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A SIMPLIFIED SEISMIC RESPONSE ANALYSIS METHOD OF CYLINDRICAL LIQUID STORAGE TANKS

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SUMMARY

A one-dimensional model is described and a simplified analytical procedure is proposed for the interaction problem of tank-liquid-soil.

The non-linear behaviour of modulus and damping is accounted for by using the hyperbola model.

The structural seismic response behaviour is examined.

The influence of tank geometric parameter is also studied.

INTRODUCTION

The volumes of liquid storage tanks are very large in practice. The assumption that the foundation is rigid is unreasonable, if soil deposit natural period is long. By now, most research applied the finite element method to consider the soil deposit flexibility. Tanks and soil deposit are analysed in three-dimension and calculating work is very large. Hence, it is necessary to propose a convenient and practical method for the design of liquid storage tanks.

On the fluid-elastic vibration of cylindrical liquid storage tank and liquid, usually there are two analysis methods: beam vibration analysis method (Veletosos) and elastic thin shell analysis method. The former is convenient and practical, and the latter is more accurate but complex. Some research [6] have shown that liquid storage tanks mainly appear beam vibration behaviour in the earthquake. Because the ratio between height and radius of most tanks is small, the authors apply the model of shear cantilever beam for the responses of the tank and apply the velocity potential theories for the responses of fluid in this paper.

In this thesis, one-dimensional wave propagation method is used for analysing the seismic responses of the horizontal layers viscoelasticity soil deposit [4], and the shear cantilever beam method is used in liquid storage tanks [3]. The whole system is simplified into one-dimensional shear model. By the displacement and stress continuous condition of soil surface and tank bottom plate, the system dynamic response complex transfer functions with respect to base rock acceleration are derived, so the problem is solved.

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ANALYSIS METHOD

Analysis Model

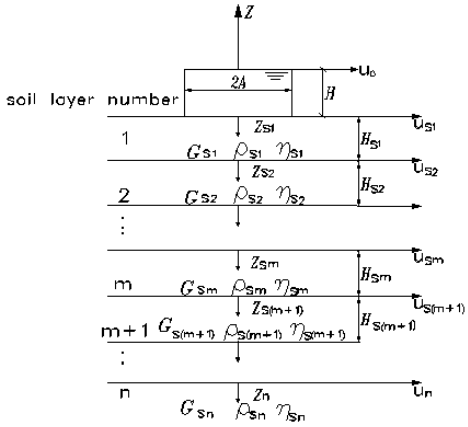

Figure 1: One-dimension system

A liquid storage tank on the horizontal layers viscoelasticity soil deposit is shown in Figure 1. The system consists of N horizontal soil layers. Each layer is characterised by the thickness, H_{sm}, mass density, ρ_{sm}, shear modulus, G_{sm}, viscosity damping, η_{sm}, horizontal displacement, u_{sm} The method is based on the assumption that: The main responses in a soil deposit are caused by the upward propagation of shear waves from the underlying rock; Each layer is homogeneous and isotropic; Each layer extends to infinity in the horizontal direction and has a halfspace as the bottom layer; The liquid storage tank generates shear cantilever beam vibration in the earthquake; The flow of the liquid in the tank is inviscid, incompressible and irrational, The tank uplift is neglected and the shell is ideally cylindrical.

Transfer Function

A liquid storage tank is given an impulsive velocity $\dot{x}_0(t)$ in the horizontal direction. Assuming that the tank wall is a shear cantilever beam, its elastic deformation is given as follows:

$$u_0(z,t)=\sum_{j=1}^{\infty} q_j(t)X_j(z) \tag{1}$$

where $X_j(z)$ is the j mode function, $q_j(t)$ is the corresponding generalised co-ordinate.

The fluid velocity potential φ must satisfy the following Laplace equation [1]. We can obtain the following dynamic fluid pressure response equation on the tank wall [3]:

$$p(a,\theta,z,t)=-\rho\frac{\partial\varphi}{\partial t}=-\rho\sin\theta\sum_{n=1,3}^{\infty}\frac{4c_n}{n\pi}Q_n(a,t)\cos\frac{n\pi(z+h)}{2h} \tag{2}$$

where $c_n = I_1\left(\frac{n\pi}{2h}a\right)\Big/ I_1'\left(\frac{n\pi}{2h}a\right)$; a is the radius of the tank; ρ is the fluid mass density;

$I_1(r)$ is the imaginary Bessel function of the first kind of order 1; h is the height of the tank;

$$Q_n(a,t)=\frac{2h(-1)^{\frac{n-1}{2}}\ddot{x}_0(t)}{n\pi}+\sum_{j=1}^{\infty}\ddot{q}_j(t)\int_{-h}^{0}X_j(z)\cos\frac{n\pi(z+h)}{2h}dz \tag{3}$$

A liquid storage tank natural vibration motion satisfies the following equation:

$$\rho_1 F\frac{\partial^2 u_0(z,t)}{\partial t^2}=\frac{GF}{K_0}\frac{\partial^2 u_0(z,t)}{\partial z^2}+S^0(z,t) \tag{4}$$

where G is shear modulus of the material of the tank, F is the tank wall cross section area. k_0 is the shear section coefficient. ρ_1 is the material mass density. $S^0(z,t)$ is the dynamic fluid pressure of the unit height on the tank wall. Hence, we can obtain the natural frequency ω_j and the corresponding mode X_j by the resolution of the natural equation(4). The results show that the difference of the modes of the tank full of fluid and void tank is very small [3], so the author substitutes the latter for the former

$$X_j(z)=\sin\frac{(2j-1)(z+h)\pi}{2h} \quad \text{(j=1,2,3)} \tag{5}$$

When the tank bottom plate is given an impulsive displacement $x_0(t)=e^{i\omega t}$, the generalised co-ordinate $q_j(t)$ is shown as follows:

$$q_j(t)=\eta_j(\omega/\omega_J)^2 e^{i\omega t}\Big/\left[1-(\omega/\omega_J)^2+2\varepsilon_j i\,\omega/\omega_J^2\right] \tag{6}$$

$$\varepsilon_j=B_j/A_j\ ;\ \eta_j=E_j/A_j\ ;\ B_j=\varepsilon\rho_1 F\int_{-h}^{0}X_j^2(z)dz=\frac{h}{2}\varepsilon\rho_1 F\ ;\ A_j=\rho_1 F\frac{h}{2}+\rho a\sum_{n=1,3\cdots}^{\infty}\frac{4c_n}{n}P_{jn}^2\ ;$$

$$E_j=\rho_1 F\frac{2h}{j\pi}\left(1-\cos\frac{j\pi}{2}\right)+\rho a\sum_{n=1,3\cdots}^{\infty}(-1)^{\frac{n-1}{2}}\frac{8c_n h}{n^2\pi}P_{jn}\ ;\ P_{jn}=\int_{-h}^{0}X_j(z)\cos n\pi(z+h)/(2h)$$

where ε_J is called the j mode dumping coefficient, and η_j is called the j mode participating coefficient.

According to wave motion theory [4], Vertical propagation of shear waves of frequency ω through the system will cause soil horizontal displacements and accelerations as follows:

$$u_{sm}(z_{sm},t)=E_m e^{i(K_{sm}z_{sm}+\omega t)}+F_m e^{-i(k_{sm}z_{sm}-\omega t)} \tag{7}$$

$$ac_{sm}(z_{sm},t)=-\omega^2\left(E_m e^{i(K_{sm}z_{sm}+\omega t)}+F_m e^{-i(k_{sm}z_{Sm}-\omega t)}\right) \tag{8}$$

where the right first term represents the incident wave travelling in the negative z_{sm} direction(upwards)and the right second term represents the reflected wave travelling in the positive z_{sm} direction(downwards), z_{sm} represents the vertical co-ordinate of layer m. $K_{sm}^2=\omega^2/V_{sm}^{*\,2}=\rho_{sm}\,\omega^2/G_{sm}^*$. V_{sm}^* is the complex shear wave velocity of the layer m. $G_{sm}^*=G_{sm}+i\omega\eta_{sm}$ represents the complex shear modulus of the layer m, The critical damping ratio, β_{sm}, is related to the viscosity η_{sm} by $\omega\eta_{sm}=2G_{sm}\beta_{sm}$.

Experiments on many soil materials indicate that G_{sm} and β_{sm} are nearly constant over the frequency range, which is of main interest in the analysis. It is therefore convenient to express the complex shear modulus in terms of the critical damping ratio instead of the viscosity:

$$G_{sm}^{*} = G_{sm}\left(1 + 2i\beta_{sm}\right) \tag{9}$$

Soil shear stress is written in the following form:

$$\tau_{sm}\left(z_{sm}, t\right) = G_{sm}\frac{\partial u_{sm}}{\partial z_{sm}} + \eta_{sm}\frac{\partial^{2} u_{sm}}{\partial z_{sm}\partial t} = iK_{sm}G_{sm}^{*}\left(E_{m}e^{iK_{sm}z_{sm}} - F_{m}e^{-iK_{sm}z_{sm}}\right)e^{i\omega t} \tag{10}$$

The equation (7), (8) and (10) are available to each layer soil. Stresses and displacements must be continuous at all interfaces. Hence, the amplitudes, E_{m+1} and F_{m+1}, of the incident and reflected wave in layer m+1, are expressed in terms of the amplitudes in layer m:

$$E_{m+1} = 0.5E_{m}\left(1 + \lambda_{m}\right)e^{iK_{sm}H_{sm}} + 0.5F_{m}\left(1 - \lambda_{m}\right)e^{-iK_{sm}H_{sm}} \tag{11}$$

$$F_{m+1} = 0.5E_{m}\left(1 - \lambda_{m}\right)e^{iK_{sm}H_{sm}} + 0.5F_{m}\left(1 + \lambda_{m}\right)e^{-iK_{sm}H_{sm}} \tag{12}$$

Where λ_m is the complex impedance ratio that is independent of frequency.

$$\lambda_{m} = K_{sm}G_{sm}^{*} \Big/ \left(K_{s(m+1)}G_{s(m+1)}^{*}\right) = \left[\rho_{sm}G_{sm}^{*} \Big/ \left(\rho_{s(m+1)}G_{s(m+1)}^{*}\right)\right]^{\frac{1}{2}} \tag{13}$$

When the tank bottom plate is given a displacement $x_{0}(t) = u_{s1}\left(z_{s1}, t\right)\big|_{z_{s1}=0} = e^{i\omega t}$, the shear stress of tank bottom plate is given as follows:

$$\tau_{t} = Q/S = GF \Big/ \left[K_{0}(a+d)^{2}\right]\sum_{j=1}^{\infty}\frac{2j-1}{2h}q_{j}(t) \quad \text{j=1,2,3} \tag{14}$$

where d is the tank wall thickness, S is the area of the tank bottom plate; Q is the shear of the tank bottom plate.

The shear stress of soil surface is written in the form:

$$\tau_{s1}(0, t) = iK_{s1}G_{s1}^{*}e^{i\omega t} \tag{15}$$

By the continuous conditions of the shear stress and displacement of soil surface: $u_{s1}(0,t) = x_{0}(t)$ $\tau_{s1}(0,t) = -\tau_{t}$, we can obtain the formulas as follows:

$$E_{1} = 0.5\left(1 + \lambda_{0}\right);\ F_{1} = 0.5\left(1 - \lambda_{0}\right) \tag{16}$$

where λ_0 is a complex constant, $\lambda_{0} = \mu\sum_{j=1}^{\infty}\frac{2j-1}{2h}q_{j}(t) \Big/ \left[(a+d)^{2} iK_{s1}G_{s1}^{*}e^{i\omega t}\right],\ \mu = \frac{GF}{K_{0}}$

Hence, the complex transfer functions of every layer of soil response (acceleration) with respect to base rock acceleration are easily derived:

$$ac\_ac_{m,n}(\omega) = ac_{sm}\left(z_{sm}, t\right) / ac_{sn}(0, t) = E_{m}e^{iK_{sm}z_{sm}} + F_{m}e^{-iK_{sm}z_{sm}} \Big/ E_{n} + F_{n} \tag{17}$$

The complex transfer functions of the tank wall dynamic fluid press, acceleration with respect to base rock acceleration:

$$p\_ac_{z,n}(\omega) = -\rho \sin\theta \sum_{n=1,3\cdots}^{\infty} \frac{4c_n}{n\pi} Q_n(a,t) \cos\frac{n\pi(z+h)}{2h} \Big/ \left[e^{i\omega t}\omega^2 (E_n + F_n)\right] \quad (18)$$

$$ac\_ac_{z,n}(\omega) = \sum_{j=1}^{\infty} X_j(z)\bar{q}_j(t) \Big/ (E_n + F_n) \quad (19)$$

where $x_0(t) = e^{i\omega t}$; $\bar{q}_j(t) = q_j(t)/e^{i\omega t}$

Seismic Response Analysis

When base rock is inputted an earthquake acceleration record $ac(t)$, by the Fourier transformation, the acceleration Fourier spectrum $ac(\omega)$ is obtained (called input spectrum). Multiplying formulas (17)~(19) by $ac(\omega)$, soil and liquid storage tank seismic response output spectrums are obtained, then their time domain responses are given by the Fourier inverse transformation.

Soil Non-linear

In this paper, the nonlinearity of modulus and damping is accounted for by the use the hyperbola model [1]:

$$G_{eq} = G_{\max} / (1 + \gamma/\gamma_r), \; \beta_{eq} = \beta_{\max} \, \gamma/\gamma_r / (1 + \gamma/\gamma_r) \quad (20)$$

where $G_{\max}$ is the maximum shear modulus, $\beta_{\max}$ is the maximum damping ratio, γ_r is the reference strain.

Soil non-linear iterative procedure can be finished in frequency domain. According to the complex transfer function of every layer of soil shear stain and acceleration spectrum of base rock, the output spectrum of shear strain is obtained. Then the maximum shear strain in the time domain is given by the method [1]. Multiplying it by 0.65, we can obtain the effective shear strain. Using an iterative procedure gets the values for modulus and damping ratio compatible with the effective strains.

EXAMPLE

According to the method in this paper, the author designs a computer program and calculates the following three examples.

Example 1: A concrete liquid storage tank on the horizontal layers soil deposit is characterised by the height, H=10.0m, the tank wall thickness, D=0.5m, radius, R=9.0m, the tank bottom plate depth, H_b=1m, mass density, $\rho = 2450 kg/m^3$, shear modulus, G=8700MPa, $\upsilon = 0.17$, shear section coefficient, K_0=2,and damping ratio, β_t=0.05. The soil deposit is characterised by the depth, H=10m, mass density, $\rho = 1900 kg/m^3$, shear modulus, G=200MPa, $\upsilon = 0.4$, and damping ration, β=0.1.

The author calculate this example by a finite element method program(the base is cylindrical, shown in Figure 2, the depth, H=10m, radius, R=25m, 3285 nodes, 2616 elements, the bottom layer is fixed).

Base rock is exited by El-centro wave, and its maximum acceleration is 0.2g, By the two methods, the maximum accelerations of the system are shown in Figure 3. First natural frequency are shown in Table 1, We can conclude the differences of two methods are allowable, this simplified method can satisfy engineering precision.

Table 1

This paper	The finite element method	Difference
7.62hz	7.30hz	4.4%

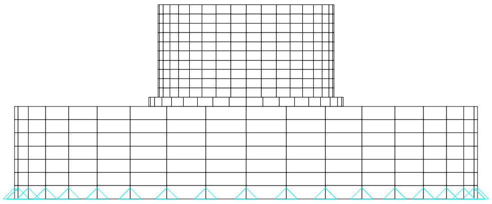

Figure 2: Tank finite element grid

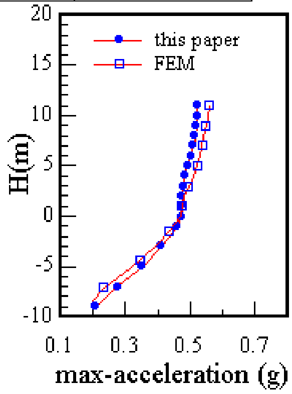

Figure 3:Maximum acceleration

Example 2: A liquid storage tank on the horizontal layers soil deposit shown in Figure 4 is characterised by the height, H=20.0m, the tank wall thickness, D=0.022m, radius, R=25.0m, mass density, $\rho = 7800\,kg/m^3$, elasticity modulus, $E = 2.1 \times 10^6\ \ kg/cm^2$, $\upsilon = 0.28$, shear section coefficient, K_0=2,and damping ratio, β_t=0.05, The fluid mass density, $\rho = 800\,kg/m^3$. The soil deposit is divided into three layers characterised by

Table 2

m	H_{sm}(m)	$\rho_{sm}(kg/m^3)$	G_{sm}(MPa)	β_{sm}
1	10	1900	Equation 21	0.07
2	10	2000	Equation 21	0.06
3	10	2100	Equation 21	0.05

The soil shear modulus is written in the following form [2]:

$$G_{sm} = V_{s0}{}^2 \rho_{sm} \left(\left(\sum_{k=1}^{sm-1} h_{sk} + h_{sm}/2 \right)^{0.25} \right) \qquad (21)$$

where V_{s0} is the shear wave velocity of the surface soil, and h_{sk} is the k layer soil thickness; V_{s0}=220 m/s. Base rock shear modulus is 550MPa, and the mass density, 2200 kg/m^3, damping ratio, $\beta = 0.04$.

Base rock is exited by El-centro wave whose maximum acceleration is 0.2g. The calculation result is shown in Figure 5~Figure 6. The system accelerations complex transfer function with respect to base rock acceleration (displacement magnify multiple) are shown in Figure 5; The system maximum acceleration in time domain are shown in Figure 6, The maximum dynamic fluid press on tank wall in time domain is given in Figure 7. In these results, the maximum acceleration on the top of tank wall is 0.93g, the maximum dynamic press on the tank wall appears 6m above the tank bottom plate, and its value is 74kPa.

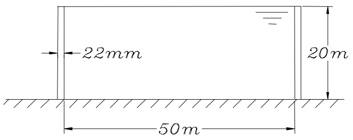

Figure 4: Tank

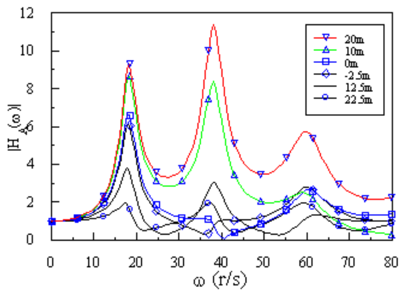

Figure 5:Acceleration transfer function

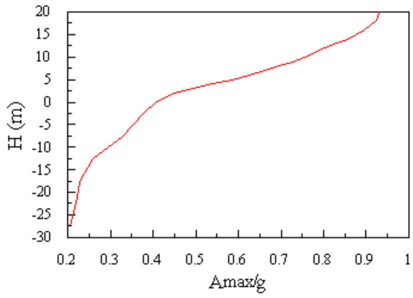

Figure 6:Maximum acceleration

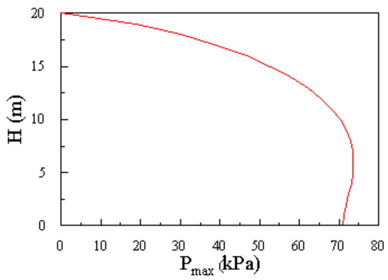

Figure 7: Max-dynamic fluid press

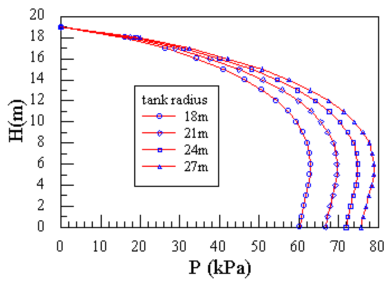

Figure 8: Max-dynamic fluid press

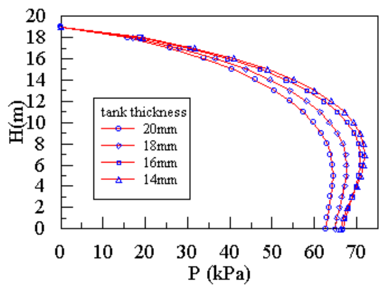

Figure 9:Max-dynamic fluid press

Example 3: When the tank shown as example 2 varity in tank wall thickness and tank radius, we calculate some instances. The results are shown as figure 8 and figure 9. We can see that the decreases of tank wall thickness results in the increase of maximum dynamic fluid press on tank wall and the increase of tank radius results in the increase of maximum dynamic fluid press on tank wall.

In summary, the results of these examples are reasonable, and any response analysis in time domain and in frequency domain is finished in less one minute (pc486). Hence, the method in this paper is convenient and practical.

CONCLUSION

By the one-dimensional analysis method of the liquid storage tank on the horizontal layers soil deposit in this paper, any maximum values of seismic response in the time domain can be obtained in the frequency domain. The whole seismic response in the time domain can also be obtained by the Fourier inverse transformation.

In addition, the random vibration response can be analysed by this method. The method proposed in the reference [5] can be adopted, if the non-linear and random of the materials are considered simultaneously.

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