HYSTERESIS MODEL FOR CONCRETE FILLED STEEL SQUARE TUBULAR BEAM-COLUMNS

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SUMMARY

Concrete filled steel square tubular (CFT) columns have been used as beam-column members of high-rise buildings in Japan recently. There are many researches on CFT beam-columns in Japan. When we perform the elasto-plastic analysis on the earthquake response, it is necessary to make clear the hysteresis models of the members, which consisting structure. However, there are not proposed preferable hysteresis model of CFT beam-columns for the elasto-plastic analysis on the earthquake response.

We proposed the skeleton curve model by dividing it into three parts. They are characterized by 1) Initial tangent stiffness, 2) Maximum strength, 3) Hysteresis to maximum strength and 4) Negative slope after maximum strength.

We compared the relations between the proposed skeleton curve model of CFT column, which forms a part of hysteresis model, and the past experimental hysteresis, and showed the hysteresis model corresponded very well

INTRODUCTION

Concrete filled steel square tubular (CFT) columns have been used as beam-column members of high-rise buildings in Japan recently. Comparing with the Steel Reinforced Concrete beam-columns, CFT columns have many structural advantages, such as high strength, large ductility and large energy absorption capacity. There are many researches on CFT columns in Japan. "Recommendations for Design and Construction of Concrete Filled Steel Tubular Structures", on the basis of the researches on CFT structures carried out, was published in 1997. However, we can find no proposed preferable hysteresis model of CFT beam-columns for elasto-plastic analysis on the earthquake response.

So we attempt to propose the preferable hysteresis model for CFT beam-columns. Hysteresis model can represent a combination of the skeleton curve and the hysteresis loop. Objective of this paper is to propose the skeleton curve model of the CFT beam-column, which forms a part of hysteresis model.

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HYSTERESIS MODEL

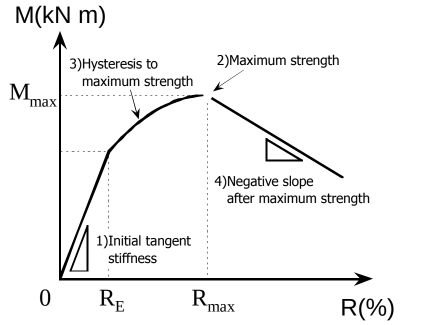
We proposed the skeleton curve model by dividing it to three parts. They are characterized by 1) initial tangent stiffness, 2) maximum strength, 3) hysteresis to maximum strength and 4) negative slope after maximum strength (see Fig.1).

Initial Tangent Stiffness

Initial tangent stiffness is estimated by combining the bending stiffness and the shear stiffness. The bending stiffness is calculated using the slope-deflection method. We obtained

$$K_b = \frac{N \cdot k}{\tan kL - kL} \quad (1)$$

where, N: axial force, L: shear span, $k^2 = N/EI$, $EI={}_sE_sI+{}_cE_cI$, ${}_sE$ and ${}_cE$: Young's modulus of steel and concrete (${}_sE$=2100t/cm^2), ${}_sI$ and ${}_cI$: moment inertia of steel and concrete. Young's modulus of concrete is calculated by using Eq. (2).



$M = QL + N\delta$, $R = \delta / L$, M: moment at the bottom of the beam-column, Q: shearing force, L: shear span, N: axial force, δ: displacement at the top of the beam-column, R: rotation angle

Figure 1: Skeleton curve model

$${}_cE = 2.1 \times 10^5 \times \left(\frac{\gamma}{2.3}\right) \times \sqrt{\frac{{}_c\sigma_B}{200}} \ \text{(kg/cm}^2\text{)} \quad (2)$$

where, weight of unit volume of concrete (γ) is 2.3(t/m^3), ${}_c\sigma_B$: compressive strength of concrete(kg/cm^2).

We assumed that the bending stiffness and the shear stiffness are not influenced each other. The shear stiffness is expressed by Eq. (3). Shape coefficient is quoted from Ref.2 and is taken into account in estimating shear stiffness.

$$K_s = \frac{{}_sG \cdot {}_s\kappa \cdot {}_sA + {}_cG \cdot {}_c\kappa \cdot {}_cA}{L} \quad (3)$$

where, ${}_sG$ and ${}_cG$: shear modulus of steel and concrete (${}_sG$=810t/cm^2), ${}_s\kappa$ and ${}_c\kappa$: shape coefficient of steel pipe and concrete (${}_s\kappa$=0.436 and ${}_c\kappa$=0.843), ${}_sA$ and ${}_cA$: area of steel and concrete. Shearing modulus of concrete is calculated by using Eq. (4).

$${}_cG = 0.9 \times 10^5 \times \left(\frac{\gamma}{2.3}\right) \times \sqrt{\frac{{}_c\sigma_B}{200}} \ \text{(kg/cm}^2\text{)} \quad (4)$$

Calculated initial tangent stiffness is multiplied by reduction factor (β=0.85). We obtained proposed initial tangent stiffness (${}_QK_R$). A plot of the comparison between experimental stiffness and calculated stiffness is given in Fig.2.

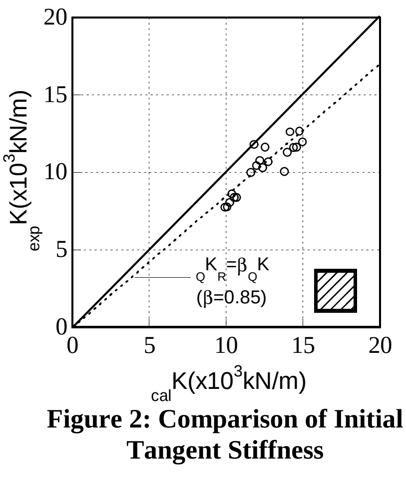


Figure 2: Comparison of Initial Tangent Stiffness

$${}_QK_R = \beta \cdot {}_QK = \beta \cdot \frac{Q}{\delta} = \beta \cdot \frac{1}{1/K_b + 1/K_s} \quad (5)$$

where, Q : shearing force, δ : displacement at the top of the beam-column.

Maximum Strength

We estimated the point of maximum strength by combining the quantities of strength and the rotation angle.

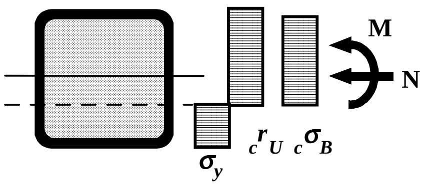


Figure 3: Stress Distribution

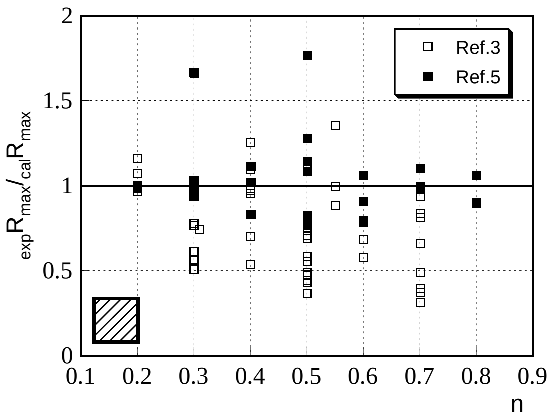


Figure 4: Rotation Angle at Maximum Strength

The quantities of maximum strength are estimated as the full plastic strength. Full plastic moment (strength of a composite cross section) is computed assuming the rectangular stress distribution of yield stress σ_y for the steel tube, and of compressive strength $_c\sigma_B$ for the concrete, as shown in Fig.3 ($_cr_U$=1.0).

The rotation angle at maximum strength ($_{cal}R_{max}$) is represented by using the number of repetition times at one rotation angle (n_c), axial load ratio (N/N_0) and width-to-thickness ratio (D/t) (see Eq. (6)). The accuracy of the equation is shown in Fig.4. The rotation angle of experimentation ($_{exp}R_{max}$) is normalized by the value of calculation ($_{cal}R_{max}$). The experimental data is a quotation from the database, which included in Ref.3.

$$_{cal}R_{\max} = (3.0 - 0.3n_c)\left\{1.5 - 1.1\left(\frac{N}{N_0}\right) - 0.01\left(\frac{D}{t}\right)\right\} \tag{6}$$

where, n_c: number of repetition times at one rotation angle, $N_0 = {_sA}\,{_s\sigma_y} + {_cA}\,{_c\sigma_B}$, $_s\sigma_y$: yield stress of steel tube

Hysteresis to Maximum Strength

Hysteresis to maximum strength is defined as a quadratic curve. The quadratic curve is defined as assuming that 1) the curve through one-third point of rotation angle at maximum strength, and 2) the slope of the curve is zero at the maximum strength (see Fig.5).

$$M = a \cdot R^2 + b \cdot R + c \tag{7}$$

$$a = -\frac{9}{4}\left(M_{\max} - \frac{{_MK'_R}}{3}R_{\max}\right)\frac{1}{R_{\max}{}^2} \tag{8}$$

where, $_MK_R = {_QK_R}\,L + N$, The prime mark means $K' = K\,L$ / 100.

However, when the rotation angle at maximum strength is large, it is difficult to estimate the limit rotation angle of elasticity by using one-third point of rotation angle at maximum strength. So we express the equation of hysteresis to maximum strength, on a decision of a (Eq.8), like this.

When $a \le -5$

$$M = \left\{-\frac{9}{4}\left(M_{\max} - \frac{{_MK'_R}}{3}R_{\max}\right)\frac{1}{R_{\max}{}^2}\right\}R^2 + \left\{\frac{9}{2}\left(M_{\max} - \frac{{_MK'_R}}{3}R_{\max}\right)\frac{1}{R_{\max}}\right\}R + \left(-\frac{5}{4}M_{\max} + \frac{3}{4}{_MK'_R}\cdot R_{\max}\right) \tag{9}$$

When $a > -5$

$$M = \left\{ -\frac{M_{\max} - {}_M K_R' \cdot 0.25}{(R_{\max} - 0.25)^2} \right\} R^2$$

$$+ \left\{ 2 \cdot \frac{M_{\max} - {}_M K_R' \cdot 0.25}{(R_{\max} - 0.25)^2} \cdot R_{\max} \right\} R + \left(M_{\max} - \frac{M_{\max} - {}_M K_R' \cdot 0.25}{(R_{\max} - 0.25)^2} \cdot R_{\max}{}^2 \right) \tag{10}$$

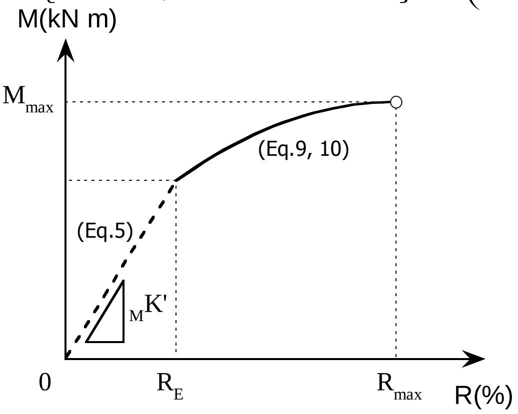


Figure 5: Hysteresis to maximum strength

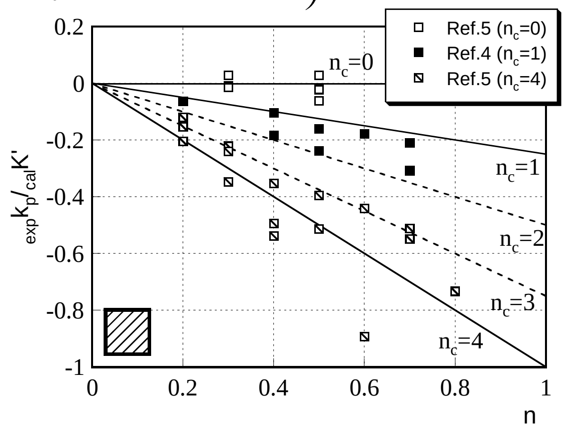


Figure 6: Negative Slope after Maximum Strength

Table 1: Hysteresis model

<table>
<tr><th></th><th>Equation</th><th></th><th>Equation</th></tr>
<tr><td>Tangent initial stiffness</td><td>$_Q K_R = \beta {}_Q K = \beta \frac{1}{1/K_b + 1/K_s} \quad (\beta = 0.85)$
$K_b = \frac{N \cdot k}{\tan kL - kL} \quad \left(k^2 = N / EI\right)$
$K_s = \frac{{}_s G \cdot {}_s \kappa \cdot {}_s A + {}_c G \cdot {}_c \kappa \cdot {}_c A}{L}$
${}_c \kappa \cong 0.843 \quad , \quad {}_s \kappa \cong 0.436$</td><td rowspan="2">Hysteresis to maximum strength</td><td rowspan="2">$a = -\frac{9}{4}(M_{\max} - \frac{{}_M K_R'}{3} R_{\max}) \frac{1}{R_{\max}{}^2}$
$When \quad a \le -5$
$M = \left\{ -\frac{9}{4}(M_{max} - \frac{{}_M K_R'}{3} R_{max}) \frac{1}{R_{max}{}^2} \right\} R^2$
$+ \left\{ \frac{9}{2}(M_{max} - \frac{{}_M K_R'}{3} R_{max}) \frac{1}{R_{max}} \right\} R$
$+ \left(-\frac{5}{4} M_{max} + \frac{3}{4} {}_M K_R' \cdot R_{max} \right)$
$When \quad a > -5$
$M = \left\{ -\frac{M_{max} - {}_M K_R' \cdot 0.25}{(R_{max} - 0.25)^2} \right\} R^2$
$+ \left\{ 2 \cdot \frac{M_{max} - {}_M K_R' \cdot 0.25}{(R_{max} - 0.25)^2} \cdot R_{\max} \right\} R$
$+ \left\{ M_{\max} - \frac{M_{max} - {}_M K_R' \cdot 0.25}{(R_{max} - 0.25)^2} \cdot R_{\max}{}^2 \right\}$
${}_M K_R = {}_Q K_R \cdot L + N$
The prime mark means $K' = K \cdot L / 100$</td></tr>
<tr><td rowspan="2">Maximum strength</td><td rowspan="2">${}_{cal} R_{max} = (3.0 - 0.3 n_c) \left\{ 1.5 - 1.1 \left(\frac{N}{N_0} \right) - 0.01 \left(\frac{D}{t} \right) \right\}$
$When \quad 0 \le x_n < t$
$N = -\{D^2 - (D - 2t)^2 - 2D \cdot x_n\}_s \sigma_y$
$M = D \cdot x_n (D - x_n)_s \sigma_y$
$When \quad t \le x_n < D - t$
$N = 2(2x_n \cdot t - D \cdot t)_s \sigma_y + (D - 2t)(x_n - t)_c \sigma_B$
$M = \{D \cdot t(D - t) + 2(x_n - t)(D - t - x_n)t\}_s \sigma_y$
$+ 0.5(D - 2t)(x_n - t)(D - t - x_n)_c \sigma_B$
$When \quad D - t \le x_n \le D$
$N = \{D^2 - (D - 2t)^2 - 2D(D - x_n)\}_s \sigma_y + (D - 2t)^2{}_c \sigma_B$
$M = D \cdot x_n (D - x_n)_s \sigma_y$</td></tr>
<tr><td>Negative slope</td><td>$k_p = -\frac{1}{4} n_c \cdot \frac{N}{N_0} \cdot {}_M K'$</td></tr>
</table>

Negative Slope after Maximum Strength

Negative slope (k_p) after maximum strength is represented by the number of repetition times at one rotation angle, axial load ratio and initial tangent stiffness. We obtain

$$k_p = -\frac{1}{4} n_c \cdot \frac{N}{N_0} \cdot {}_M K' \tag{11}$$

where, k_p: negative slope after maximum strength (kNm/%), ${}_MK'$: calculated initial tangent stiffness (kNm/%), n_c: number of repetition times at one rotation angle

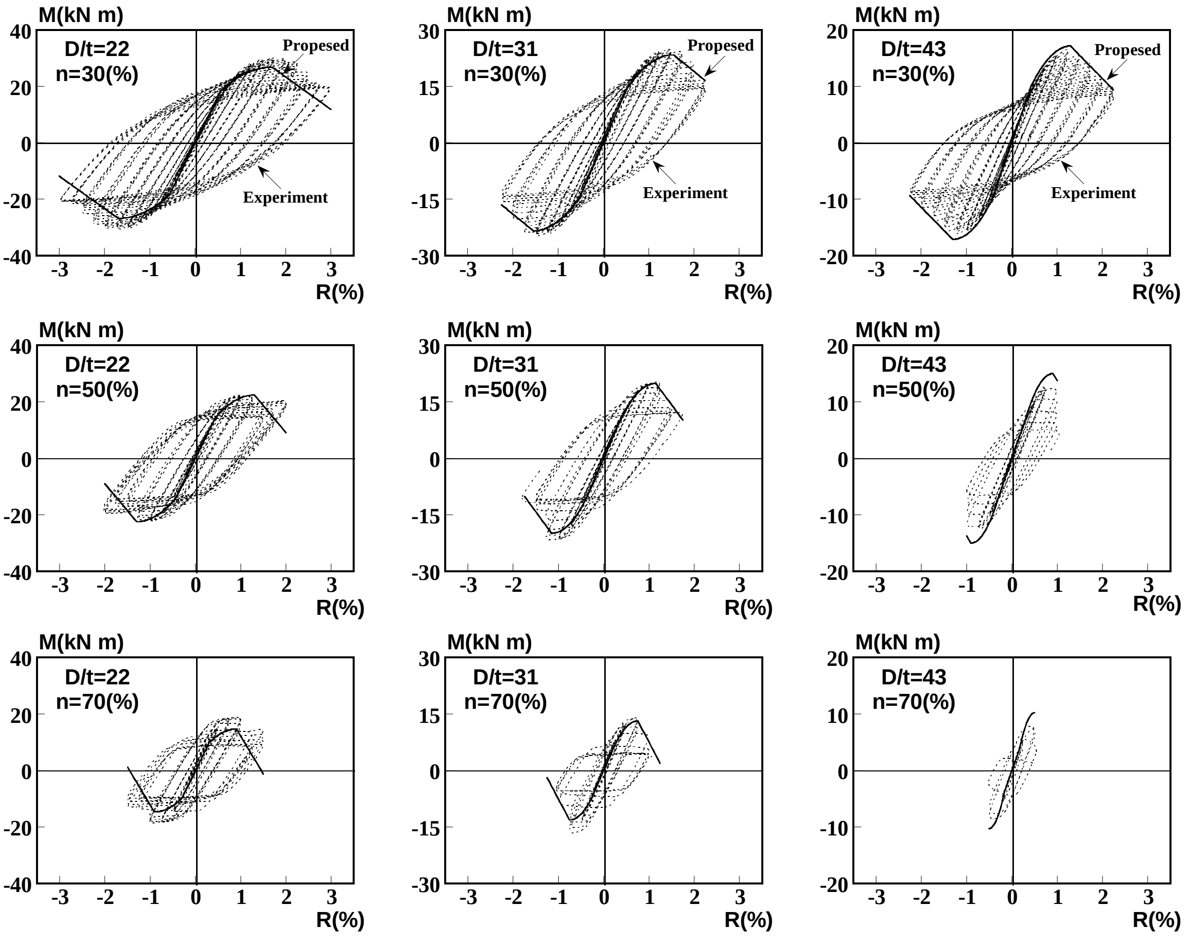


Figure 7: Comparison between Experimental Hysteresis and Proposed Skeleton Curve (Ref.5)

The negative slopes of the monotonic loading specimen, which the number of repetition times (n_c) is zero, are nearly zero. With an increase of the repetition times, the negative slope increases. The accuracy of the equation is shown in Fig.6.

COMPARISON BETWEEN HYSTERESIS MODEL AND TEST RESULTS

The hysteresis skeletons curve, which dividing to three parts and characterized by 1) Initial tangent stiffness, 2) Maximum strength, 3) Hysteresis to maximum strength and 4) Negative slope after maximum strength, is arranged in Table 1.

We show the comparison between experimental hysteresis and proposed skeleton curve (see Fig.7, 8). The data of experiment is the experimental results of Ref.4 and Ref.5. In the experiment of Ref.5, shear span ratio (L/D=5) is maintained uniform, and width-to-thickness ratio (D/t=22, 31, 43) and axial load ratio (0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8) are varied parametrically. In the experiment of Ref.4 width-to-thickness ratio (D/t=33.3) is maintained uniform, and shear span ratio (L/D=3, 4.5, 6, 9, 12) and axial load ratio are varied parametrically.

The solid lines show the proposed skeleton curve. The dotted line is the experimental behavior. Compared with experimental hysteresis, the hysteresis model corresponded well (see Fig.7, 8)

CONCLUSIONS

We proposed the skeleton curve model of CFT beam-column, which forms a part of hysteresis model, by dividing it to three parts, and it corresponded very well with experimental hysteresis.

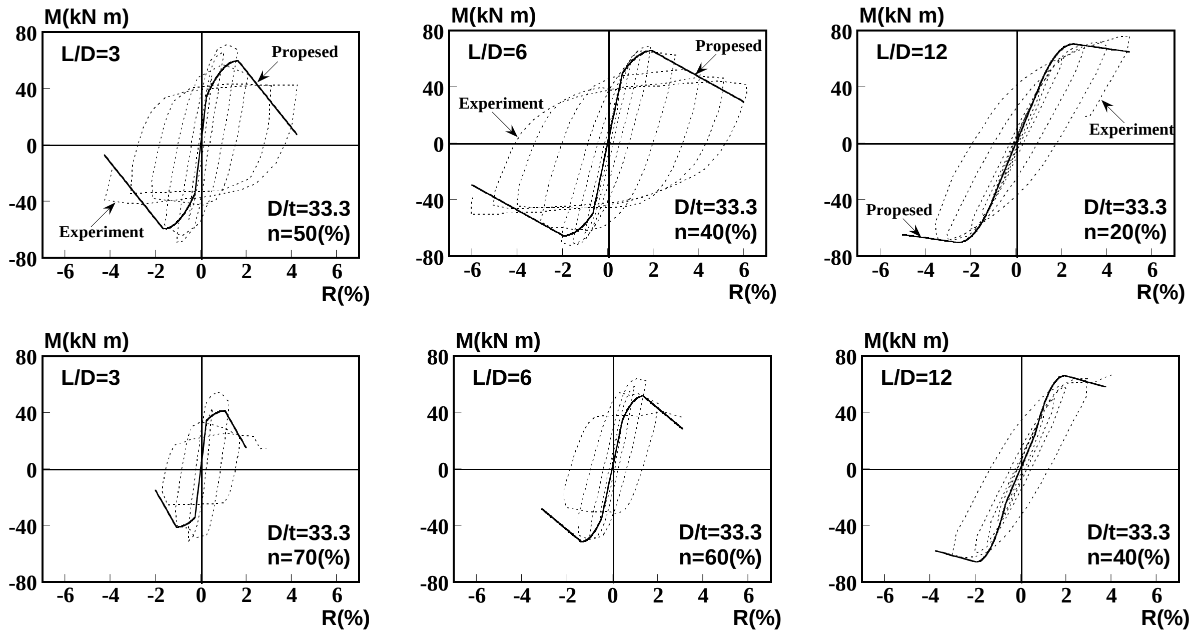


Figure 8: Comparison between Experimental Hysteresis and Proposed Skeleton Curve (Ref.4)

1. Initial tangent stiffness is estimated by combining bending stiffness and shear stiffness. Bending stiffness is calculated using the slope-deflection method. Shape coefficient is taken into account in estimating shear stiffness.
2. The point of maximum strength is obtained by combining the quantities of strength and the rotation angle. The quantities of maximum strength are estimated as the full plastic strength. And, the rotation angle at the maximum strength is represented by the number of repetition times at one rotation angle, axial load ratio and width-to-thickness ratio.
3. Hysteresis to maximum strength is defined as a quadratic curve. The quadratic curve is defined as assuming that 1) the curve through one-third point of rotation angle at maximum strength, and 2) the slope of the curve is zero at the maximum strength.
4. Negative slope after the maximum strength is represented by the number of repetition times at one rotation angle, axial load ratio and initial tangent stiffness
5. Compared proposed skeleton curves with the experimental results of Ref.4 and Ref.5, and showed the hysteresis model corresponded very well.

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