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SEISMIC RESPONSE ANALYSIS FOR TELECOMMUNICATION TOWERS BUILT ON THE BUILDING

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SUMMARY

The seismic response spectrum method for a secondary system is developed, to consider dynamic interactions with the primary system. The proposed seismic response evaluation method is divided into two steps. The first step is to transform the seismic input such as floor response spectra values at the period of the secondary system from the specified design spectra at the ground. In this step, we present the modal synthesis method in which the modal properties of the combined system are determined from the modal characteristics of the primary system and the equivalent single-degree-of freedom oscillators of the secondary system. Furthermore, the method considers as transient effect of the primary system on the response of the secondary system. On the other hand, the second step is the seismic response calculation of the secondary system using the relative acceleration response spectra values. To consider rigid body mode and closed spaced modes, the present modal combination rule includes two kinds of correlation coefficients between the input acceleration and relative accelerations and among the relative accelerations. To illustrate the present method, seismic response analysis of a tower-building model is carried out, and the accuracy of the method is discussed.

INTRODUCTION

For the seismic design of the secondary system, such as telecommunication towers or equipment mounted on the building (the primary system), response spectrum method is often used. In this method, at the first step, it is common to evaluate the seismic floor response spectra (FRSs) from the specified design spectra of the ground motion. The FRSs are the maximum response series of single-degree-of-freedom (S-DOF) oscillators which have different damping ratios and natural frequencies and which is assumed to be mounted on the floor of the primary system. From the generated FRSs, at the second step, the responses of secondary system are evaluated by means of some modal combination rules. In order to obtain seismic response of the secondary system accurately, the FRSs should be evaluated exactly at the first step, and the total response should be combined from each modal response reasonably at the second step. In these points of views, many methods were developed as follows.

In the evaluation of FRSs, the compatible power spectrum density function (PSDF) is calculated from the design spectra, then the PSDF of the floor where the secondary system is attached is converted from the ground PSDF and the modal properties of the combined system, finally the FRSs is evaluated from the floor PSDF. The compatible method between PSDF and design spectra (or between PSDF and FRSs) was proposed by Kaul (1978) or Unruh and Kana (1981). Their methods are based on the stationary process, and also consider infinite earthquake duration by adjustments to damping factor of S-DOF oscillators [Routhenbluth, 1969]. In turn, in the PSDF transformation from ground to floor, the combined modal property should be evaluated accurately. For example, when the weight of secondary system is relatively large, the dynamic interaction between primary andsecondary system is significant. From this point of view, many modal synthesis methods were proposed, where the modal properties of combined system are calculated from primary and secondary one individually e.g. [Singh and Suarez, 1986,1987]. As mentioned above, in the previous studies, in the evaluation of FRSs dynamic interaction between primary and secondary system is considered perfectly, however, non-stationary or transient response effects are neglected.

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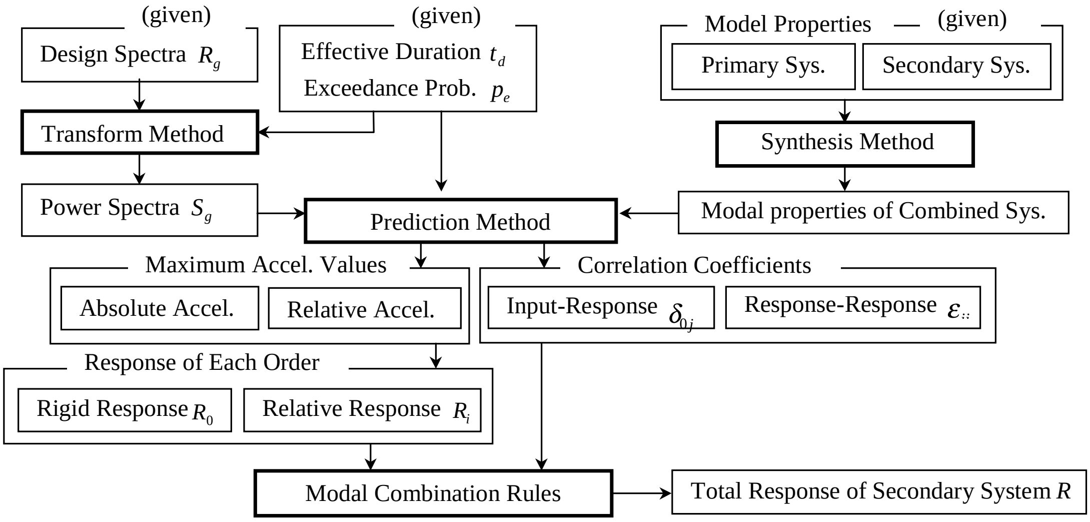


Fig. 1 Flow Chart of Seismic Analysis for Secondary System

Theoretically, the assumption of stationary response gives overestimated result compared with the real response, and it is significant when the natural period of combined system is long, or when the earthquake duration is small.

On the other hand, in the modal combination rule of response, the square root of the sum of the squares (SRSS) rule [Goodman et al., 1953] is well known and is still used quite widely, but is not adequate in the case of the response with closely spaced frequencies. The double sum (DSUM) rule [Rosenblueth, 1969] and the complete quadratic combination (CQC) rule [Der Kiureghian, 1981] give reasonable estimation even when each mode is closed, but is not adequate when the input motion is a narrow-band process or when the rigid body mode is influential. To solve these problems, Hadjian (1981) and Singh and Mehta (1983) proposed alternative rules using relative acceleration response spectra and maximum input acceleration (or ZPA; zero period acceleration). Also, A.K.Gupta et.al.(1984) proposed a rule in which total responses are synthesized from the rigid body mode and the damped periodic mode. Der Kiureghian and Nakamura (1993) improved the original CQC rule, to avoid the truncation error. In the case of the secondary system, the input motion is the response at the floor of the primary system, therefore, it generally becomes a narrow-band process.

In this paper, a seismic response spectrum method for a secondary system is proposed, considered as a dynamic interaction problem with a primary system. Firstly, we present a method to predict the maximum response for the primary-secondary system (combined system), considering infinite earthquake duration or transient response. In this method, the maximum absolute or relative acceleration responses of the combined system can be obtained, under the assumption of the stationary ground motion. Secondly, using these values, the modal combination rule for the maximum responses of the secondary system is presented. This method is based on the modal superposition theory and is also used for maximum values of the relative acceleration and input acceleration. The seismic analysis for the secondary system in this study is shown in Fig.1.

MAXIMUM RESPONSE OF THE COMBINED SYSYTEM

Eigenvalues Analysis

Exact formulation of the eigenvalue problem are shown by Singh and Suares (1987), for a multi-degree of freedom (M-DOF) primary system with a S-DOF secondary system. In this paper, at first, this formulation is extended to a M-DOF primary system with a M-DOF secondary system.

When N_p SDOF oscillators equivalent to a secondary system are attached to the M-DOF primary system (number of freedom: N_s) and the combined system is subject to a ground motion $\ddot{x}_g(t)$, the equations of motion become

$$\begin{bmatrix} \mathbf{0} & \mathbf{M}_c \\ \mathbf{M}_c & \mathbf{C}_c \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{x}}_c \\ \dot{\mathbf{x}}_c \end{Bmatrix} + \begin{bmatrix} -\mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_c \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{x}}_c \\ \mathbf{x}_c \end{Bmatrix} = -\begin{Bmatrix} \mathbf{0} \\ \mathbf{M}_c \quad \ddot{\mathbf{x}}_G \end{Bmatrix} \tag{1}$$

or,

$$\mathbf{D}\dot{\mathbf{z}} + \mathbf{E}\mathbf{z} = -\ddot{\mathbf{z}}_G \tag{2}$$

where

$$\mathbf{M}_c = \begin{bmatrix} \mathbf{M}_p & \mathbf{0} \\ \mathbf{0} & \lceil m_j^s \rfloor \end{bmatrix}, \quad \mathbf{C}_c = \begin{bmatrix} \mathbf{C}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \sum_{j=1}^{N_s} 2h_j^s \omega_j^s m_j^s \left[\mathbf{v}_j \mathbf{v}_j^{\mathrm{T}}\right], \quad \mathbf{K}_c = \begin{bmatrix} \mathbf{K}_p & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \sum_{j=1}^{N_s} m_j^s \omega_j^{s\,2} \left[\mathbf{v}_j \mathbf{v}_j^{\mathrm{T}}\right] \tag{3}$$

in which $\mathbf{M}_c$, $\mathbf{C}_c$ and $\mathbf{K}_c$ are the mass, damping and stiffness matrix of the combined system, the matrices of subscript p are those of the primary system, ω_j^s, h_j^s and m_j^s are the natural frequency, damping ratio and modal mass of the j-th mode of the secondary system. □□□ denotes a diagonal matrix, and a superposed T denotes a transpose. And, the vector $\mathbf{v}_j$ is defined as follows if the j-th mode oscillator is assumed to attach to node k of the primary system:

$$\mathbf{v}_j^{\mathrm{T}} = \{\cdots, \overset{k-1}{0}, \overset{k}{1}, \overset{k+1}{0}, \cdots, \overset{N_p+j-1}{0}, \overset{N_p+j}{-1}, \overset{N_p+j+1}{0}, \cdots\} \tag{4}$$

The magnitude of modal mass m_j^s is given as,

$$m_j^s = \sum_{i=1}^{N_s} M_i^s \beta_j^s \phi_{ij}^s \tag{5}$$

in which M_i^s is a mass of node i, β_j^s is the j-th participation factor and ϕ_{ij}^s are the mode shape values of the secondary system, respectively.

The eigenvalue equation of equation (1) becomes,

$$\left(p_i \mathbf{D} + \mathbf{E}\right)\begin{Bmatrix} p_i \boldsymbol{\phi}_i^c \\ \boldsymbol{\phi}_i^c \end{Bmatrix} = \{\mathbf{0}\}, \quad i = 1 \sim 2\left(N_p + N_s\right) \tag{6}$$

To obtain the modal properties of the combined system, we need to solve equation (6). However, there are two problems, solving this eigenvalue equation. One is the numerical inaccuracy which could occur in the solution of equation (6) due to ill-conditioning of the matrices caused by the lightness of the secondary system. And the other is the number of the order in equation (6) is two times as large as the number of degree of freedoms of the primary and secondary system, therefore, the computational cost to solve equation (6) is relatively large. In practice, the lower modes of the combined system which affect to the response can be synthesized from the first few modes of the primary and secondary systems individually. Thus, to avoid these difficulties, we can introduce the following transformation in equation (6):

$$\begin{Bmatrix} p_i \boldsymbol{\phi}_i^c \\ \boldsymbol{\phi}_i^c \end{Bmatrix} = \mathbf{T}\hat{\boldsymbol{\phi}}_i = \begin{bmatrix} \mathbf{U} & 0 \\ 0 & \mathbf{U} \end{bmatrix}\hat{\boldsymbol{\phi}}_i, \quad \mathbf{U} = \begin{bmatrix} \boldsymbol{\Phi}_p & \\ & \lceil 1/\sqrt{m_j^s} \rfloor \end{bmatrix} \tag{7}$$

in which $\boldsymbol{\Phi}_p$ is the mode matrix of the primary system in which is arranged the lower modes (number of mode: m_p), and which is normalized by $\boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{M}_p \boldsymbol{\Phi}_p = \mathbf{I}$ ($\mathbf{I}$ is a unit matrix). Also, the number of oscillators as the secondary system is considered as the first lower mode (number of mode: m_s). In practice, m_p and m_s can be selected to include in the range of frequency interest.

Substituting equation (7) in equation (6), premultiplying by $\mathbf{T}^{\mathrm{T}}$ and utilizing equation (4) and the orthonormal properties of $\boldsymbol{\Phi}_p$, we obtain

$$\left(p_i \hat{\mathbf{D}} + \hat{\mathbf{E}}\right)\hat{\boldsymbol{\phi}}_i = \{\mathbf{0}\} \tag{8}$$

where

$$\hat{\mathbf{D}} = \mathbf{T}^{\mathrm{T}}\mathbf{D}\mathbf{T} = \left[\begin{array}{c|cc} \mathbf{0} & \multicolumn{2}{c}{\mathbf{I}} \\ \hline \mathbf{I} & \lceil 2h_i^p \omega_i^p \rfloor & 0 \\ & 0 & 0 \end{array}\right] + \sum_{j=1}^{m_s} 2m_j^s h_j^s \omega_j^s \left[\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{w}_j \mathbf{w}_j^{\mathrm{T}} \end{array}\right] \tag{9a}$$

$$\hat{\mathbf{E}} = \mathbf{T}^{\mathrm{T}}\mathbf{E}\mathbf{T} = \left[\begin{array}{c|cc} -\mathbf{I} & \multicolumn{2}{c}{\mathbf{0}} \\ \hline \mathbf{0} & \lceil \omega_i^{p\,2} \rfloor & 0 \\ & 0 & 0 \end{array}\right] + \sum_{j=1}^{m_s} m_j^s \omega_j^{s\,2} \left[\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{w}_j \mathbf{w}_j^{\mathrm{T}} \end{array}\right] \tag{9b}$$

$$\mathbf{w}_j^{\mathrm{T}} = \mathbf{v}_j^{\mathrm{T}}\mathbf{U} = \{\overset{1}{\phi_{k1}^{p}},\ \overset{\cdots}{\cdots},\ \overset{M_p}{\phi_{kM_P}},\ \overset{M_p+1\ldots}{0},\ \cdots,\ 0,\ \overset{M_p+j}{-1\Big/\sqrt{m_j^s}},\ 0,\ \cdots\} \tag{10}$$

Solving the complex eigen equation in equation (8), we can obtain the eigenvaluses p_j and eigen-vectors $\mathbf{\Phi}_j^c$, $\hat{\mathbf{\Phi}}_j^c$ of the combined system. That is, the modal properties of the combined system are synthesized from given primary and secondary one individually.

The Moment Functions of Absolute and Relative Acceleration Responses

In section 3, a modal combination rule for the response of the secondary system is described, in which two kind of maximum response values are used: those are the maximum absolute acceleration of the floor where the secondary system is attached, and the maximum relative acceleration of secondary system. To evaluate these maximum values, we will derive the moment functions of the absolute and relative acceleration in this subsection. Using these moment functions, maximum acceleration responses are evaluated by means of a method shown in the next subsection.

The equation of motion (1) can be decoupled with the help of the standard transformation as,

$$\begin{Bmatrix} \dot{\mathbf{x}} \\ \mathbf{x} \end{Bmatrix} = \mathbf{\Phi}_c \boldsymbol{\xi}(t) = \mathbf{T}\hat{\mathbf{\Phi}}_c \boldsymbol{\xi}(t) \tag{11}$$

in which $\boldsymbol{\xi}$ is the vector of the principal coordinates. Substituting equation (11) into equation (1) and premultiplying by $\mathbf{\Phi}_c^{\mathrm{T}}$, we obtain $2(m_p + m_s)$ decoupled equations

$$d_i \dot{\xi}_i(t) + e_i \xi_i(t) = -\Gamma_i \ddot{x}_g(t), \quad i = 1 \quad ` 2\left(m_p + m_s\right) \tag{12}$$

where

$$\Gamma_i = \boldsymbol{\phi}_i^{c\,\mathrm{T}} \mathbf{M}_c \tag{13}$$

in which Γ_i is the component vector of the participation factor of the combined system.

It can be shown that the absolute acceleration response subjected to the ground motion at node k of the combined system is given as follows:

$$a_k = -\mathrm{Re}\left[\sum_{i=1}^{2m} p_i \phi_{ki}^c \alpha_i \xi_i\right] \tag{14}$$

and the first derivative of the response with respect to time $\dot{a}_k$ is given as follows:

$$\dot{a}_k = \mathrm{Re}\left[\sum_{i=1}^{2m} p_i \phi_{ki}^c {\alpha_i}^2 \xi_i + \sum_{i=1}^{2m} p_i \phi_{ki}^c \alpha_i \beta_i \ddot{x}_g\right] \tag{15}$$

where

$$\alpha_i = e_i / d_i, \quad \beta_i = \Gamma_i / d_i \tag{16}$$

in which m is the total number of the mode of the combined system $\left(m = m_p + m_s\right)$.

The moment function of the absolute acceleration response will be presented below. Now, using the parameters in equation (16), the unit impulse response function of ξ_i in equation (12) can be obtained as follows:

$$h_i(t) = -\beta_i \exp\left[-\alpha_j t\right] \tag{17}$$

By means of equations (14) or (15), (17), assuming the ground motion $\ddot{x}_g$ as a stationary process. The first three moment functions are derived as

$$\sigma_a{}^2(t) = \mathrm{E}\left[a_k^2(t)\right] = \mathrm{Re}\left[\int_{-\infty}^{+\infty}\left\{\sum_{i=1}^{2m}\sum_{j=1}^{2m} p_i p_j \phi_{ki}^c \phi_{kj}^c \alpha_i \alpha_j A_{ij}(t)\right\} \times S_g(\omega) d\omega\right] \tag{18}$$

$$\sigma_{a\dot{a}}(t) = \mathrm{E}\left[a_k(t)\dot{a}_k(t)\right] = \mathrm{Re}\left[\int_{-\infty}^{+\infty}\left\{-\sum_{i=1}^{2m}\sum_{j=1}^{2m} p_i p_j \phi_{ki}^c \phi_{kj}^c \alpha_i \alpha_j^2 A_{ij}(t) - \sum_{i=1}^{2m}\sum_{j=1}^{2m} p_i p_j \phi_{ki}^c \phi_{kj}^c \alpha_i \alpha_j \beta_j B_{ig}(t)\right\} \times S_g(\omega) d\omega\right] \tag{19}$$

$$\sigma_{\dot a}{}^2(t) \;=\; \mathrm{E}\left[\dot a_k{}^2(t)\right] \;=\; \mathrm{Re}\left[\int_{-\infty}^{+\infty}\left\{\sum_{i=1}^{2m}\sum_{j=1}^{2m} p_i p_j \phi_{ki}^c \phi_{kj}^c \alpha_i^2 \alpha_j^2 A_{ij}(t) \;+\; \sum_{i=1}^{2m} p_i \phi_{ki}^c \alpha_i \beta_i \left(\sum_{j=1}^{2m} p_j \phi_{kj}^c \alpha_j^2 \left(B_{jg}(t) + B_{gj}(t)\right)\right)\right.\right.$$

$$\left.\left. +\; \left(\sum_{i=1}^{2m} p_i \phi_{ki}^c \alpha_i \beta_i\right)^2 \right\} \times\; S_g(\omega) d\omega\right] \tag{20}$$

where

$$A_{ij}(t) \;=\; \frac{\beta_i \beta_j}{(\alpha_i - i\omega)(\alpha_j + i\omega)}\left(\mathrm{Exp}[-i\omega t] - \mathrm{Exp}[-\alpha_j t]\right)\left(\mathrm{Exp}[i\omega t] - \mathrm{Exp}[-\alpha_j t]\right) \tag{21a}$$

$$B_{jg}(t) \;=\; \frac{-\beta_j}{\alpha_j - i\omega}\left(1 - \mathrm{Exp}\left[\left(-\alpha_j + i\omega\right)t\right]\right), \quad B_{gj}(t) \;=\; \frac{-\beta_j}{\alpha_j + i\omega}\left(1 - \mathrm{Exp}\left[\left(-\alpha_j - i\omega\right)t\right]\right) \tag{21b}$$

in which $S_g(\omega)$ is the PSDF of $\ddot x_g$, and $\sigma_a{}^2(t)$, $\sigma_{a\dot a}(t)$ and $\sigma_{\dot a}{}^2(t)$ are the 0-th, 1-st and 2-nd moment functions of the absolute acceleration response at node k of the combined system, respectively. The three moment functions are dependent of time t, namely, the absolute response is the non-stationary process as transient response. Practically, integrals in equations (18) through (20) are calculated numerically in the range of frequency of interest.

The first three moments of the relative acceleration responses also can be derived in the same way as the above formulations. When a modal mass of secondary system as node ℓ is attached to the floor of primary system as node k in the combined system, the relative acceleration response $a_{\ell k}^r$ of node ℓ in terms of node k is

$$a_{\ell k}^r \;=\; a_\ell - a_k \;=\; -\,\mathrm{Re}\left[\sum_{i=1}^{2m} p_i \left(\phi_{\ell i}^c - \phi_{ki}^c\right)\alpha_i \xi_i\right] \tag{22}$$

The results of the first three moment functions are obtained by replacing ϕ_{ki}^c to "$\phi_{\ell i}^c - \phi_{ki}^c$" in equations (18) through (20), respectively.

Prediction of Maximum Response

The first three moment functions of the transient responses are formulated in the previous subsection. In this subsection, the prediction method of the maximum response of the non-stationary process is presented.

In the random vibration theory, a up and down-crossings of response level as $a(t) = \pm b$ occur in accordance with Poisson's process. For a non-stationary response, if the response $a(t)$ is a symmetric and zero-mean random process, the distribution of $a_{\max}$ can be written as follows [Amin and Gungor,1971]

$$\mathrm{Prob}\left[a_{\max} \le b, \quad 0 \le t \le t_d\right] \;=\; 1 - p_e(t_d) \;=\; \exp\left[-2\int_0^{t_d} v_b{}^+(t)dt\right] \tag{23}$$

in which t_d is the duration, p_e is the exceedence probability and $v_b{}^+(t)$ is an up-crossing rate. The up-crossing rate $v_b{}^+(t)$ can be calculated from joint density function of response and its derivative at time t as follows [Amin and Gungor, 1971]:

$$v_b{}^+(t) \;=\; \int_0^\infty \dot a \;\; f_{a\dot a}(b, \dot a; t) \;\; d\dot a \tag{24}$$

in which $f_{a\dot a}$ is the joint probability density function, and the probability density function of $a(t)$ is also defined as $f_a(a;t)$.

If the $f_{a\dot a}$ and f_a are Gaussian functions, equation (24) is derived as

$$v_b{}^+(t) \;=\; \frac{1}{2\pi}\frac{\sigma_{\dot a}(t)}{\sigma_a(t)}\sqrt{1 - \rho_{a\dot a}{}^2(t)}\,\exp\left[-\frac{1}{2\left(1 - \rho_{a\dot a}{}^2(t)\right)}\frac{b^2}{\sigma_a{}^2(t)}\right] \;+\; \frac{1}{\sqrt{8\pi}}\frac{\sigma_{\dot a}(t)}{\sigma_a{}^2(t)}\rho_{a\dot a}(t) b \exp\left[-\frac{1}{2}\frac{b^2}{\sigma_a{}^2(t)}\right] \tag{25}$$

where

$$\rho_{a\dot a}(t) \;=\; \frac{\sigma_{a\dot a}(t)}{\sigma_a(t)\sigma_{\dot a}(t)} \tag{26}$$

in which $\rho_{a\dot{a}}$ is the correlation coefficient for $a(t)$ and its derivative at time t.

In the preceding subsection, ${\sigma_a}^2$, $\sigma_{a\dot{a}}$ and ${\sigma_{\dot{a}}}^2$ are represented in equations (18) through (20), and these functions must be evaluated numerically. Thus, substituting equation (25) into equation (23), the up-crossing lebels b as the given exceedance probability p_e cannot be obtained as a closed-form. We should, therefore, solve the nonlinear equation at b, as follows:

$$g(b) \;=\; \exp\left[-2\int_0^{t_d} {\nu_b}^+(t)dt\right] \;-\; \left(1-p_e\left(t_d\right)\right) \tag{27}$$

A prediction of the maximum response is b, when $g(b)$ equals to 0 as the given p_e. Meanwhile, the authors solved this nonlinear equation by means of the Newton-Raphson scheme in the later examples.

MODAL COMBINATION RULE

Based on the modal superposition theory, we propose a modal combination rule, as follows:

$$R^2 \;=\; \sum_i \sum_j \varepsilon_{ij} R_i R_j \;+\; 2\sum_j \delta_{0j} R_0 R_j \;+\; R_0^2 \tag{28}$$

in which R is a total response, R_i and R_j are the i-th and the j-th modal responses calculated from relative acceleration spectra, R_0 is a rigid response calculated from the maximum input acceleration. ε_{ij} is a modal correlation coefficient between the i-th and the j-th relative modal responses, and δ_{0j} is a modal correlation coefficient between the j-th relative modal response and the input acceleration. These modal correlation coefficients are defined as follows:

$$\varepsilon_{ij} \;=\; \int_0^{t_d} \frac{\sigma_{ij}(t)}{\sigma_i(t)\sigma_j(t)}\; dt \tag{29}$$

$$\delta_{0j} \;=\; \int_0^{t_d} \frac{\sigma_{0j}(t)}{\sigma_0(t)\sigma_j(t)}\; dt \tag{30}$$

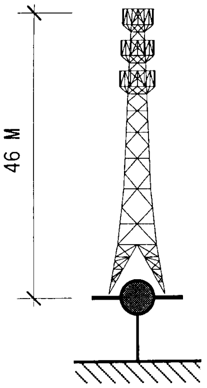
in which ${\sigma_i}^2(t)$ and ${\sigma_j}^2(t)$ are the moment functions of relative acceleration response, and ${\sigma_0}^2(t)$ is the moment function of input acceleration (or acceleration response at the floor the secondary system attached). These functions are obtained in section 2. Also, in the same way of section 2, σ_{ij} and σ_{oj} can be obtained.

ILLUSTRATIVE EXAPLES

The numerical examples are presented for an existing telecommunication tower as a secondary system attached to the SDOF primary system shown in Fig.2. The tower is modeled as a three-dimensional symmetrical truss structure with linear members, and the first three translational undamped natural periods are 0.61, 0.17 and 0.11 second. To consider various design conditions, natural periods of the primary system are set 0.15, 0.31, 0.61, 1.22 and 2.5 second as parameters corresponding to the natural period rates of the primary to secondary system (Tp/Ts) to 1/4, 1/2, 1/1, 2/1 and 4/1. Mass rate of the primary to secondary system is fixed to 100, and damping ratios of the primary and secondary system with respect to all modes are set to 0.05 and 0.02, respectively.

To evaluate applicability of the proposed approach, the results are compared with the time history analyses of the combined system. The design spectrum for the proposed method and a sample of artificial earthquakes are shown as Figs. 3 and 4. The effective duration of earthquakes t_d is set to 8.2 second from the Husid plot [A.K.Gupta, 1990], in which the rate of strong motion contribution is defined as 90%. Also, the Monte Carl simulation (MCS) of time history analyses were carried out using 100 artificial earthquake motion, to evaluate the accuracy of the proposed method.

In the first step of the proposed method, the FRSs for secondary systems are evaluated from the specified design spectra, the effective duration of the ground motion and the primary and secondary eigen properties. In the below examples, the exceedance probability p_e set to 0.15 and the effective duration t_d were fixed through the analyses. Fig.5 shows the 0-th moment functions of building response normalized by the asymptote, calculated in equation (18). It is clear that the asymptotic level is reached much more slowly in the primary system with larger natural period, especially in the case of Tb/Tt=4/1 the response of building dose not reached to stationary in the duration. Table 1 shows the maximum acceleration of input to secondary system (that is the maximum absolute acceleration response of the primary system) and the 1-st and 2-nd relative acceleration response of



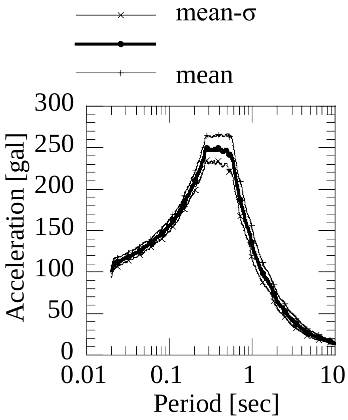


Fig 2. A Tower-Building Model   Fig 3. The Design Spectra

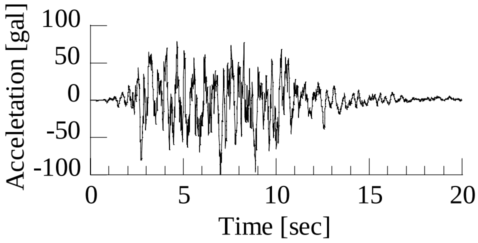


Fig 4. A Sample of Artificical Earthquakes

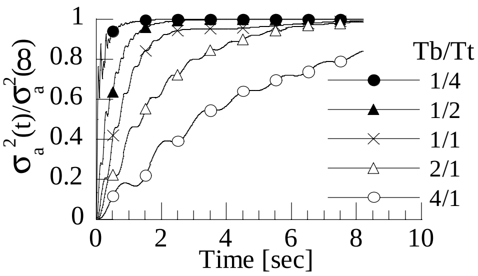


Fig 5. The 0-th Moment Functions of Building Response (Normalized by Asymptote)

secondary system, and are the comparison of the results by the proposed method with those by method assuming stationary response and MCS. The stationary method is used the compatible response/power spectrum transformation proposed by Unruh and Kana (1981) and the stationary transfer function of the combined system. The results of the proposed method show good agreement with the average of MCS except the 1-st and 2-nd relative accelerations in Tp/Ts=1/1 and 1/4. Especially, for the maximum input acceleration to secondary systems, the accuracy of the proposed method is higher than that of the stationary method. Exception occurs in the case in which the modal frequency between the secondary and primary system is close, and in this case the proposed method gives conservative results.

TABLE 1. CALCULATED FRS

T_p/T_s*)	Input acceleration[gal]		
	Proposed	Stationary	Time Analysis**)
1/4	179	187	170/179/188
1/2	246	272	230/246/262
1/1	200	247	186/204/223
2/1	106	145	93/105/120
4/1	52	90	46/53/62
T_p/T_s	The 1st relative acceleration[gal]		
	Proposed	Stationary	Time Analysis**)
1/4	410	375	343/397/459
1/2	554	533	490/565/652
1/1	1926	1684	1242/1565/1972
2/1	116	108	96/113/132
4/1	24	22	19/23/27
T_p/T_s	The 2nd relative acceleration[gal]		
	Proposed	Stationary	Time Analysis**)
1/4	1431	1280	951/1174/1452
1/2	146	143	127/145/167
1/1	28	29	27/31/35
2/1	6.6	6.4	6.2/7.2/8.3
4/1	2.2	2.0	1.9/2.2/2.6

*) The period rate of primary to secondary system
**) mean-σ /mean/mean+σ

In the second step, the responses of the secondary system are evaluated from the input acceleration for the first step. Figs.4 and 5 show the maximum acceleration response and axial stress of brace members of the tower calculated in equations (28) through (30). To evaluate the efficiency of the modal correlation coefficients, the results of the modal combination rules without ε_{ij} and without δ_{0j} are also described. In Tp/Ts=4/1, 2/1 and 1/2, the responses obtained by the proposed method agree well with the one of MCS. But, in the case of Tp/Ts=1/1 and 1/4, the proposed method gives conservative results of MCS. The main reason of these no agreement seems to be overestimation of FRSs at first step. On the other hand, as concerns the modal correlation, as shown in Figs.4 and 5 (especially in the case of Tb/Ts=1/2 or 2/1), evaluations of the total response with all terms agree well with those of MCS, however, the evaluations without δ_{0j} gives erroneous results. Thus the coefficient δ_{0j} is effective to improve the accuracy of evaluated total response. In turn, the coefficient ε_{ij} is not evaluated in the example because the closed modes of the tower do not exist. But, the coefficients ε_{ij} become much effective in the case of the system with closed space modes [Wilson, et al., 1981].

CONCLUSIONS

Seismic response spectrum analysis for secondary systems is presented, which is divided into two steps. The first step is to be evaluated seismic input as FRSs for secondary systems. Using the proposed method, one can obtain the maximum acceleration of the combined system as seismic input for secondary system, considering the infinite earthquake duration or the transient response under the stationary ground motion. The second step

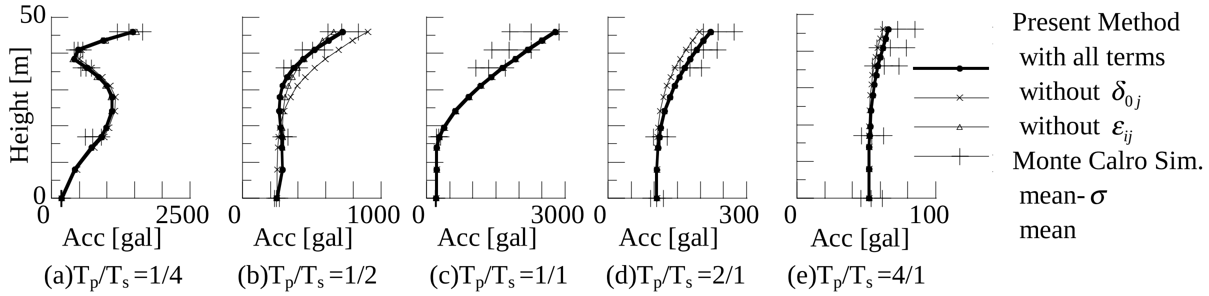


Fig 5. Maximum Acceleration of Tower

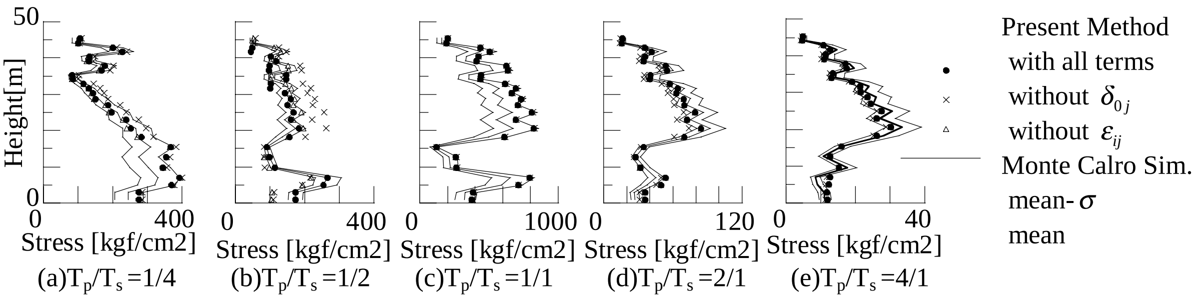


Fig 6. Maximum Stress of Brace Members of Tower

consists of the modal combination rule to evaluate maximum responses of secondary systems from seismic input given by the first method. The proposed rule is used for the evaluation of the relative modal accelerations and the input acceleration, and two kinds of correlation are considered: one among the relative modal responses and one between the relative modal response and input acceleration. To demonstrate the proposed methods, the authors applied the proposed method to a telecommunication tower built on the buildings. The results show that the numerical solution by the proposed methods is similar to the results of Monte Calro simulation (MCS) except the case in which the modal frequency between the secondary and primary system is close. In the exception case, the method gives conservative results. As shown in results, the correlation coefficient between the relative response and input acceleration is effective to evaluate total response of secondary system accurately.

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