SHEAR MODULUS OF SOILS UNDER CYCLIC LOADING AT SMALL AND MEDIUM STRAIN LEVEL

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SUMMARY

The main purpose of this paper is to investigate the stiffness of soils under cyclic loading at small and medium strain level. The influence of some important factors on the shear modulus of soils such as: the strain level, the plasticity index and the mean effective stress are discussed in detail. Behaviour of different kind of soils is also compared based on available laboratory data including some experimental results obtained by the authors.

INTRODUCTION

In many practical cases the ground response under seismic loading is evaluated using the well-known equivalent linear method in which compatible values of shear modulus and damping ratio are chosen according to the shear strain level in soil deposit. In this simplified approach the developed pore water pressure and the residual soil displacements cannot be calculated. However, if a given problem do not involve large strains ($\gamma > 10^{-2}$) the equivalent linear method can be considered acceptable in a practical point of view. This work deals in the range of small to medium strain levels ($10^{-6} \leq \gamma \leq 10^{-2}$).

The use of this method of analysis needs reliable strain-dependent shear modulus and damping ratio curves.

In this paper only the factors affecting the shear modulus of soils are treated and discussed in detail. The text is essentially divided into two parts. In the first part, the stiffness of soil at small strain level ($\gamma \approx 10^{-6}$) is analysed. In the second part, the investigation proceeds with the study of the shear modulus degradation at higher strain level until 10^{-2}.

INITIAL SHEAR MODULUS

In recent years many studies were performed to investigate the behaviour of soil at small strain level. The initial shear modulus G_0 (for $\gamma \approx 10^{-6}$) is a very important parameter not only for seismic ground response analysis but also for a variety of geotechnical applications.

A considerable number of empirical relationships have been proposed for estimating initial shear modulus for different kind of soils: [Hardin and Black, 1969], [Iwasaki and Taksuoka, 1977], [Marcusson and Wahls, 1972; Kokusho, 1972; Kokusho and Esashi, 1981 and Nishio et al., 1985 in Ishihara, 1996] and [Biarez et al., 1999].

All of these relationships are based on two experimental evidences: the shear wave velocity (v_s) is a linear function of void ratio (e) and depends on the mean effective stress (p') with a power of n/2, as proposed originally by [Hardin and Richart, 1963]:

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$$v_s = C \cdot (B - e) \cdot p'^{n/2} \tag{1}$$

in which, B, C and n are constants depending on the type of soil.

The relationship between the total mass density (ρ) and the void ratio of soil can be obtained by:

$$\rho = \rho_s \cdot \frac{1}{1+e} \tag{2}$$

in which, ρ_s is the mass density of solid particles.

On the basis of the theory of wave propagation it is well-known that the value of G_0 can be obtained from the shear wave velocity:

$$G_0 = \rho \cdot v_s^2 \tag{3}$$

By combining the results of the previous equations (1), (2) and (3), it will give the typical formula as:

$$G_0 = \rho_s \cdot \frac{1}{1+e} \cdot C^2 \cdot (B-e)^2 \cdot p'^n = A \cdot \frac{(B-e)^2}{1+e} \cdot p'^n = A \cdot F(e) \cdot p'^n \tag{4}$$

More recently, another empirical void ratio function of the form $F(e) = e^{-x}$ was proposed by other researchers: [Lo-Presti, 1998] and [Biarez et al., 1999]. This empirical function is based on experimental data and can be explained using the simple theory of Hertzian contacts for perfect spheres. According to the study of [Biarez and Hicher, 1994] a different arrangement of a group of identical spheres would be characterised by a different void ratio and a coefficient G(e) which express the arrangement of the spheres.

By considering an idealised continuous medium of spheres with identical sizes, [Santos, 1999] showed that the initial shear modulus can be expressed as a function of G(e):

$$G_0 \propto G(e)^{-\frac{2}{3}} \cdot p'^{\frac{1}{3}} \tag{5}$$

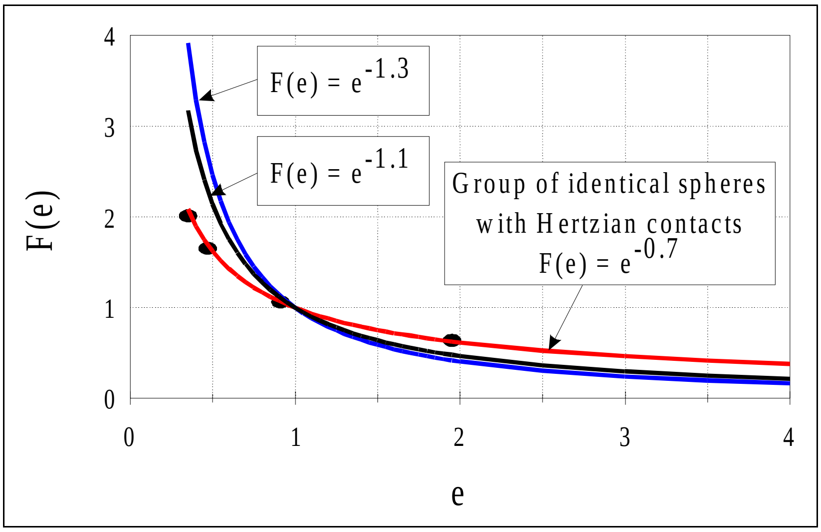

Figure 1: F(e) as a function of the void ratio for an idealised continuous medium of perfect spheres

It can be deduced that the void ratio function F(e) is proportional to $G(e)^{-\frac{2}{3}}$ and can be approximated by an exponential function (Figure 1):

$$F(e) \propto G(e)^{-\frac{2}{3}} \cong e^{-0.7} \tag{6}$$

According to [Cascante and Santamarina, 1996] the exponent n can be an indicator of the type of contact. For example, n=1/3 for contacts between spheres, whereas n=1/2 for cone-to-plane contacts. For real soils, which is a random package of particles of different sizes and shapes it was found that the exponent n can be taken equal to 1/2 for clays and also in a simplified way for sands [Biarez et al., 1999].

Based on available laboratory data mentioned before and including some experimental results obtained by the authors (resonant column tests), [Santos, 1999] proposed two unified curves that represent the lower and upper bound values of G_0:

$$G_0 = 4000 \cdot e^{-1.3} \cdot p'^{0.5} \quad , \text{ for the lower bound} \tag{7}$$

$$G_0 = 8000 \cdot e^{-1.1} \cdot p'^{0.5} \quad , \text{ for the upper bound} \tag{8}$$

These curves and the experimental data are represented in Figure 2 for a given value of p'=100kPa. The void ratio functions are also of the form $F(e) = e^{-x}$ and are plotted in Figure 1.

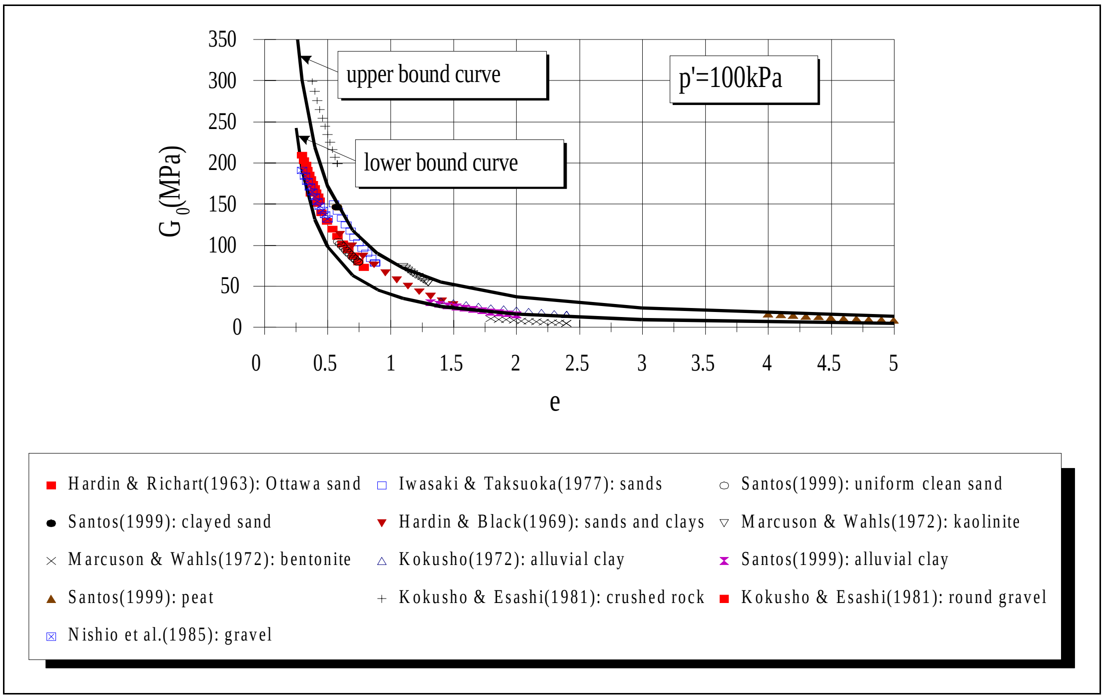

Figure 2: G_0 as a function of e

It is remarkable to observe that, in general, the experimental data lie between the two bound curves. Some discrepancies were observed only in one particular case for a crushed rock material. It is also important to emphasise that the data plotted in Figure 2 represent many experimental results obtained by different authors using several type of testing technique (seismic test, resonant column test, cyclic torsional shear test, cyclic triaxial test with local measurements). So, the proposed curves seem to be a consistent tool that can be applied for sands and clays as a guide for practical purposes. For gravels, the proposed bound curves may not be recommended because the exponent n shows also some dependency on the coefficient of uniformity, [Lo Presti, 1998]. Similar conclusions can be taken for any other value of p'.

The influence of some important factors on the initial shear modulus can be deduced from the data reproduced in Figure 2 and represented schematically in Figure 3: percentage of fines, material grading, plasticity index (PI) and stress history (overconsolidation ratio, OCR). At such small shear strain level, the effects of loading frequency (f) and the number of cycles (N) are not significant and can be neglected.

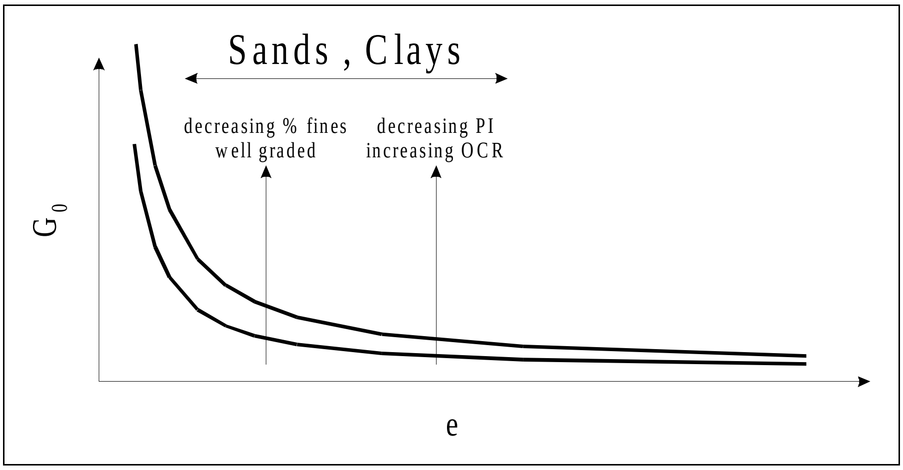

Figure 3: Factors affecting the relationship G_0-e

SHEAR MODULUS DEGRADATION FACTOR

In recent years, due to the improvements in laboratory testing (local deformation measurement and stress-path control tests), many reliable experimental data has accumulated allowing a considerable advance in the knowledge of the stress-strain behaviour of soils.

It has been demonstrated that the behaviour of soils can be described using the concept of kinematic regions in stress space. In the simplify scheme proposed by [Jardine, 1992] the current effective stress point is surrounded by two sub-yield surfaces Y1 and Y2. Inside the surface Y2 (Zone II) the response is non-linear but fully recoverable (elastic). It appears that inside Zone II a small region (Zone I-inside surface Y1) can exist in which the response is linear elastic. The surface Y2 defines the threshold at which drained or undrained cyclic loading starts to affect the soil significantly. When the Y2 surface is reached significant plastic straining begins to occur until the Y3 yield surface (Zone III).

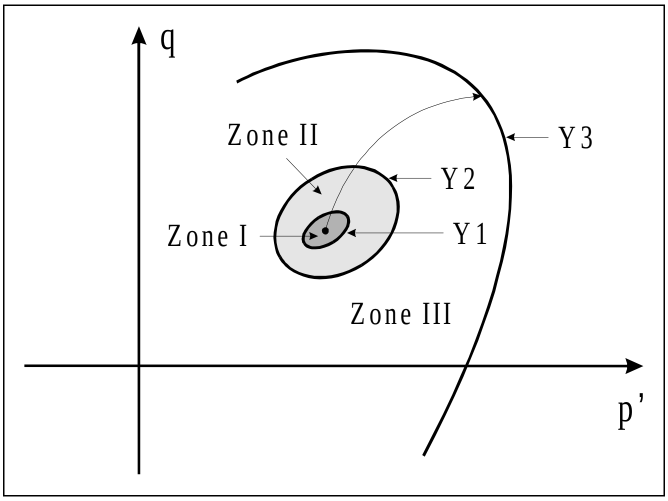

Figure 4: Scheme of multiple yield surfaces [Jardine, 1992]

Based on a similar idea [Vucetic, 1994] explained some fundamental aspects of cyclic response of saturated soils. In Figure 5, which are represented the strain-dependent shear modulus degradation factor (G/G_0) and damping ratio (ξ) of soils that author suggested the definition of two level of cyclic shear strain: the linear threshold shear strain, γ_t^e (also called the nonlinearity threshold by [Vucetic and Dobry, 1991] and [Ishihara, 1996]) and the volumetric threshold shear strain, γ_t^v. The latter is the most important and represent the limit beyond which the soil structure starts to change irreversibly: in drained conditions permanent volume change will take place, whereas in undrained conditions pore water pressure will build up.

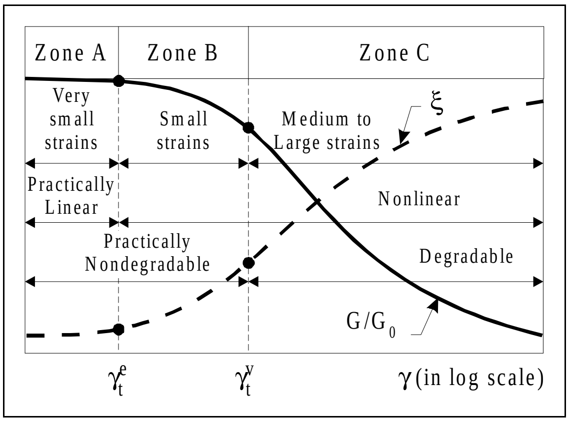

Figure 5: Typical G/G_0-γ and ξ-γ curves after [Vucetic, 1994]

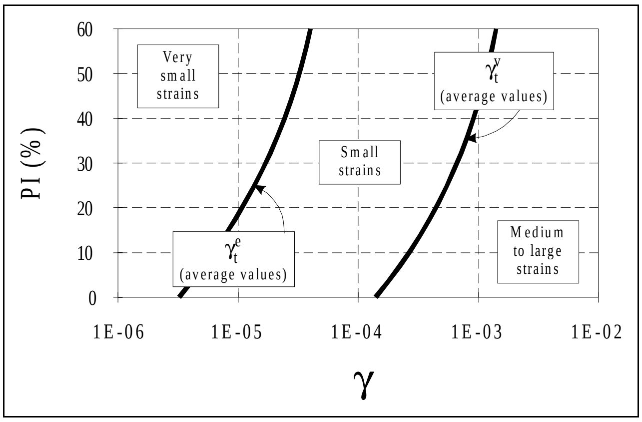

Figure 6: Threshold shear strains as a function of PI after [Vucetic, 1994]

The agreement of the two models can be clearly seen in Table 1:

Table 1: Typical soil behaviour

[Jardine, 1992]	[Vucetic, 1994]	Soil behaviour
Zone I	Zone A	Linear elastic
Zone II	Zone B	Non-linear elastic
Zone III	Zone C	Elastoplastic

[Vucetic, 1994] suggested that γ_t^v depends on soil microstructure and can be possibly correlated to the soils' PI. [Vucetic and Dobry, 1991] and [Vucetic, 1994] proposed G/G_0 degradation curves and approximate range of γ_t^v according to the plasticity index of soil (Figures 6 and 7).

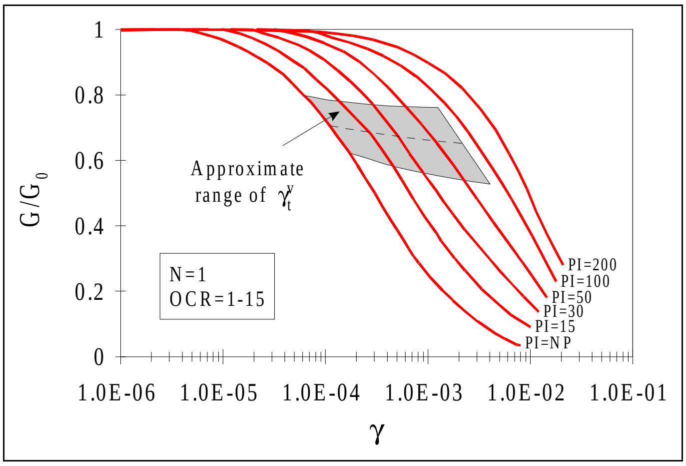

Figure 7: Stiffness degradation curves from [Vucetic and Dobry, 1991]

Indeed, the stiffness degradation curve must be affected by other factors such as: stress history, mean effective stress, number of cycles, loading frequency, etc. For example, Figure 8 shows clearly that for sands the influence of the mean effective stress can be quite important at low confining pressure conditions:

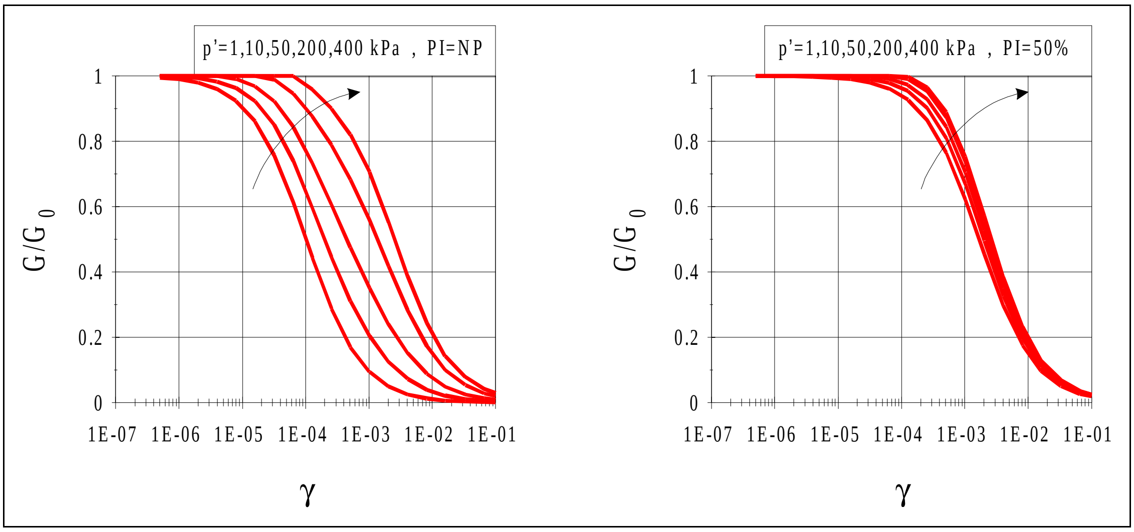

Figure 8: Stiffness degradation curves from [Ishibashi and Zhang, 1993]

At this moment, it is clear that the stiffness degradation curve can be affected by many factors and cannot be described only by the plasticity index of soil. But it seems that all the influences of these factors can be considered in a simple form when using the concept of volumetric threshold shear strain. In other words, a unique G/G_0 curve can be defined for normalised shear strains γ/γ_t^v, as originally proposed by [Santos, 1999]. Based on this key idea, in Figure 9 are represented the results of Figure 7 [Vucetic and Dobry, 1991] in normalised shear strains scale, using the average values of γ_t^v (according to PI's values as indicated in Figure 6).

As expected, there is almost a perfect coincidence of all the previous curves for PI=NP, 15, 30 and 50%. Besides, the shear strain level $\gamma=\gamma_t^v$ correspond approximately to a stiffness degradation factor of $G/G_0=0.7$ (Figure 9), i.e.:

$$\gamma_t^v \cong \gamma(G / G_0 = 0.7) \qquad (9)$$

In practice, the volumetric threshold shear strain is not easy to determine and its value increase with the plasticity index and the strain rate in cohesive soils and with the mean effective stress in cohesionless soils. For normalisation purposes a reference shear strain γ_t^r is suggested and defined as:

$$\gamma_t^r = \gamma(G / G_0 = 0.7) \qquad (10)$$

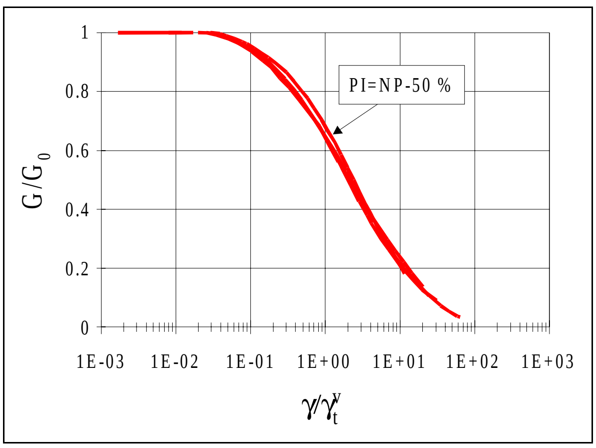

Figure 9:Results of [Vucetic et al., 1991] in γ* scale

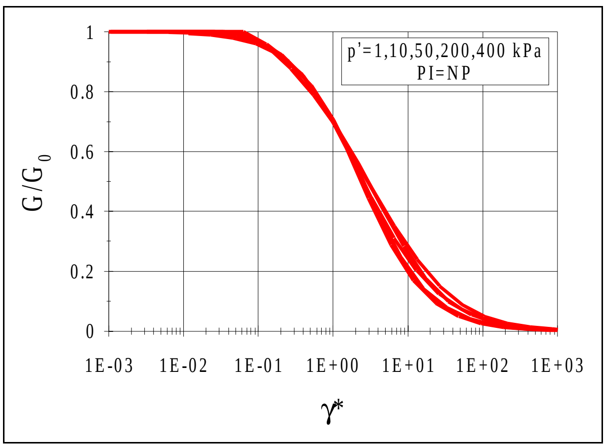

Figure10:Results of [Ishibashi et al., 1993] in γ* scale

The same key idea can be used to explain the influence of the mean effective stress in G/G_0 curves for sands. The previous results of [Ishibashi and Zhang, 1993] (Figure 8) are plotted in Figure 10 in normalised shear strains scale $\gamma^*=\gamma/\gamma_t^r$. The curves are again almost coincident.

These encouraging results show the possibility to define almost a unique relationship between G/G_0 and γ^*. [Santos, 1999] proposed two simple equations to define the lower and upper bound values of G/G_0 as a function of γ^* (for $10^{-6}\leq\gamma\leq10^{-2}$):

$$\text{lower bound} \begin{cases} 1 & ,\gamma^* \leq 10^{-2} \\ \dfrac{1-\tanh[0.48\cdot\ln(\gamma^*/1.9)]}{2} & ,\gamma^* > 10^{-2} \end{cases}$$

$$\text{upper bound} \begin{cases} 1 & ,\gamma^* \leq 10^{-1} \\ \dfrac{1-\tanh[0.46\cdot\ln(\frac{\gamma^*\text{-}0.1}{3.4})]}{2} & ,\gamma^* > 10^{-1} \end{cases} \qquad (11)$$

The previous values of [Vucetic and Dobry, 1991] and [Ishibashi and Zhang, 1993] are plotted in Figure 11 and compared with the proposed curves:

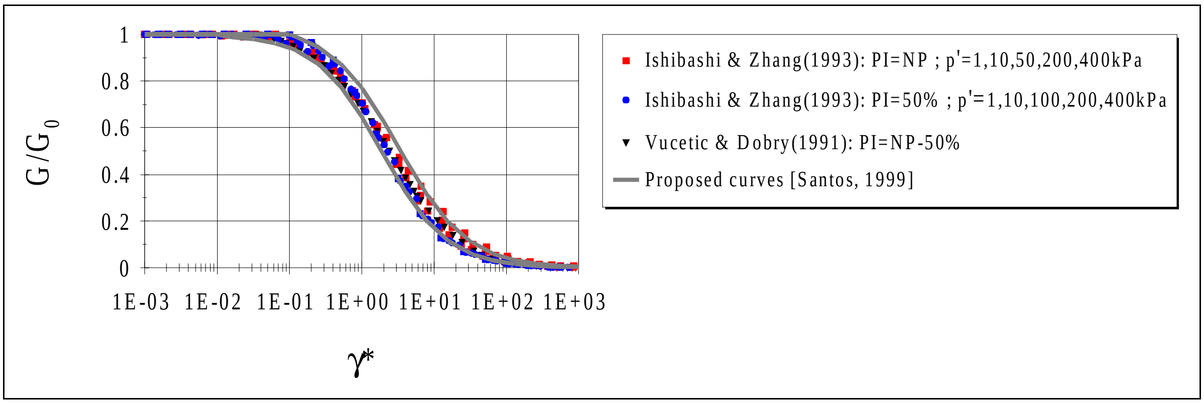

Figure 11: Proposed stiffness degradation curves in γ^* scale

All of the results seem to be in good agreement with the proposed stiffness degradation curves in normalised shear strains scale. Some experimental results obtained by the authors using the resonant column and the cyclic torsional shear test (with N=10) for Toyoura sand and for an alluvial clay (PI=41%) are also in good agreement with the proposed curves, [Santos, 1999].

CONCLUSIONS

At small deformation new unified relationships between the initial shear modulus (G_0), the void ratio and the mean effective stress, are proposed for soils (sand, silt, clay and peat). Several well-known semi-empirical correlations including some experimental results obtained by the authors (resonant column tests) are analysed and compared with the new unified relationships.

At higher strain level, the investigation proceeds with the study of the shear modulus degradation. Based on recent works of some authors [Jardine, 1992] and [Vucetic, 1994] the concept of volumetric threshold strain is used to explain the influence of the plasticity index and the mean effective stress on the shear modulus of soils. A new reference shear strain (γ_t^r) is defined for normalisation purposes. New unified stiffness degradation curves (G/G_0) are proposed in normalised shear strains scale (γ^*). Previous stiffness degradation (G/G_0) curves proposed by [Vucetic and Dobry, 1991] and [Ishibashi and Zhang, 1993] are in good agreement with the new unified curves in normalised shear strains scale. The influence of loading frequency, number of cycles and stress history on the relationship G/G_0-γ^* need to be investigated in future developments.

In conclusion this paper shows the possibility to define some simple unified relationships and curves that allow the assessment of shear modulus of soils under cyclic loading at small to medium strain level ($10^{-6} \leq \gamma \leq 10^{-2}$), for practical design purposes.

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