CLOSED-OPEN-LOOP OPTIMAL CONTROL OF BUILDING STRUCTURES SUBJECTED TO EXPONENTIALLY ATTENUATING HARMONIC LOADING

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SUMMARY

In the past two decades researchers in structural control community were mainly focused on the development of closed-loop control strategies since dynamic loads exerted on structures such as earthquakes are not known in prioror. In this paper, a closed-open-loop control algorithm is introduced by monitoring the dynamic loads in real time. An estimator is used to predict the parameters of load model. As a pilot study, only exponentially-attenuating dynamic loads are considered herein. The feed forward gain factor is derived and its sensitivity to the parameters in external loads is studied. An illustrative single-story building is used to demonstrate the pros and cons of the new algorithm. Analytical results show that the algorithm is significantly superior to the closed-loop control in reducing the dynamic responses of a structure when subjected to an impulsive type of excitations such as blast and near-fault earthquake loads. The algorithm is also more efficient in use of external energy to suppress vibration in structures. Results of this study also indicate that the closed-loop control is effective to suppress the vibration level of structures when subjected to stationary disturbances.

INTRODUCTION

Significant progress has been made to the development of algorithms for active control of civil engineering structures subjected to environmental loads (Housner *et al.*, 1994). Among various algorithms, optimal linear quadratic regulators were studied most extensively and have been applied in small-scale and full-scale structures (Soong *et al*., 1991). The algebraic Riccati algorithm was first studied by several investigators. These research works were mainly focused on feedback control law because most environmental loads such as earthquake and wind are not known in prioror. Recognizing that, at any particular time *t*, the knowledge of an external excitation prior to that time instant *t* may be available, Yang *et al*. (1987) proposed an instantaneous optimum active control algorithm. Later studies by Cheng *et al*. (1991) revealed that the feedback gain matrix in the instantaneous algorithm is very sensitive to incremental time intervals used in response analysis. A generalized optimal active control algorithm was consequently developed by Cheng *et al*. (1991), with negligence of the Euler equation governing the optimal solution between two boundary values.

This brief background indicates that the external excitation term has never been dealt with satisfactorily in the development of active control algorithms. As a result, most algorithms are approximately developed with feedback gain only. The end result of a closed-loop control law is mainly to introduce damping into a structure, a feature that is also associated with any passive damper. Increase in damping can effectively reduce the dynamic responses of structure subjected to a gradually increasing disturbance. For an impulsive earthquake ground motion with a high velocity and/or acceleration pulse, such as near-fault earthquake records, damping may not be effective in mitigation of the peak responses. To reduce the peak displacement and acceleration of a structure, nonlinear control laws have been recently developed by Wu (1995) and Tomasula *et al*. (1996). These nonlinear laws were derived with higher-order performance indices and can suppress the peak responses to a certain degree.

As a first step to explore the closed-open-loop control of structures, only exponentially-attenuating harmonic loads are considered herein. These load models can be adopted to simulate the blast wave and shock effect on a

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building structure. They may also represent one frequency component of a non-stationary earthquake ground motion. In addition, the exact solution derived for this type of loads can be used to verify the accuracy of other control algorithms designed for general dynamic loads. This paper is aimed to answer the following questions: 1) Under what circumstances is a closed-loop control algorithm effective regardless of the external excitation to a structure? 2) Does a closed-open-loop control always consume less/more amount of external energy than a corresponding closed-loop control to achieve the same performance level?

CLOSED-OPEN-LOOP CONTROL ALGORITHM

Consider a building structure modeled by an *n*-degree-of-freedom lumped mass-spring-dashpot system. The matrix equation of motion of the structural system with a control mechanism can be written as

$$M\cdot\ddot{X}(t) + C\cdot\dot{X}(t) + K\cdot X(t) = E\cdot f_o e^{-\alpha t}\cos(\beta t) + Du(t) \quad (1)$$

in which **M**, **C** and **K** are, respectively, the $n\times n$ mass, damping and stiffness matrices, and **X**(t) is the *n*-dimensional displacement with respect to the base of the structure. The *n*-dimensional vector **E** defines location of the external excitation of peak force f_o and, $\alpha(>0)$ and β represent the decaying rate and frequency of the excitation, respectively. The $n\times m$ matrix **D** defines location of the control force and **u**(t) is the *m*-dimensional control force vector. In the state- space, the above equation can be written in the form

$$\dot{Z}(t) = AZ(t) + Bu(t) + Hf_o e^{-\alpha t}\cos(\beta t), \quad Z(o) = Z_o \quad (2)$$

in which

$$Z(t) = \begin{Bmatrix} X(t) \\ \dot{X}(t) \end{Bmatrix}, \quad A = \begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} O \\ M^{-1}D \end{bmatrix} \text{ and } H = \begin{bmatrix} O \\ M^{-1}E \end{bmatrix}. \quad (3)$$

A quadratic performance index is usually chosen for study in structural control. It can be expressed into

$$J = \frac{1}{2}\int_o^{t_f} \left[Z^T(t)QZ(t) + u^T(t)Ru(t)\right]dt \quad (4)$$

where **Q** and **R** denote, respectively, a $2n\times 2n$ positive semi-definite matrix and an $m\times m$ positive definite matrix. In Eq. (4), the superscript ^{T} indicates vector or matrix transpose, and the time interval $[0, t_f]$ is defined to be longer than that of the external excitation. The minimum performance index J, defined by Eq. (4) and subject to the constraint represented by Eq. (2), is reached using a closed-open-loop control law when (Soong 1990)

$$u(t) = -R^{-1}B^T\left[P(t)Z(t) + T(t)\right] \quad (5)$$

in which the superscript $^{-1}$ denotes the inverse of a matrix. The gain matrix P(t) is the solution of a Riccati equation for closed-loop control which can be considered as constant for structural applications (Yang *et al.*, 1987). The open-loop control part satisfies

$$\dot{T}(t) = \left(PBR^{-1}B^T - A^T\right)T(t) - PHf_o e^{-\alpha t}\cos(\beta t), \quad T(t_f) = 0. \quad (6)$$

The external excitation, $f_o e^{-\alpha t}\cos(\beta t)$, is the real part of a complex exponential function $f_o e^{-(\alpha+i\beta)t}$ in which $i = \sqrt{-1}$ is an imaginary unit. The open-loop part of control law can thus be written as the real part of $\mathbf{S}(t) f_o e^{-(\alpha+i\beta)t}$. The gain matrix **S**(t) in general is complex and can be derived as

$$S(t) = \Phi\left(I - e^{-[(\alpha+i\beta)I+\Lambda](t_f-t)}\right)\Phi^{-1}\left[(\alpha+i\beta)I + \left(PBR^{-1}B^T - A^T\right)\right]^{-1}PH \quad (7)$$

where $\mathbf{\Lambda}$ is a $2n\times 2n$ diagonal matrix and $\mathbf{\Phi}$ is a $2n\times 2n$ matrix. Each element in diagonal of the matrix $\mathbf{\Lambda}$ and each column in the matrix $\mathbf{\Phi}$ denote, respectively, an eigenvalue and corresponding eigenvector of the matrix ($\mathbf{PBR}^{-1}\mathbf{B}^T - \mathbf{A}^T$). The open-loop control $\mathbf{T}$(t) finally takes the form

$$\mathrm{T(t)} = \mathrm{Re}[\mathrm{S(t)}] \mathrm{f}_o \mathrm{e}^{-\alpha t} \cos(\beta t) + \mathrm{Im}[\mathrm{S(t)}] \mathrm{f}_o \mathrm{e}^{-\alpha t} \sin(\beta t) \tag{8}$$

where Re[ ] and Im[ ] denote, respectively, the real and imaginary part of a complex number in the bracket.

It is noted that the uncontrolled building structure is stable and in general so is the structure controlled with a closed-loop algorithm ($\mathbf{T}$(t) = 0). The system matrices $\mathbf{A}$ and $\mathbf{A}$-$\mathbf{BR}^{-1}\mathbf{B}^T\mathbf{P}$ of both structures thus have complex eigenvalues of negative real part when damped vibration is considered. This means that the eigenvalues of the system described by Eq. (6) are of positive real part and they cannot be equal to $-(\alpha+i\beta)$. Therefore, Eq. (7) represents the sole solution of the system.

To understand how the gain matrix S(t) varies with other parameters and to study the performance and energy consumption of the open-loop control, a single-story frame structure is studied (Soong, 1990). The equation of motion of the simple system is

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = f_o e^{-\alpha t} \cos(\beta t) + u(t), \; x(o) = \dot{x}(o) = 0 \tag{9}$$

where ξ and ω are, respectively, the damping ratio and natural frequency of the uncontrolled structural system. For this study, two weighting matrices $\mathbf{Q}$ and $\mathbf{R}$ are selected as follows

$$\mathrm{Q} = \begin{bmatrix} \omega^2 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } \mathrm{R} = \frac{\gamma}{\omega^2} \tag{10}$$

where γ is a dimensionless coefficient that determines the relative importance of control effectiveness (response reduction) and economy (control force requirements). $\gamma=\infty$ represents the uncontrolled case. Under these conditions, the closed-loop gain matrix $\mathbf{P}$ can be expressed into (Wu, 1995)

$$\mathrm{P} = \begin{bmatrix} 2\xi\omega\gamma(\nu_1\nu_2 - 1) & \gamma(\nu_1 - 1) \\ \gamma(\nu_1 - 1) & 2\xi\gamma(\nu_2 - 1)/\omega \end{bmatrix} \tag{11}$$

in which $\nu_1 = \sqrt{1 + 1/\gamma}$ and $\nu_2 = \sqrt{1 + 0.5(\nu_1 - 1)/\xi^2}$.

PARAMETRIC STUDIES ON THE GAIN MATRIX S(T)

From Eq. (5), one can see that only the second element of the 2×1 vector $\mathbf{S}$(t) of the single-story frame structure is needed to determine the control force required. This element, denoted by S_2(t), can be derived from Eq. (7) and its stationary solution is presented in two parts as

$$\frac{Re[S_2]}{R} = \frac{[\omega^2(\nu_1 - 1) + 2\xi\omega(\nu_2 - 1)\alpha][\alpha^2 - \beta^2 + 2\xi\omega\nu_2\alpha + \nu_1\omega^2] + 4\xi\omega\beta^2(\alpha + \xi\omega\nu_2)(\nu_2 - 1)}{(\alpha^2 - \beta^2 + 2\xi\omega\nu_2\alpha + \nu_1\omega^2)^2 + 4\beta^2(\alpha + \xi\omega\nu_2)^2}, \tag{12}$$

$$\frac{Im[S_2]}{R} = \frac{\beta\{2\xi\omega(\nu_2 - 1)(\alpha^2 - \beta^2 + 2\xi\omega\nu_2\alpha + \nu_1\omega^2) - 2(\alpha + \xi\omega\nu_2)[\omega^2(\nu_1 - 1) + 2\xi\omega(\nu_2 - 1)\alpha]\}}{(\alpha^2 - \beta^2 + 2\xi\omega\nu_2\alpha + \nu_1\omega^2)^2 + 4\beta^2(\alpha + \xi\omega\nu_2)^2}. \tag{13}$$

The transient solution of S_2(t) is lengthy and usually not necessary to be included in structural control since the ratio S_2(t)/R remains constant, then oscillates for several cycles and finally drops to zero at the terminal instant.

It dies out faster than $e^{-\alpha t}$, depending on the equivalent damping of a closed-loop control, and thus has little influence on the performance of the proposed control algorithm as long as the terminal instant t_f in Eq. (4) is sufficiently long.

As one can see from Eqs. (12) and (13), the ratio S_2/R is a function of γ, ξ, α/ω and β/ω. When β/ω=0, the structural system is subjected to a monotonically-decreasing load and the imaginary part of the gain matrix S_2 vanishes. To see the effect of the exciting frequency β on the open-loop control gain factor, both $Re[S_2]/R$ and $Im[S_2]/R$ are plotted in Figs.1(a,b) for a constant structural damping ξ=0.0124 and control coefficient γ=1. It can be clearly observed that the open-loop control gain factor strongly depends on the exciting frequency β only for low decaying rate α. The rate of change in the imaginary part of gain factor is maximized around β/ω=1, which is the resonant case. This result indicates that, for rapidly-decaying excitations, a single gain factor can be used to approximately represent those corresponding to various exciting frequencies. Such a design is particularly effective for low frequency excitations.

PERFORMANCE OF CLOSED-OPEN-LOOP CONTROL ALGORITHM

For the frame structure exemplified in the parametric studies, the optimal closed-open-loop control force can be derived from Eq. (5) as follows:

$$u(t) = -\left[\omega^2(\nu_1 - 1)x(t) + 2\xi\omega(\nu_2 - 1)\dot{x}(t)\right] - \frac{Re[S_2]}{R} f_o e^{-\alpha t}\cos(\beta t) - \frac{Im[S_2]}{R} f_o e^{-\alpha t}\sin(\beta t) \quad (14)$$

After introducing Eq. (14) into Eq. (9), the displacement x(t) and velocity $\dot{x}(t)$ can be determined. Their detailed derivation and results are not shown in this paper due to the limited space. The power consumed by the actuator can then be determined by

$$p(t) = -u(t)\dot{x}(t). \quad (15)$$

The displacements and velocities of the structure are presented in Figs. 2(a,b) along with control forces and power consumption of actuator. They are compared with those of uncontrolled structure and of the structure controlled with the closed-loop law. It can be observed that both displacements and velocities are reduced the most when the structure is controlled with a closed-open-loop algorithm. The faster the excitation decays, the more effective this algorithm. In comparison with the closed-loop control, the new algorithm can suppress the peak displacements more than 50% while the maximum control force required is only increased about 25%. This extra reduction in peak response per unit control force is attributable to non-concurrence between the maxima of response and excitation. As a result, power consumed by actuator with the new algorithm is even less as evidenced in Figs. 2(a,b). In particular, when an uncontrolled structure vibrates in a resonant mode as shown in Fig. 2(a), control with the new algorithm can completely suppress the vibration with the same control force as required with a closed-loop algorithm but a significantly less amount of power required.

It has been proved that a closed-loop control algorithm is optimal in mitigating structural responses under white noise excitations. Figure 2(a) shows that the closed-loop control strategy can also be employed to effectively reduce the responses when the structure is subject to harmonic loading. This result indicates the effectiveness of the conventional closed-loop control scheme for any stationary excitations regardless of their frequency bandwidth.

MONITORING OF EXTERNAL EXCITATIONS

This study only concerns the exponentially-attenuating loads, e.g., $f(t) = f_o e^{-\alpha t}\cos(\beta t)$. Two parameters, α and β, can characterize a time-varying feature of such loads. Since the parameter β does not significantly affect the performance of the algorithm for rapidly-decaying excitations (Fig. 1), it can just be assigned an average value based on engineering judgment. Only the parameter α needs to be monitored in real time in order to implement a closed-open-loop control algorithm.

Suppose the external excitation f(t) is sampled with a time interval Δt. The parameter α can then be estimated by

$$\hat{\alpha} = -\frac{f(\Delta t) - f_o}{\Delta t \cdot f_o}. \tag{16}$$

To show the effects of the above estimation and the gain factor approximation in terms of β on the performance of the proposed algorithm, the displacement, velocity, control force and energy consumption of the single-story structure for the two cases (β/ω=1) as shown in Figs. 2(a,b) are presented in Figs. 3(a,b) when the gain factor of the open-loop control is calculated with β/ω=0 and Eq. (16) is used to estimate the parameter α. In comparison with Figs. 2(a,b), one can find that all response quantities are very close to those corresponding to the gain factors determined with the actual β value. Their difference decreases rapidly as the decaying rate of the excitation increases. This indicates that the new algorithm can also be applied to effectively control the dynamic responses of a structure under an impulsive type of earthquake loads such as near-fault effect.

CONCLUSIONS

A closed-open-loop control strategy has been applied to structural control. The open-loop part of the control algorithm is implemented in real time by means of instrumentation. Based on this study, the proposed algorithm is significantly superior to the closed-loop control when the decaying rate of the dynamic loads is greater than the product of damping ratio and natural frequency of the building structure. Specifically it is more efficient to reduce the peak response of the structure per unit energy consumption when the structure is subjected to impulsive types of dynamic loads. The proposed estimator on the parameters in external loads is very effective as demonstrated with the illustrative example. Since the performance of the algorithm is not so sensitive to the pitch of excitations, the algorithm is expected to work reasonably well in controlling seismic responses of a structure. This study also shows that the conventional closed-loop control is effective in reducing the peak responses of structures under stationary excitations.

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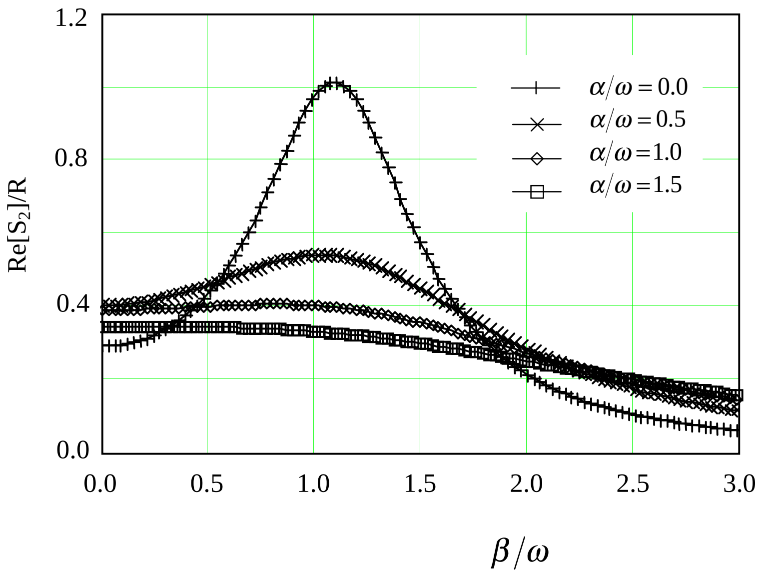
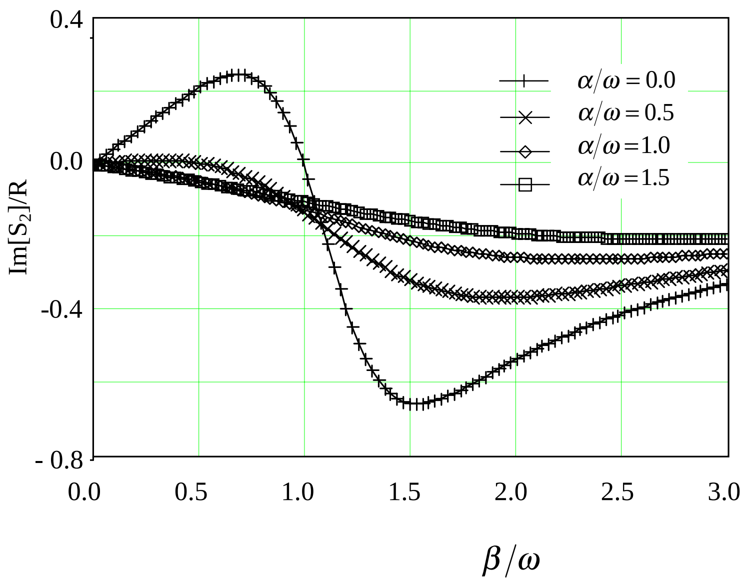


(a) Real Part



(b) Imaginary Part

Fig. 1 Open-loop control gain S_2/R

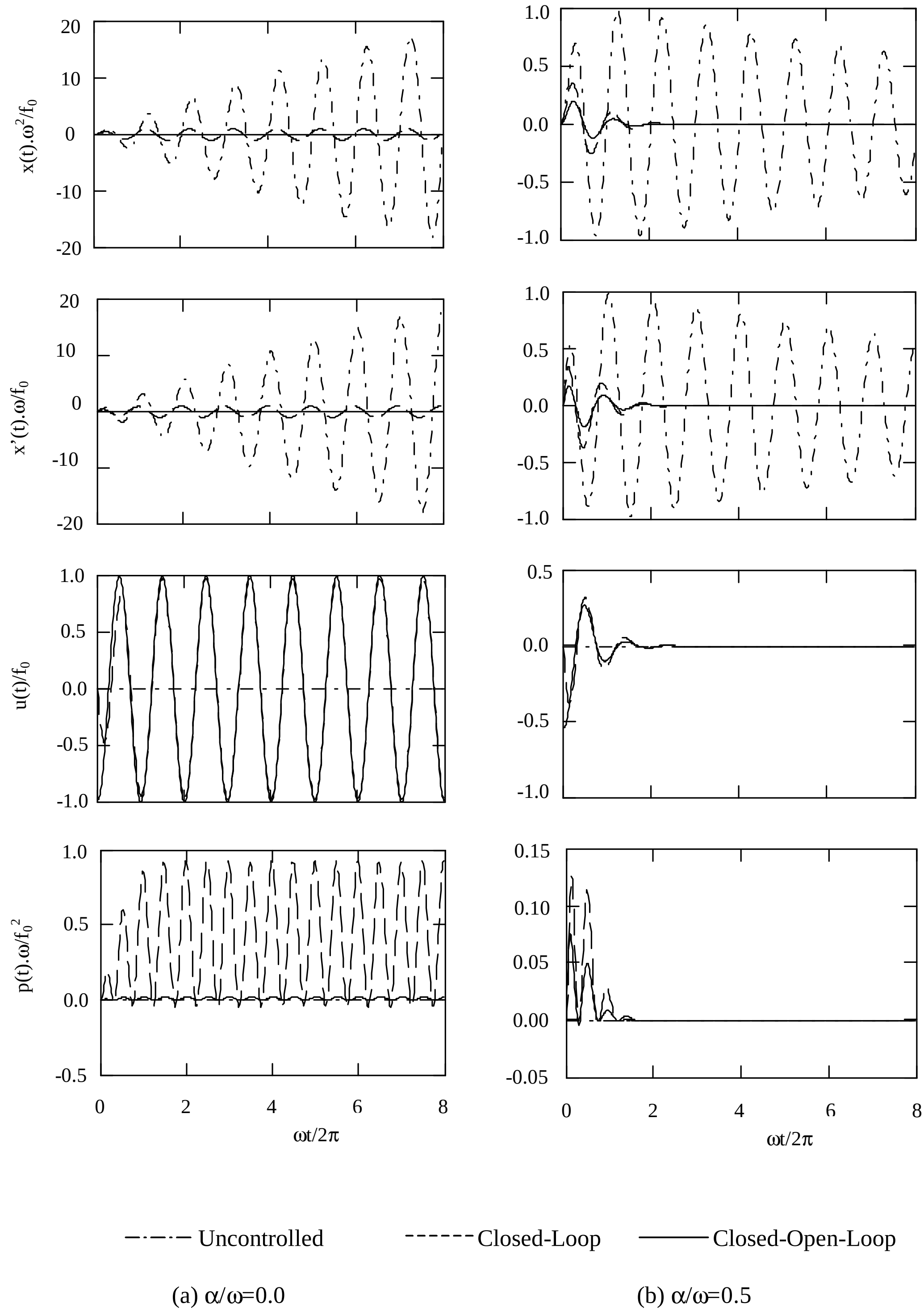
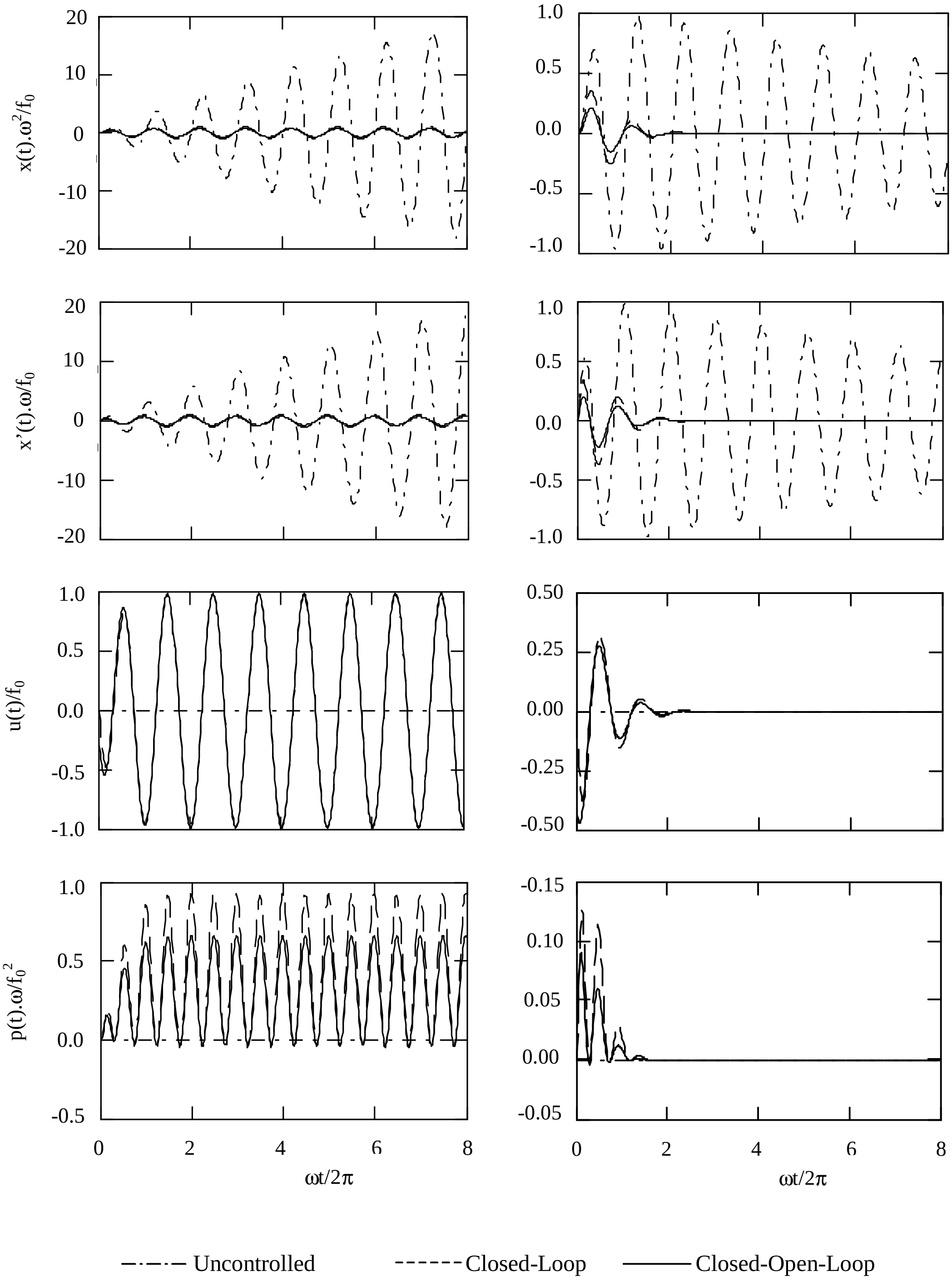


Fig. 2 Displacement, velocity, control force and power consumption associated with accurate open-loop control gain



(a) $\alpha/\omega=0.0$ (b) $\alpha/\omega=0.5$

Fig. 3 Displacement, velocity, control force and power consumption associated with approximate open-loop control gain