depth of soil was spanned by the model. Although, Port and Rokko Islands are several kilometres apart, the geologic setting is very similar leading to the similar site response. It is important to differentiate between a similar geologic setting and a similar soil profile. Chiba and Lotung have similar soil profiles (sand and clay) but a different geologic setting and their site response varies. On the other hand, Port and Rokko Island have similar soil profiles and the same geologic setting leading to a similar site response. In the future, we hope to enlarge this investigation to include more sites and more data to further explore the fundamental aspects of site response.

ACKNOWLEDGMENTS

This research has been supported by Port and Harbor Research Institute of Yokosuka, Japan, NSF Grant # CMS-9727002, and the NSF Graduate Fellowship program. The Port Island and Rokko Island data was made available by the Kobe City Government. The Chiba Array data are made available by Institute of Industrial Science, University of Tokyo. We want to thank the Electric Power Research Institute, in particular H.T. Tang, for making the Lotung data available through a Cooperative Research and Development Agreement with the National Institute of Standards and Technology.

REFERENCES

Aki, K. and P. Richards. (1980) *Quantitative Seismology: Theory and Methods. Vol. 1.* W.H Freeman, San Francisco, CA.

Anderson, D.G. (1993). *Geotechnical synthesis for the Lotung large-scale seismic experiment*, Report TR-102362, Palo Alto: Electric Power Research Institute.

Anderson, D. G. and Tang, Y. K. (1989). "Summary of soil characterization program for the Lotung large-scale seismic experiment." *Proceedings: EPRI/NRC/TPC workshop on seismic soil-structure interaction analysis techniques using data from Lotung, Taiwan, Report NP-6154*.

Astrom, K.J. and Soderstrom, T. (1974). "Uniqueness of the maximum likelihood estimates of the parameters of an ARMA model." *IEEE transactions on automatic control*, AC-19, pp. 769-773.

Beck, J.L. (1978). "Determining Models of Structures from Earthquake Records." *Earthquake Engineering Research Laboratory, Report 78-01*. Pasadena, California.

Bohlin, T. (1987). "Model validation." *Encyclopedia of systems and control* (ed. Singh, M.) Oxford: Pergamon Press.

Ghanem, R. G., Gavin, H., and Shinozuka, M. (1991) *Experimental Verification of a number of structural system identification algorithms*. p. 302. Technical Report NCEER-91-0024. Buffalo: National Center for Earthquake Engineering Research.

Glaser, S.D. (1998). *System Identification and Its Application to Soil Dynamics*. Geotech. Eng. Report No. UCB/GT 98-01. Dept. of Civil and Env. Eng., Univ. of California, Berkeley.

Glaser, S.D. (1995). “System Identification and Its Applications to Estimating Soil Properties,” *Journal of Geotechnical Engineering*, 553-560.

Hubral, P., S. Treital., and P. Gutowski. (1980). “A Sum Autoregressive Formula for the Reflection Response,” *Geophysics*, **45**(11), 1697-1705.

Iwasaki, Y. and M. Tai. (1996). “Strong Motion Records at Kobe Port Island,” *Special Issue of Soils and Foundations*. Japanese Geotechnical Society. 29-40.

Kanasewich, E. (1981). *Time Sequence Analysis in Geophysics*. Edmonton, Alberta: University of Alberta Press.

Katayama, T., F. Yamazaki, S. Nagata, and L. Lu, (1990). *Development of Strong Motion Database for the Chiba Seismometer Array*. Earthquake Disaster Mitigation Engineering, Inst. of Ind. Science, Univ. of Tokyo. Report No. 90-1 (14).

Ljung, L. (1987). *System Identification: Theory for the User*. Prentice Hall, Inc. Englewood Cliffs, N.J.

Polhemus, N. and A. Cakmak. (1981). “Simulation of Earthquake Ground Motions using ARMA models,” *Automatica*. **26**(4), 721-737.

Popescu, T. D., and Demetriu, S. (1990). “Analysis and simulation of strong earthquake ground motions using ARMA models.” *Automatica*, 26(4), pp. 721-737.

Priestley, M.B. (1992). *Spectral Analysis and Time Series*. London: Academic Press.

Robinson, E. and S. Treital. (1978). “The Fine Structure of the Normal Incidence Synthetic Seismogram,” *Geophysic J. R. Astr. Soc*. **53**. 289-309.

Safak, E. (1988). *Analysis of Recordings in Structural Engineering: adaptive filtering, prediction and control.* Open file report 88-647. Menlo Park, CA: United State Geologic Survey.

Shannon, C.E. (1949). “Communication in the presence of noise.” *Proceedings of the institute of radio engineers*, 37, pp. 10-21.

Tang, H. T. (1987). *Large-scale soil-structure interaction*. Report NP-5513-SR. Palo Alto: Electric Power Research Institute.

UCB/EERC-95/01. (1995). *Geotechnical Reconnaissance of the Effects of the January 17, 1995, Hyogoken-Nanbu Earthquake*, Japan. Earthquake Engineering Research Center. University of California, Berkeley.

Wen,. K.L. and Yeh, Y.T. (1984). “Seismic velocity structure beneath the SMART 1 array.” *Bulletin of the institute of earth science*, Academia Sinica, Vol. 4.