0432

# SYSTEM IDENTIFICATION OF THE BASE ISOLATED STRUCTURE BY PREDICTION ERROR METHOD USING RECORDED SEISMIC RESPONSE DATA UNDER HYOGOKEN-NANBU EARTHQUAKE

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## SUMMARY

A procedure for nonlinear system identification using prediction error identification method with state-space description (PEM-SS) is presented. System identifications of the base isolated structure by this method using recorded seismic response data under Hyogoken-Nanbu earthquake are carried out. As the result, PEM-SS using nonlinear MDOF model is found to be very effective to identify the dynamic characteristics of the base isolated structure. The results using several nonlinear SDOF and MDOF models indicate that MDOF model considering both bilinear force displacement relation and viscous damping of the base isolated story is most appropriate for this structure.

## INTRODUCTION

Hyogoken-Nanbu earthquake in 1995 produced an important set of strong-motion recordings from a large number of buildings, including the base isolated structures. It seems that some of these recordings indicate response nonlinearities. Therefore, it is obvious that improvement of the identification methods, which are able to estimate nonlinear behaviour of structures accurately, is more important. Many kinds of identification methods, which are based upon time domain analysis and state-space description of target models, have been proposed. These time domain methods are able to be applied easily to the identification using nonlinear model than frequency domain methods. State-space model is convenient to describe MIMO system and to consider random noises such as modelling errors, immeasurable disturbances, and measurements errors. Extended Kalman filter (EKF) method is one of them, and is broadly applied to nonlinear system identification. However, in the case of many unknown model parameters, the identification using EKF requires both long-length data and appropriate initial values of the parameters, to prevent them from leading to the problem that divergence of error covariance or the parameters. Prediction-error identification method with state-space description (PEM-SS), which is used in this study, is one of them too. However, system identification using PEM-SS requires smaller numbers of initial values of the parameters than that using EKF. Therefore, this method is thought to be more practical to identify real existing structure that indicates response nonlinearities.

In this study, a procedure for nonlinear system identification using PEM-SS is presented, and system identifications of the base isolated structure by PEM-SS using recorded seismic response data under Hyogoken-Nanbu earthquake are carried out.

## METHOD OF IDENTIFICATION

### Prediction-error method (PEM) with LTI State-Space models:

Consider a system described by

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}(\theta)\boldsymbol{x}_k + \boldsymbol{B}(\theta)\boldsymbol{u}_k + \boldsymbol{\Gamma}\boldsymbol{v}(k) \tag{1}$$

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$$\boldsymbol{y}_k = \boldsymbol{C}\boldsymbol{x}_k + \boldsymbol{v}(k) \tag{2}$$

where $\boldsymbol{x}_k$ and $\boldsymbol{y}_k$ are the system state vector and the measurement vector. $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are the time-invariant matrices. $\boldsymbol{\theta}$ is a vector of the model parameters. $\boldsymbol{\Gamma v}$ and $\boldsymbol{v}$ are the process and measurement noises.

The predicted output at time $k$ using above model are given by

$$\hat{\boldsymbol{y}}_{k+1} = \boldsymbol{C}\left[(\boldsymbol{A}+\boldsymbol{\Gamma}\boldsymbol{C})\hat{\boldsymbol{x}}_k + \boldsymbol{\Gamma}\boldsymbol{y}_k + \boldsymbol{B}\boldsymbol{u}_k\right] \tag{3}$$

By defining the prediction error vector $\boldsymbol{\varepsilon}_k$ and matrix $\boldsymbol{E}$ as follows:

$$\boldsymbol{\varepsilon}_k = \boldsymbol{y}(k) - \hat{\boldsymbol{y}}(k), \boldsymbol{E} = \left[\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \ldots, \boldsymbol{\varepsilon}_N\right] \tag{4}$$

where $N$ is the number of data.

The model parameter $\boldsymbol{\theta}$ are then estimated by minimization of a defined scalar-valued index function $J$:

$$J(\boldsymbol{\theta}) = det\left(\frac{1}{N}(\boldsymbol{E}^T \times \boldsymbol{E})\right) \rightarrow min. \tag{5}$$

**Extension of PEM for Nonlinear System Identification:**

For a nonlinear structural system of MDOF, the equation of motion can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{Q}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ldots, t, \boldsymbol{\theta}) = \boldsymbol{F}(t) \tag{6}$$

where $\boldsymbol{M}$ and $\boldsymbol{D}$ respectively are the mass and damping matrices, $\boldsymbol{x}$ is the relative displacement vector to the ground, $\boldsymbol{F}$ is the input force, and $\boldsymbol{Q}$ is the nonlinear restring force vector. Then state-space equation can be written as

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ddot{\boldsymbol{x}}, \ldots, t, \boldsymbol{\theta}, \boldsymbol{F}(t)) \tag{7}$$

$$\boldsymbol{y} = \boldsymbol{C}(\boldsymbol{x}) \quad (8$$

Following two techniques are used for nonlinear system identification.

<u>Technique-A</u>: divide the restring force $\boldsymbol{Q}$ into $\boldsymbol{K}$, which are in proportion to $\boldsymbol{x}$, and the residuals $\boldsymbol{R}$ as follows

$$\boldsymbol{Q} = \boldsymbol{K}\boldsymbol{x} + \boldsymbol{R} \tag{9}$$

Then equation of motion can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} + \boldsymbol{K}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ldots, t, \theta)\boldsymbol{x} = F(t) - \boldsymbol{R}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ldots, t, \theta) \tag{10}$$

<u>Technique-B</u>: consider the restring force as the input force. Then equation of motion can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{D}\dot{\boldsymbol{x}} = \boldsymbol{F}(t) - \boldsymbol{Q}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \ldots, t, \boldsymbol{\theta}) \tag{11}$$

By these techniques, the equation of motion for the nonlinear structural system can be converted into an equivalent one for the linear time-varying structural system. Therefore, state equations are obtained as follows

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} \qquad \text{: Technique-A} \tag{12}$$

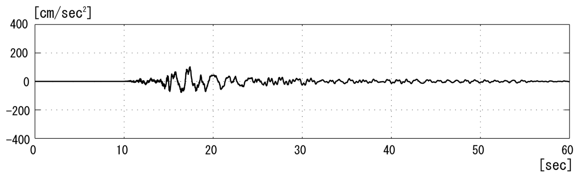
$$\dot{x} = Ax + B(t)u \qquad : \text{Technique-B} \tag{13}$$

In this paper, Technique-A was applied to the identification using the bilinear force-displacement relation model. Technique-B was applied to the identification using the trilinear force-displacement relation model. By transforming these continuous state equations into discrete ones, identification of nonlinear system using PEM-SS is able to be performed in a similar way of LTI state-space model.

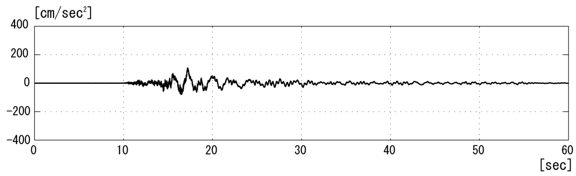
## IDENTIFICATION OF DYNAMIC CHARACTERISTICS OF THE BASE ISORATED STRUCTURE

### Outline of The Structure and Recorded Seismic Response Data:

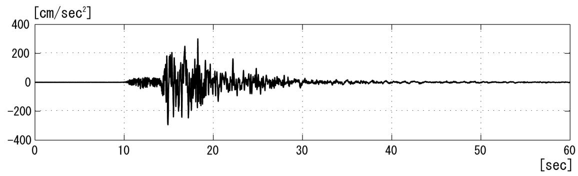
West Building, which is identified in this study, is located in Kobe City, Hyogo Prefecture, approximately 30 km north from the earthquake epicentre. It is a 6story steel encased reinforced concrete frame structure supported on 54 lead-rubber isolators, 66 rubber isolators, and steel dampers. The acceleration sensors were installed on the foundation, first story, and sixth story. To identify the dynamic characteristics, a set of recordings obtained during Hyogoken-Nanbu Earthquake of magnitude 7.2 (JMA) January 17,1995, were used. Figure 1 shows the time histories of absolute accelerations of the foundation, the first story, and the sixth story in E-W direction.




**(a) Absolute Acceleration of sixth story**




**(b) Absolute Acceleration of first story**




**(c) Absolute Acceleration of Foundation**

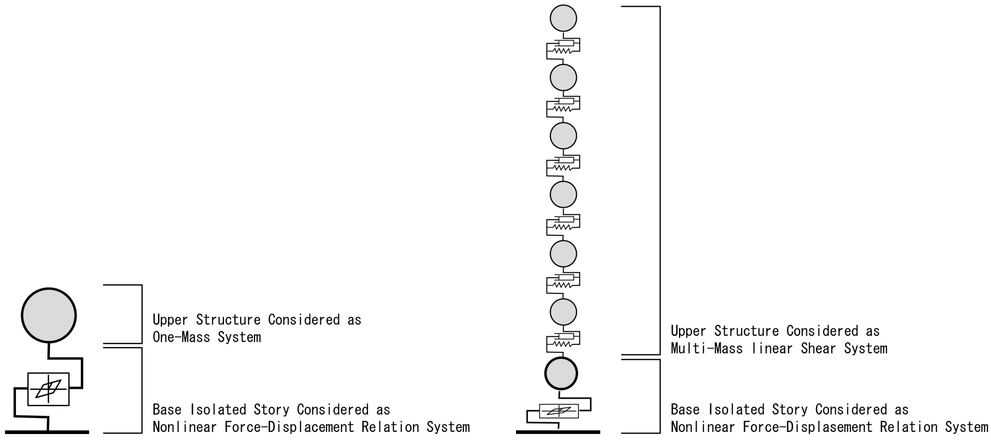
**Figure 1: Time Histories of Acceleration Response**

The peak foundation acceleration was 300 gal (0.31g), the peak of the first story and the sixth story accelerations were 106 gal (0.11g) and 103 gal (0.11g), respectively. Time history of recorded acceleration of the first story corresponds well to that of sixth story. So it is considered that the upper structure was shaken in almost the same phase and amplitude.

For the identification, recorded seismic response data of E-W direction were used. The recorded absolute acceleration response at the foundation was used as input. The relative velocity responses calculated from the first story, the sixth story, and the foundation acceleration responses were used as output variables.

### Identification using Bilinear Force-Displacement Relation Models:

Several structural models, which are shown in Figure 2, were applied for the identification. In all models, the restring forces of base isolated stories were supposed to be a bilinear force-displacement relation, and mass of the every story was supposed to be a known parameter.




**(a) SDOF model** **(b) MDOF model**

**Figure 2: Bilinear Force-Displacement Relation Models**

In Case-A (1), (2), the upper structure was supposed to be a rigid body, i.e., the structure was supposed to be SDOF model. In other cases, the upper structure was supposed to be multi mass linear shear system. In Case-B (1), (2), the stiffness and the damping coefficient of the upper structure were supposed to be known parameter, in Case-C (1), (2), only the damping coefficient of the upper structure was supposed to be known parameter. In Case-D (1), (2), all parameters of the upper structure without the mass were supposed to be unknown parameters.

Additionally the results are provided in each case under the following conditions:

Case-* (1): neglecting the viscous damping of the base isolated story
Case-* (2): considering the viscous damping of the base isolated story

Estimated and initial values of model parameters, and the value of index function $J_n$ are shown in Table 1. In Table 1, the viscous damping coefficient, the primary stiffness, and the yeiled displacement of base isolated story are described as $c_0$, ${}_1k_0$, $X_{y,}$ respectively. The value that secondary stiffness divided by primary stiffness are described as $\alpha_{1.}$ And the stiffness and the viscous damping of i story is described as $k_i$, $c_i$.

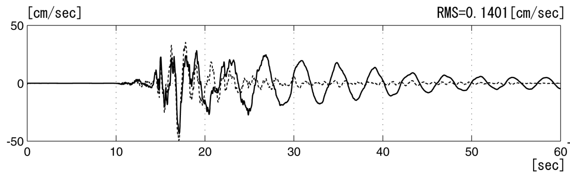
**Table 1: Results of Identification using Bilinear Force-Displacement Relation Models**

| | | Desinged values | Initial values | SDOF model | | MDOF model | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Case-A (1) | Case-A (2) | Case-B (1) | Case-B (2) | Case-C (1) | Case-C (2) | Case-D (1) | Case-D (2) |
| Parameters of Base Isolated Story | $J_n$ | - | - | 0.01 | 0.01 | 0.76 | 0.05 | 0.58 | 0.05 | 23.91 | 50.59 |
| | ${}_1k_0$[t/cm] | - | 500 | 252 | 192 | 753 | 832 | 874 | 765 | 672 | 986 |
| | $c_0$[t*sec/cm] | - | 50 | - | 57 | - | 72 | - | 69 | - | -41 |
| | $X_y$[cm] | - | 1.00 | 2.55 | 0.97 | 2.99 | 0.93 | 1.88 | 0.88 | 2.49 | 1.42 |
| | $\alpha$ | - | 0.50 | 0.75 | 0.82 | 0.23 | 0.39 | 0.25 | 0.05 | 0.77 | 0.45 |
| Parameters of Upper Structure | $k_1$[t/cm] | 65509 | 60000 | - | - | - | - | 64607 | 70108 | 55028 | 42219 |
| | $k_2$[t/cm] | 57592 | 50000 | - | - | - | - | 29632 | 27096 | 22565 | 40885 |
| | $k_3$[t/cm] | 11931 | 10000 | - | - | - | - | 12956 | 13068 | 16589 | 12665 |
| | $k_4$[t/cm] | 9014 | 9000 | - | - | - | - | 8840 | 8987 | 11225 | 11225 |
| | $k_5$[t/cm] | 7487 | 7000 | - | - | - | - | 9412 | 9443 | 6137 | 6924 |
| | $k_6$[t/cm] | 5986 | 5000 | - | - | - | - | 6784 | 6561 | 5387 | 5523 |
| | $c_1$[t*sec/cm] | - | 72 | - | - | - | - | - | - | 635 | 725 |
| | $c_2$[t*sec/cm] | - | 60 | - | - | - | - | - | - | 104 | 155 |
| | $c_3$[t*sec/cm] | - | 12 | - | - | - | - | - | - | -12 | -33 |
| | $c_4$[t*sec/cm] | - | 10.8 | - | - | - | - | - | - | 126 | 40 |
| | $c_5$[t*sec/cm] | - | 8.4 | - | - | - | - | - | - | 345 | 182 |
| | $c_6$[t*sec/cm] | - | 6 | - | - | - | - | - | - | 59 | 43 |

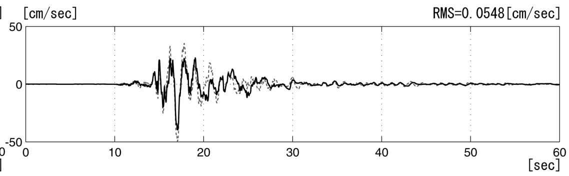
Generally speaking, in case of identification of actually existing structure, the real or true values of the parameters are often unknown. Therefore, the accuracy of the estimated parameters is examined by comparing the index values. In Case-D (1), (2), the values of index function are higher than those of any other cases, and the damping coefficients of third story are estimated to be a negative values. Those results seem to suggest that the values of index function converged to the local minimum, since the applied models have too many unknown parameters.

As was shown in Table 1, the estimated yield displacements are quite different between Case-* (1) and Case-* (2). In the Case-* (1), in which the viscous damping of base isolated story is neglected, the yield displacements are estimated close to 3cm that is the designed value of the yield displacement of the steel dampers. In other cases, in which the viscous damping of base isolated story is considered, the yield displacements are estimated about 1cm. In those cases, the estimated yield displacements seem to converge those of the lead-plug in the rubber isolators.

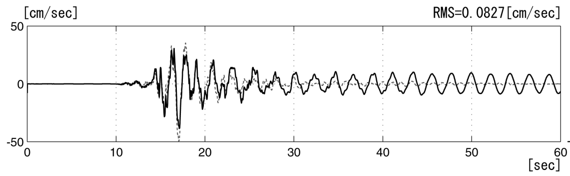
Comparison between calculated structural response using the estimated parameters and observed ones was made for the evaluation of the accuracy of the estimated values. Time histories of velocity responses at the first story and root-mean-squares of residuals between the observed velocities and calculated ones at the same story are shown in Figure 3.



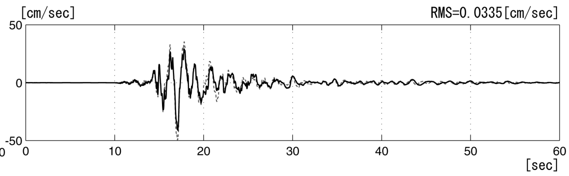

(Case-A (1))



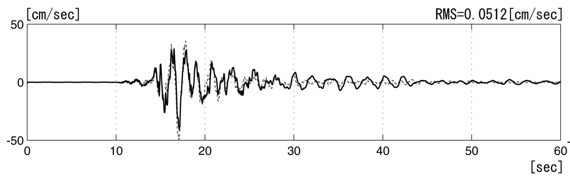

(Case-A (2))



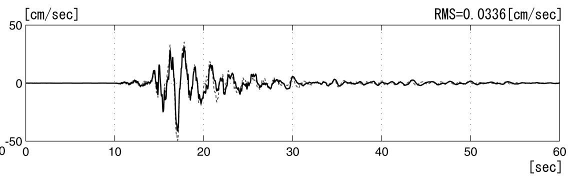

(Case-B (1))




(Case-B (2))




(Case-C (1))




(Case-C (2))

**Figure 3: Time Histories of Estimated and Observed Velocity Responses: Dashed Line is Observed Responses; Rigid Line is Estimated Responses**

In the cases of neglecting viscous damping of base isolated story, time histories calculated from estimated parameters correspond to observed ones during 10 to 30 sec. However, calculated time histories do not correspond to observed ones during 30 to 60 sec. In proportion as both degree of freedom and unknown parameters of model are increasing, calculated time history will be similar to observed one. In the case of considering the viscous damping of base isolated story, calculated time histories show good agreement with observed ones on the whole. In particular, calculated time histories from the identifications using MDOF model show good agreement with the observed ones. In all cases, time history of Case-B (2) shows best agreement with observed one. The result of Case-C (2) is almost equal to that of Case-B (2).

As a result, MDOF model, in which considering the viscous damping of the base isolated story, is most appropriate for the identification of this base isolated structure. The calculated values of root-mean-square of the residuals support the result.

**Identification using Trilinear Force-Displacement Relation Models:**

From the results of the identification using bilinear force-displacement relation model, which carried out above sections, it can be considered that there are two yield displacements in the base isolated story of the structure. It seems reasonable to suppose that the actual hysteretic mechanism of the base isolated story will be described as trilinear force-displacement relation. Therefore, identifications with MDOF models, in which the restring force of the base isolated story is supposed to be a trilinear force-displacement relation, are carried out. From the results of above section, the upper structure of the base isolated structure supposed to be a multi-mass linear shear model. Viscous damping of the upper structure supposed to be proportional damping, and the each stiffness of upper structure was considered known parameters.

In section 2.2, the results of the cases, in which the viscous damping of base isolated story is considered, were excellent. However, in case of the identification using trilinear force-displacement relation model, because of the possibility that the whole damping are included in the hysteretic damping, the following results of two cases are provided:

Case-F (1): identification using MDOF model neglecting the viscous damping of base isolated story.
Case-F (2): identification using MDOF model considering the viscous damping of base isolated story.

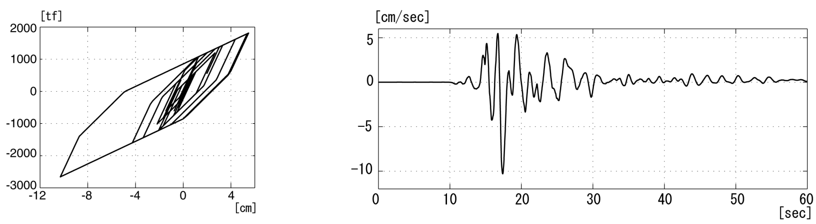
In both cases, the unkown parameters without viscous damping coefficient $c_0$ are primary stiffness ${}_1k_0$, primary yeild displacement $X_{y1}$, secondary yeild displasement $X_{y2}$, the value that secondary stiffness divided by primary stiffness $\alpha_1$, and the value that tertiary stiffness divided by primary stiffness $\alpha_2$ of base isolated story.

The results of the identification are shown in Table 2.

**Table 2: The Results of The Identification using Trilinear Force-Displacement Relation Models**

| | Initial values | MDOF model | |
|---|---|---|---|
| | | Case-F (1) | Case-F (2) |
| $J_n$ | - | 0.8260 | 0.4183 |
| ${}_1k_0$[t/cm] | 500 | 892 | 796 |
| $c_0$[t*sec/cm] | 50 | - | 60 |
| $X_{y1}$[cm] | 1.00 | 0.70 | 0.79 |
| $X_{y2}$[cm] | 3.00 | 4.82 | 2.69 |
| $\alpha_1$ | 0.50 | 0.39 | 0.46 |
| $\alpha_2$ | 0.20 | 0.14 | 0.22 |

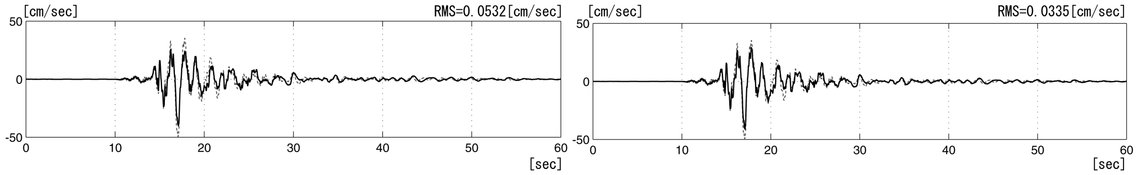
The estimated values of secondary yield displacement in Case-F (2) are closer to 3 cm that is a designed value of the yield displacement of steel damper than those in Case-F (1). The value of index function in Case-F (2) is smaller than that in Case-F (1). Therefore, it is concluded that the results of the identification in Case-F (2) is more accurate. The estimated hysteresis loop and time history of relative displacement response, which are calculated using the estimated parameters, are shown in Figure 4.




**(a) Hysteresis Loop** **(b) Time history of Relative Displacement Response**

**Figure 4: Estimated Hysteresis Loop and Time History of the Base Isolated Story**

To evaluate the accuracy of the estimated values, response analysis using estimated parameters is carried out. Time histories of observed velocity responses and calculated ones at the first story, and the root-mean-squares of residuals between observed velocity responses and calculated ones at the same story are shown in Figure 5.




(Case-F (2)) (Case-B (2))

**Figure 5: Time Histories of Estimated and Observed Velocity Responses: Dashed Line is Observed Responses; Rigid Line is Estimated Responses**

Time history calculated from the identification of Case-F (2) seems to be more similar to the observed one than that of Case-F (1). However, the root-mean-square of residual of Case-B (2), that shows best agreement with the observed one, is smaller than that of Case-F (2). The reason for this result seems to be that the identification using trilinear model will be required to estimate greater number of model parameters than using bilinear model. In this study, only two channels of output (relative velocity response of first- and sixth-story) were used for identification. It can be considered that more channels of output will be required for the accurate identification using trilinear model.

## CONCLUSION

A procedure for nonlinear system identification using prediction error identification method with state-space description (PEM-SS) was presented. System identifications of the base isolated structure by this method using recorded seismic response data under Hyogoken-Nanbu earthquake were carried out. As the result, PEM-SS using nonlinear MDOF model is found to be very effective to identify the dynamic characteristics of the base isolated structure. The results using several nonlinear SDOF and MDOF models indicate that MDOF model considering both bilinear force displacement relation and viscous damping of the base isolated story is most appropriate for this structure.

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