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A NEW ALGORITHM FOR REAL-TIME SUB-STRUCTURE PSEUDO-DYNAMIC TESTS

Veit BAYER[1], Uwe E DORKA[2] And Ulrich FÜLLEKRUG[3]

SUMMARY

The real-time sub-structure pseudo-dynamic test (real-time sub-structure PSD test) is a new testing procedure that is able to simulate the interaction between a sub-structure and its main structure. Applications are in qualification testing of structural control devices like tuned mass dampers (TMDs) or active mass dampers (AMDs), or payloads in aerospace systems like satellites (sub-structure) in rockets (structure). A stable, implicit time stepping scheme is required which must be formulated as acceleration control. In order to avoid iteration, a sub-stepping method is applied during each time step. Results of an analytical study on a vibrating 2-DOF sub-system on a 2DOF structure show the effects of the number of sub steps and of an error force compensation that is necessary to compensate the unavoidable equilibrium error at the end of each time step. This study is representative of aerospace applications. One – to – one application to buildings with TMDs is possible.

INTRODUCTION

The pseudo-dynamic or PSD test has been used successfully to simulate the behaviour of structures under earthquakes for some time now. It was developed particularly for large scale tests where the mass effects are simulated in the computer, thus allowing for a slow motion simulation with high observability. The motion was in fact a discontinuous one where the solution of the time stepping scheme (typically the central difference method) was achieved with high accuracy for every time step with a full stop at its end (see e.g. [Nakashima, 1984]). The first rapid continuous PSD tests were performed at the Ruhr-University Bochum, Germany in 1987 [Dorka, Heiland 1991] on single degree of freedom systems. Here, a time magnification factor of 10 was introduced into the integration scheme and the procedure ran continuously. Now, the accuracy of the solution at the end of every time step depends on various factors like speed of loading, reaction time of the hydraulic system etc. and cannot be achieved arbitrarily high. An error develops at the end of every time step that, if not compensated for, tends to destabilize the process.

A next step in the development of the PSD test was the sub-structure PSD test where only a part of the structure is tested and the rest is modeled in the computer. For this, the central difference method is often not suitable anymore because of its natural numerical stability limit. The operator splitting method [Nakshima, Kaminosono, Ishida, Ando, 1990] is now the method of choice for this type of test. Here, the solution is split into an analytical and a testing part. The analytical part is solved by an unconditionally stable scheme, like the Newmark method with parameters for constant average acceleration. For the testing part, the central difference method is still applied

Application of the sub-structure PSD method in real time is still under development. So far, this method which is not really "pseudo"-dynamic anymore, has only been applied to non-oscillating sub-structures. It will be of great advantage, though, in the qualification testing of structural control systems, like tuned or active mass dampers (TMDs or AMDs) which are oscillatory sub-systems. It also has applications beyond building structures. This paper is based on a research project financed by the "Deutsche Forschungsgemeinschaft" (DFG) that looks into the application of this procedure for qualification testing of payloads (satellites) in launch vehicles [Dorka, Füllekrug, Ji, Gschwilm 1998]. In such applications, the structure (building or rocket) is modeled in the

[1] *University of Rostock, Dept. of Civil Engineering. Wismar, Germany;email: veit.bayer@bau.uni-rostock.de*
[2] *University of Rostock, Dept. of Civil Engineering. Wismar, Germany*
[3] *German Aerospace Centre (DLR), Institute of Aeroelastics. D-–37073 Göttingen, Germany*

computer and the sub-structure (AMD, TMD or satellite) is placed on a shaking table providing the link. In the case that the sub-structure has eigenfrequencies close to the ones of the structure (as for TMDs), the sub-structure PSD test is the only available method that is in principle capable of a realistic simulation under such circumstances.

DESCRIPTION OF THE SUB-PSD ALGORITHM

With oscillating sub-structures, only an unconditionally stable implicit time stepping algorithm is appropriate. To avoid high frequency oscillations due to the usual equilibrium iteration at the end of the time step, the so called "sub-step" technique is introduced. Here, every time step is divided into a number of equally spaced sub-steps. A ramp function is generated from the explicit part of the algorithm at the beginning of the time step and applied over the sub-steps. Then, the implicit part of the algorithm is added in each sub-step using the measured restoring forces. The more sub-steps are introduced, the more will this method approach the exact value at the end of the time step. Thus, this method approximates the correct value like an iteration but without the oscillations. The sub-step technique can be seen as a digital version of the analog force feed back applied in [Thewalt, Mahin, 1987].

For substructures mounted on shaking tables (TMDs, AMDs, aerospace payloads etc.), the usual displacement formulation of the time stepping scheme is not appropriate. An acceleration formulation is required. The dynamic equation of equilibrium at time t+Δt is given as:

$$\underline{M} \cdot \ddot{\underline{u}}^{t+\Delta t} + \underline{C} \cdot \dot{\underline{u}}^{t+\Delta t} + \underline{K} \cdot \underline{u}^{t+\Delta t} = \underline{F}_m^{\,t+\Delta t} + \underline{L}^{\,t+\Delta t} + f(\underline{E}^t) \qquad (1)$$

with $\underline{M}$, $\underline{C}$, $\underline{K}$ denoting the mass, damping and stiffness matrices, respectively, $\underline{L}$ being the load vector, $\underline{u}$ the displacement vector and $\underline{F}_{\underline{m}}$ the coupling force vector between structure and sub-structure. $f(\underline{E})$ is the error compensation which is a function of the equilibrium error that can be calculated at the end of each time step. Applying the Newmark method, eq. (1) can be cast into the recursive form:

$$\ddot{\underline{u}}^{t+\Delta t} = \underline{M}_e^{-1} \cdot [\underline{F}_e^{\,t} + f(\underline{E}^t) + \underline{F}_m^{\,t+\Delta t} + \underline{L}^{t+\Delta t}] \qquad (2)$$

$$\dot{\underline{u}}^{t+\Delta t} = \dot{\underline{u}}^t + [(1-\delta) \cdot \ddot{\underline{u}}^t + \delta \cdot \ddot{\underline{u}}^{t+\Delta t}]\Delta t \qquad (3)$$

$$\underline{u}^{t+\Delta t} = \underline{u}^t + \Delta t \cdot \dot{\underline{u}}^t + [(\frac{1}{2} - \alpha) \cdot \ddot{\underline{u}}^t + \alpha \cdot \ddot{\underline{u}}^{t+\Delta t}]\Delta t^2 \qquad (4)$$

where α, δ are the Newmark constants, the dot denotes derivation with respect to time and

$$\underline{M}_e = \underline{M} + a_7 \cdot \underline{C} + a_8 \cdot \underline{K} \qquad (5)$$

$$\underline{F}_e^{\,t} = -\underline{C} \cdot [\dot{\underline{u}}^t + a_6 \cdot \ddot{\underline{u}}^t] - \underline{K} \cdot [\underline{u}^t + a_9 \cdot \dot{\underline{u}}^t + a_{10} \cdot \ddot{\underline{u}}^t] \qquad (6)$$

where a_6 to a_{10} are combinations of Δt with the Newmark constants as

$$a_6 = (1-\delta)\Delta t\,;\ a_7 = \delta\Delta t\,;\ a_8 = \alpha\Delta t^2\,;\ a_9 = \Delta t\,;\ a_{10} = (1/2 - \alpha)\Delta t^2 \qquad (7)$$

Equation (2) is a linear control equation for every time step of the form:

$$\ddot{\underline{u}}^{t+\Delta t} = \underline{M}_e^{-1} \cdot [\underline{F}_e^{\,t} + f(\underline{E}^t) + \underline{L}^{t+\Delta t}] + \underline{M}_e^{-1} \cdot \underline{F}_m^{\,t+\Delta t} \qquad (8)$$

The first part of the sum in eq. (8) consists of values that are known at the beginning of the time step only. It can be applied as ramp over all sub-steps. The second part of this sum contains the coupling forces between structure and sub-structure which vary over the time step. Their values are measured at every sub-step and fed into the sub-stepping loop. This process repeats with every time step.

The error forces $\underline{E}^t$ depend on the number of sub-steps (analytical error) and errors from the testing system (overshoot or undershoot) and cannot be eliminated completely. Because $\underline{E}^t$ is an equilibrium error, it does not only affect the accuracy of the solution but also its stability. To minimize this error, a PID algorithm is introduced:

$$f(\underline{E}^t) = P \cdot [\underline{E}^t + \Delta t \cdot I \sum_{k=0}^{t} \underline{E}^k + (D/\Delta t) \cdot (\underline{E}^t - \underline{E}^{t-\Delta t})] \quad (9)$$

This algorithm is widely known in control technology. The three factors (P - proportional, I - integral, D - differential) can be optimized for stability and accuracy of control loops. Untested for applications in sub-structure PSD tests, a first approach was the parameter combination P = -1, D = I = 0. This is equivalent to applying the error force with reversed sign in the next time step as was done with success in rapid continuous PSD testing [Dorka, Heiland, 1991].

ANALYTICAL STUDY OF THE SUB-PSD ALGORITHM

In order to investigate the effectiveness of the proposed algorithm, a simple 4-mass system was studied with 2 masses representing the structure (building or rocket) and the other 2 masses the sub-structure (TMD or payload). Table 1 gives the masses and spring stiffnesses.

These parameters were tuned to a model specially designed for testing the developed algorithm at the German Aerospace Centre (DLR) in Göttingen. The test specimen is shown in Figure 1 (in the appendix). Table 2 gives the eigenfrequencies and damping coefficients identified from the computational model.

In the present study, the length of the time step, number of sub-steps and the simple error compensation mentioned above were studied analytically under a transient base excitation representative of a decoupling of two rocket stages. Over- and undershoot were introduced by increasing or decreasing the calculated control signal (acceleration). Apart from the observed range of frequency and the dynamic characteristic of the test specimen, this can be compared to a building subject to ground acceleration, equipped with a 2 DOF – Tuned Mass Damper.

Table 1: Masses and spring stiffnesses of analytical system.

	Structure		**Sub-structure**	
Mass [kg]	18.137	3.735	4.915	0.516
Stiffness [N/m]	65467	59188	136179	23086

Table 2: Eigenfrequencies and damping coefficients of structure, sub-structure and combined system.

	1. Mode	***2. Mode***	***3. Mode***	***4. Mode***
		Combined Systems		
Frequency [HZ]	7.14	15.71	34.19	44.57
Damping [%]	2.01	1.97	3.17	3.98
		Structure		
Frequency [HZ]	8.55	22.42		
Damping [%]	1.90	2.34		
		Sub-Structure		
Frequency [HZ]	24.04	37.10		
Damping [%]	2.44	3.39		

The results as they were calculated using the Sub-PSD algorithm have been assessed on the basis of the coupling force F_m in comparison to the correct analytical coupling force F_e. The latter was calculated using the combined 4-mass system. Figure 2 shows the applied transient acceleration at the bottom of the structure and the spectrum of the analytical coupling force F_e. The spectrum shows three frequency ranges of interest. The first two coincide with the first and second eigenfrequency of the combined system at 7.1 HZ and 15.7 HZ, respectively. The third one is a sinusoidal rigid body motion with a frequency of about 19 HZ implied by the base excitation that causes no vibrational response from the system.

Figure 3 shows the effect of the number of sub-steps k on the stability of the solution. The top graphs show a typical instability at $k = 2$ and the bottom graphs a stable solution at $k = 9$ with adequate accuracy. The time step was 4 ms and no error force compensation was applied.

Not only the number of sub-steps but also the size of the time step and the error force compensation have an influence, not so much on the stability, but mainly on the accuracy. This can be seen in Figure 4 where the results are presented for three different time steps with and without error force compensation. One can see that a small time step increases the stability of the solution and the accuracy mainly for the higher mode. The rigid body motion is accurate as soon as the solution is stable. The simple error force compensation increases the stability for all time step sizes and increases the accuracy especially for the higher mode considerably. For this example, 8 to 9 sub-steps give a good result (see plot of F_m vs. time in Fig. 2).

Even 3 sub-steps are sufficient in this case if a Δt of 1ms can be applied. This may cause some difficulty, though, with the testing equipment that often is not fast enough to run a control cycle of 3 kHZ with sufficient accuracy. Hydraulic equipment in particular has a constant delay time of at least 1 ms. Thus, only 4ms time steps with 4 sub-steps can be applied at the most. This will take away some accuracy in the response of higher modes, but a vibration with a period of up to 12.5 s can be sampled accurately. This may be just sufficient for application to earthquake engineering.

Furthermore, the influence of undershoot and overshoot in the testing equipment was studied. This error can be minimized by fine-tuning the actuator controller, especially by using adaptive PID controllers, but it cannot be completely avoided. The upper plots of Figure 5 present the results for 10% overshoot. As can be seen clearly, overshoot might have a significant effect on the stability of the test. In this case, destabilization evolves around a frequency of about 40 HZ. The same run was performed including the simple error force compensation of eq.(9) with $P = -1$, $I = D = 0$ and plotted in the bottom graphs of Fig. 5. The error force compensation yields completely stable behaviour with the peak at 40 HZ completely gone and sufficient accuracy in the time domain.

The effect of undershoot (see Figure 6, above) is less significant. However, a high-frequency resonance still can be seen in the spectrum, which will causes inaccuracy in the time series. Again applying the error force compensation, the peak at about 40 Hz vanishes practically completely (Figure 6, below).

SUMMARY AND CONCLUSIONS

The real-time sub-structure PSD test is a new testing procedure that is able to simulate the interaction between a sub-structure and its main structure. Applications are in qualification testing of structural control devices like tuned mass dampers (TMDs) or active mass dampers (AMDs), or payloads in aerospace systems like satellites (sub-structure) in rockets (structure). A stable, implicit time stepping scheme is required which must be formulated as acceleration control. In order to avoid iteration, a sub-stepping method is applied during each time step. Moreover, a simple compensation for unavoidable equilibrium errors is applied at the end of each time step.

An analytical study on a vibrating 2-DOF sub-system on a 2DOF structure showed the effects of the number of sub-steps on the stability of the solution and accuracy of the results. The influences of undershoot and overshoot were also studied. It was shown that the error force compensation significantly improves the obtained results.

It can be concluded that the proposed algorithm is suitable for a wide range of real-time sub-structure testing applications, from aerospace to earthquake engineering. The simple error force compensation is already very effective, especially in the presence of overshoot. Further studies are under way on other combinations of PID parameters for this error compensation.

Presently, the application of the proposed algorithm to high-rise buildings, equipped with tuned mass dampers as oscillatory sub-systems, subjected to earthquake ground acceleration is studied.

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APPENDIX: FIGURES

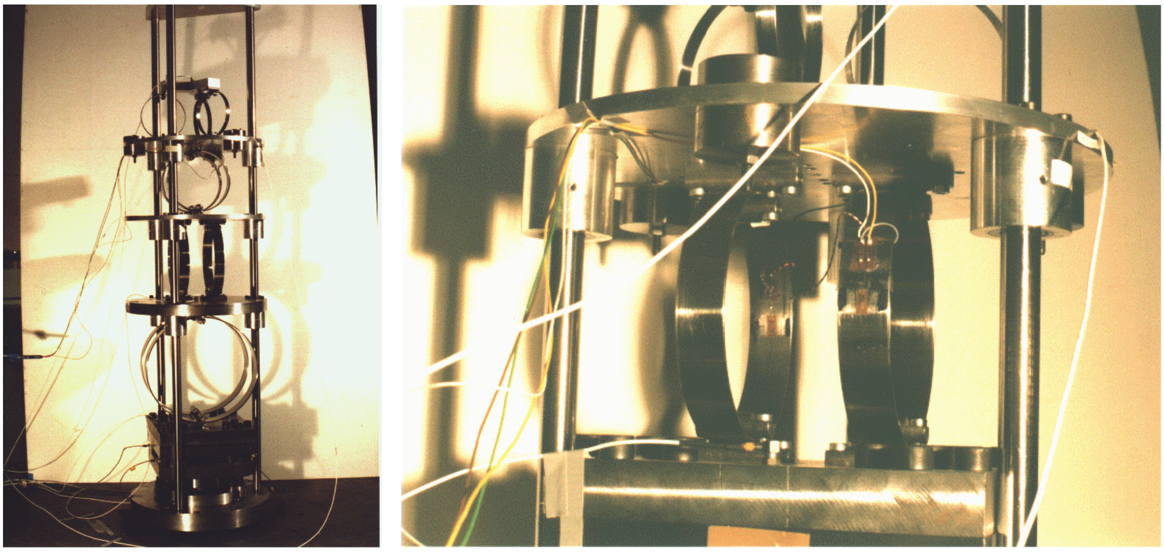

Figure 1: Overview and detail of the test specimen.

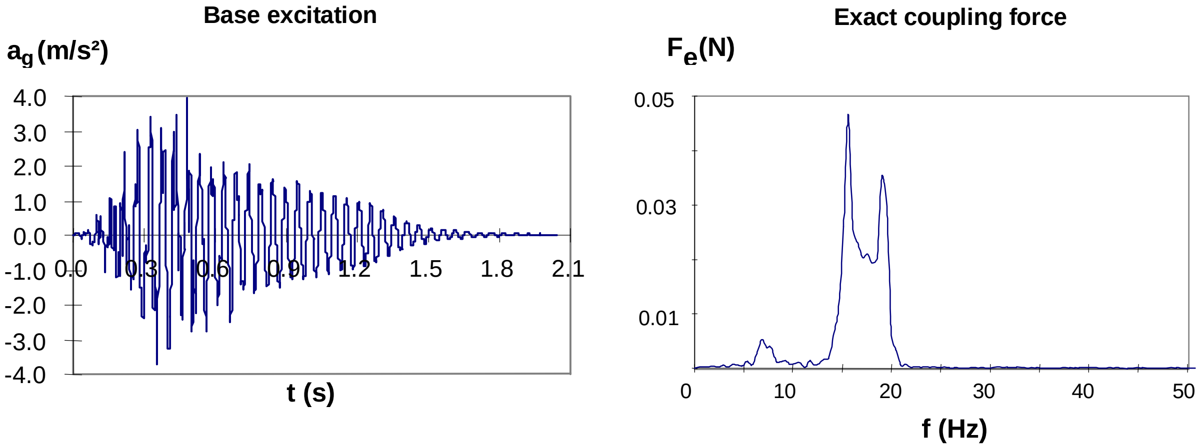

Figure 2: Applied transient and spectrum of correct coupling force.

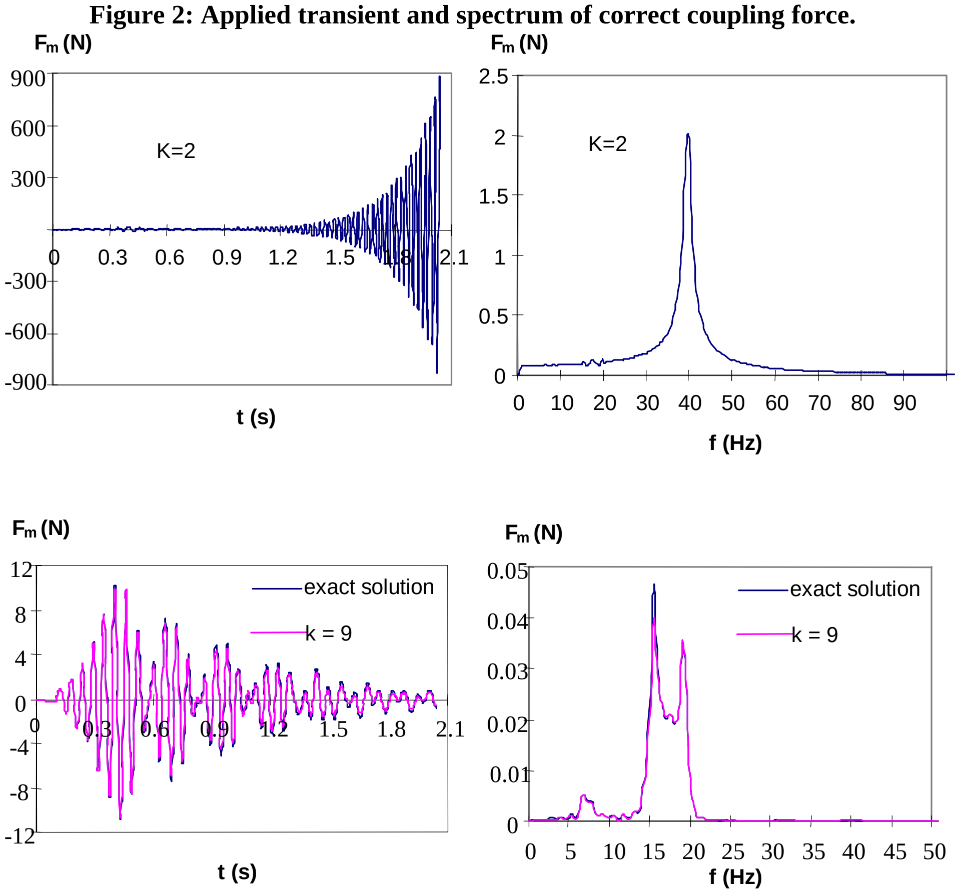

Figure 3: Influence of number of sub-steps on the stability (Δt=4ms, no error force compensation).

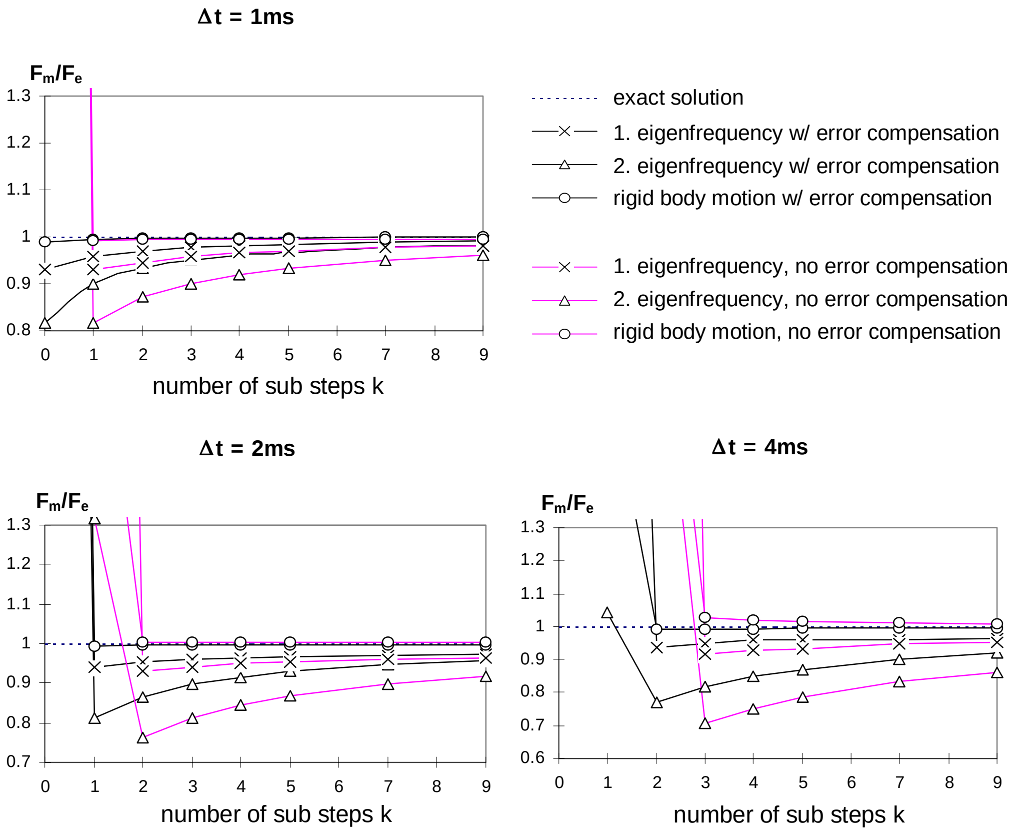

Figure 4: Influence of time step, sub-step and error force compensation on accuracy and stability.

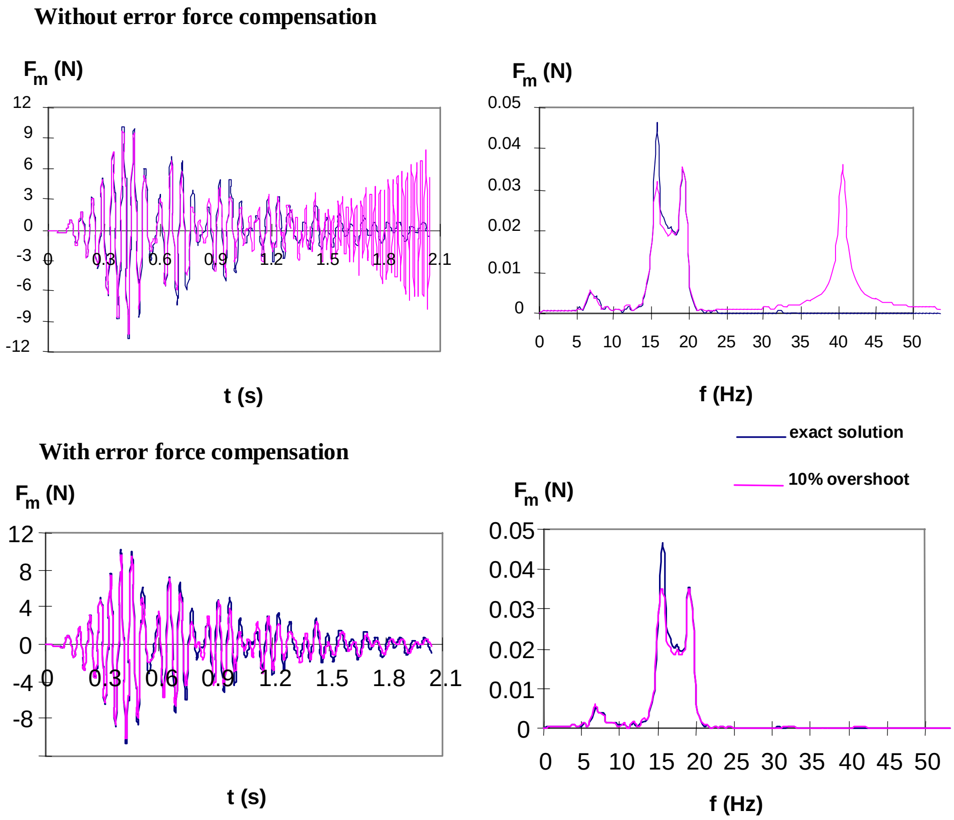

Figure 5: Influence of 10% overshoot on accuracy and stability (Δt=4ms, k=3).

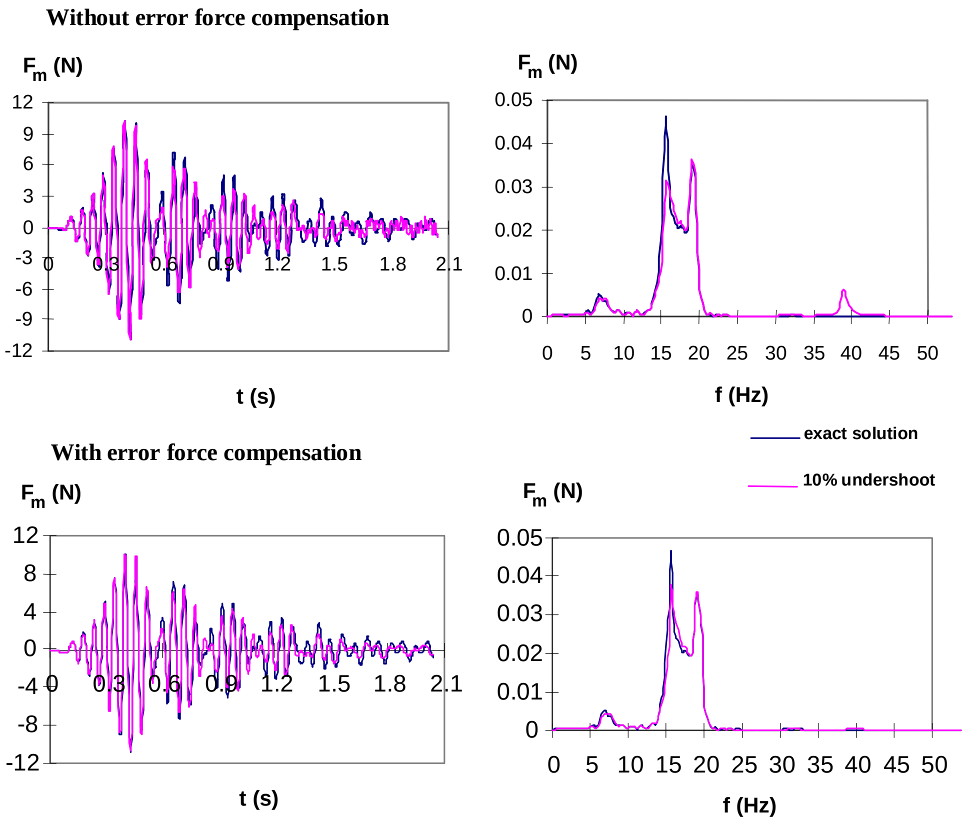

Figure 6: Influence of 10% undershoot on accuracy and stability (Δt=4ms, k=3).