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BEHAVIOR OF PIPELINE WITH INITIAL BOW UNDER THE ACTION OF AXIAL LOAD

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SUMMARY

In this paper, the pipeline with initial bow under the action of axial load were analyzed. Also 15 steel pipelines with length l = 200cm and diameter d = 8.9cm were tested. The tested pipelines are subjected to the incrementally longitudinal load until yielding. The purpose of these investigations is to obtain the fundamental load–deformation behaviors of pipeline with initial bow and to investigate the flexibility improvement as well as hazard reduction of pipeline system during earthquake excitation. According to the theoretical analysis and experimental test, it is found that both of the axial force–longitudinal deformation and axial force–lateral deformation relationships are nonlinear for a pipeline with initial bow. These results indicate that if a pipeline can be made with a little curve, not only the member flexibility will be changed but also the system ductility will be improved.

For illustration, a designed simple pipeline model system subjected to ground displacement was analyzed in this paper. The results show due to the effect of initial bow the energy absorption of the system is increased and member stress is reduced. However, the change rate depends on the size (length and diameter) as well as the initial bow.

INTRODUCTION

During earthquake the pipeline system may subject to the action of longitudinal force as well as lateral force. Since the ductility of pipeline under axial load is not so good as that under lateral load, thus during earthquake most of the pipeline damage such as buckling, tension crack, and shear crack at the connection of two pipelines or at the point that two pipelines perpendicular are caused by longitudinal load. In order to improve these problems, in this investigation, the traditional straight pipelines are substituted by using the pipelines with initial bow. The fundamental concept is using the geometric characteristic of slightly curved pipeline to increase the flexibility and ductility of the system.

FORCE–DISPLACEMENT OF PIPELINE WITH INITIAL BOW

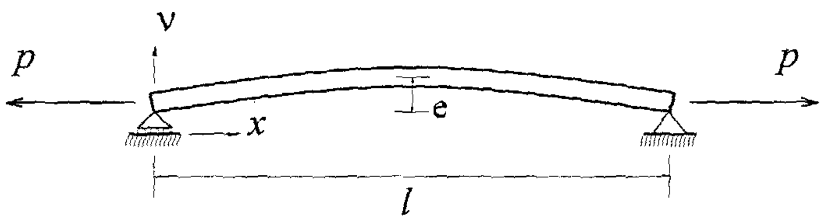

Figure 1: Simply supported pipeline subjected to longitudinal load

Figure 1 shows a pipeline with initial bow is simply supported at two ends. Under the action of axial force p, the governing equation can be expressed as

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$$EI(v'''' - v_o'''') - pv'' = 0 \tag{1}$$

where E = modulus of elasticity; I = moment inertia of pipeline cross-section; v_o = initial shape function; v = shape function after the action of axial force.

For this pipeline, assuming the initial shape function is [Tsai, 1977]

$$v_o = e\sin\frac{\pi x}{l}, (0 \le x \le l) \tag{2}$$

where l = length of the pipeline; e = initial bow at midpoint of the pipeline. After solving (1) and introducing the boundary conditions, $v(0) = v(l) = 0, v''(0) = v''(l) = 0$, we can obtain $v(x)$ and longitudinal deformation δ, i.e

$$v(x) = \frac{e}{1+\frac{p}{p_{cr}}}\sin\frac{\pi x}{l} \tag{3}$$

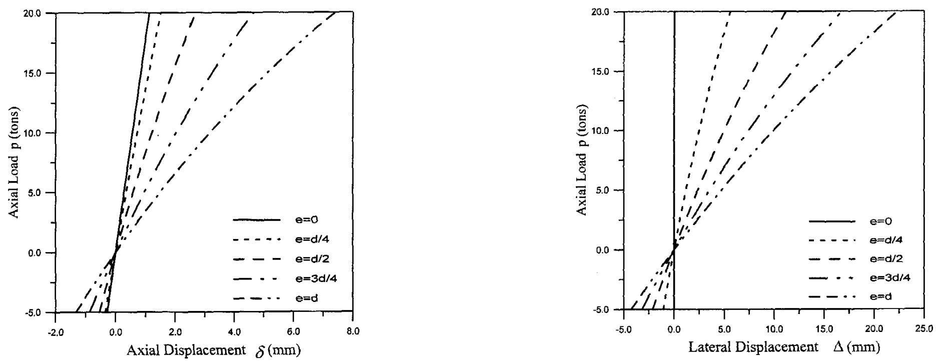
$$\delta = \frac{pl}{AE} - [\tfrac{1}{2}\int_0^l (v)^2\,dx - \tfrac{1}{2}\int_0^l (v_0)^2\,dx] = \frac{pl}{AE}[1 + \frac{4e^2}{d^2} \times \frac{(1+\frac{p}{2p_{cr}})}{(1+\frac{p}{p_{cr}})^2}] \tag{4}$$

in which p_{cr} = the critical axial load of straight pipe($= \frac{\pi^2 EI}{l^2}$).

In eq.(4), the $p-\delta$ relationship is nonlinear. The longitudinal deformation is proportional to the square of initial bow. From equation (3), furthermore, the lateral deformation Δ can be determined, i.e

$$\Delta = v(x) - v_0(x) = -[\frac{e}{\frac{p_{cr}}{p}+1}\sin\frac{\pi x}{l}] \tag{5}$$

Using eq.(4) and (5), we can generate the axial force–longitudinal deformation as well as the axial force–lateral displacement relationships for pipeline with any initial bow. For illustration, a group of pipeline (l = 200cm, d = 10.1cm, t = 0.6cm, E = 2×10^6kg/cm^2) with different initial bow (e = 0, e = d/4, e = d/2, e = 3d/4, e = d) are analyzed in this investigation. The analysis results are shown in fig. 2. From these figures, it is clear that the $p-\delta$ and $p-\Delta$ relationships are nonlinear for any pipeline with initial bow, Also in these figures, we can see the pipeline with larger initial bow (e value) will be more flexible and the increasing rate of δ and Δ are greater.



(a) $p-\delta$ relationship **(b) $p-\Delta$ relationship**

Figure 2: Load–Deformation of pipeline under longitudinal load

For comparison, totally, fifteen steel pipeline specimens are tested. The properties of the specimen including pipe length, initial bow, elastic modulus, yielding stress, are shown in Table 1 and Table 2. The test arrangement is shown in fig. 3 (except for the wood box).

Table 1: Length and initial bow of pipeline specimens.(unit: cm)

Specimen no.	A01	A02	A03	A11	A12	A13	A21	A22	A23	A31	A32	A33	A41	A42	A43
Length	200.0	200.0	200.0	200.3	200.0	200.1	199.6	199.7	199.7	199.7	199.5	199.4	199.0	199.2	198.8
Initial bow	0.0	0.0	0.0	2.8	2.8	3.05	4.2	4.0	4.1	5.5	6.05	5.9	8.45	8.45	8.55

Table 2: Properties of steel specimens

Specimen no.	1	2	3	4	5	6
Temper	No	No	No	Yes	Yes	Yes
Elastic modulus(kg/cm^2)	2×10^6	2×10^6	2×10^6	3×10^6	3×10^6	3×10^6

Yielding stress(kg/cm^2)	3300	3450	3300	2700	2850	2700
Average elastic modulus(kg/cm^2)	2×10^6			3×10^6		
Average yielding stress(kg/cm^2)	3350			2750		

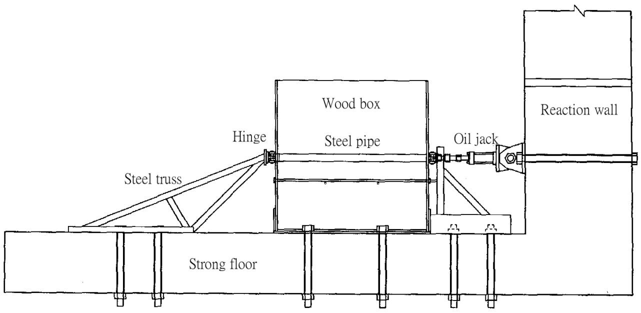
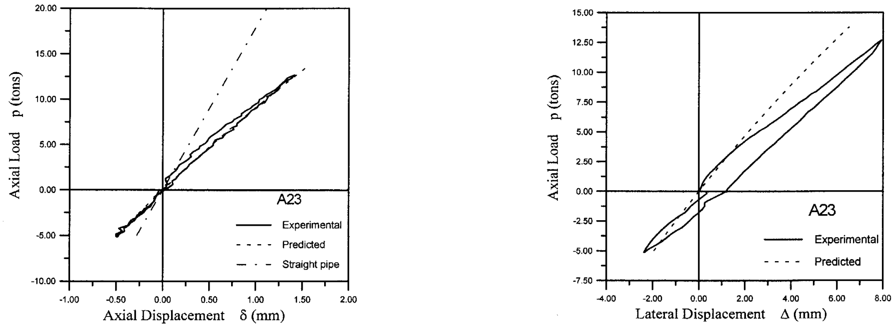


Figure 3: Test arrangement of pipeline subjected to longitudinal load

In general, the experimental tests obtain the similar nonlinear relationship of $p-\delta$ and $p-\Delta$ that generated in analysis. Fig. 4 is the typical result of test (specimen A23). In figures, the deformations obtained from test (solid line) are greater than that of analysis (broken line). The main reason is the test arrangement is more flexible.



(a) $p-\delta$ relationship **(b) $p-\Delta$ relationship**

Figure 4: Test results of specimen A23

FORCE–DEFORMATION OF PIPELINE EMBEDDED IN SOIL

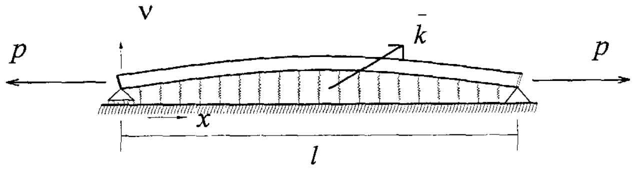


Figure 5: Model of pipeline with initial bow embedded in soil

The pipeline with initial bow embedded in soil is modeled as fig. 5. The curved pipeline is supported by elastic multi springs. When the pipeline is subjected to the action of axial force *p*, the govern equation is [Ja-Shian Chang,1996]

$$EI(v'''' - v_0'''') - pv'' + \bar{k}(v - v_0) = 0 \qquad (6)$$

where $\bar{k}$ = constant of soil spring.

From eq.(6) we can solve shape function

$$v(x) = \frac{e}{1+\frac{p}{p_{cr}'}}\sin\frac{\pi x}{l} \qquad (7)$$

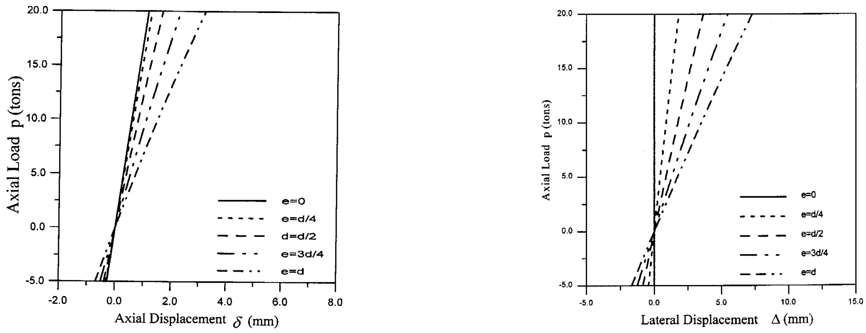
where $$p_{cr}' = \frac{\pi^2 EI}{l^2} + \frac{\bar{k} l^2}{\pi^2} \qquad (8)$$

and the axial deformation δ as well as the lateral deformation Δ

$$\delta = \frac{pl}{AE}\{1+\frac{4e^2}{d^2}\times\frac{[\frac{p_{cr}}{p_{cr}'}+\frac{p\times p_{cr}}{2(p_{cr}')^2}]}{(1+\frac{p}{p_{cr}})^2}\} \tag{9}$$

$$\Delta = -(\frac{e}{1+\frac{p_{cr}}{p}})\sin\frac{\pi x}{l} \tag{10}$$

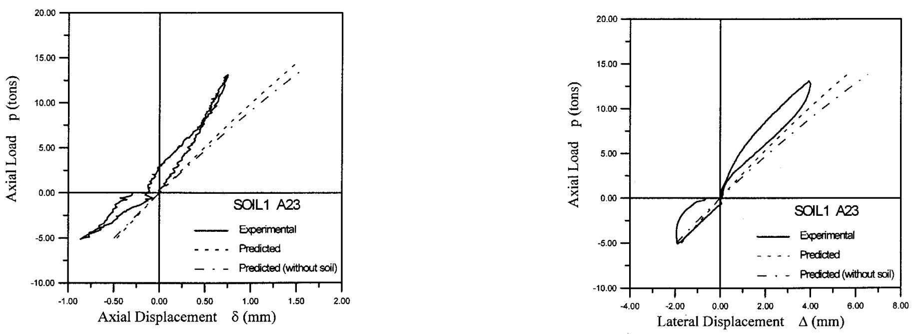
For the pipeline embedded in soil, fundamentally, the form of the solved shape function eq.(7) is analogous to that of eq.(3), except the critical load p_{cr} in eq. (3) replaced by p'_{cr}. Also from expression (9) (10), we can see the nonlinear relationship of axial force–axial deformation as well as axial force–lateral displacement. However, due to the effect of soil, p'_{cr} is greater than p_{cr} , under axial force p, both axial deformation and lateral deformation will be smaller than that without considering the soil effect. The effect of soil also can be seen from fig. 6, which is the analytical result for soil spring constant $\bar{k}$ is assumed to be 4×10^4KN/m^3. Comparing with fig. 2(a), obviously, the force–deformation relationships are similar to that not considering the soil effect, but due to the elastic constrain of soil, the deformation amount is smaller.



(a) $p-\delta$ relationship **(b) $p-\Delta$ relationship**

Figure 6: Load–Deformation of pipeline embedded in soil and subjected to longitudinal load

The comparison of analysis and test for the pipeline embedded in soil is shown in Fig. 7 (for specimen A23). The test arrangement is same as that shown previously before except the box is filled with soil (fig. 3). The constant $\bar{k}$ of soil spring is 5×10^3KN/m^3. In the figure, the trend of $p-\delta$ and $p-\Delta$ are analogous to that of analysis. However, the curve slope of experiment is larger than analysis. The surrounding constrain of wood box should be one of the reasons that make the rigidity of pipeline higher.



(a) $p-\delta$ relationship **(b) $p-\Delta$ relationship**

Figure 7: Load–Deformation comparison of analysis and test

BEHAVIOR OF PIPELINE UNDER DYNAMIC FORCE

The governing equation of a pipeline with initial bow under the action of dynamic force can be expressed as

$$EI\frac{\partial^4(v-v_0)}{\partial x^4}-p\frac{\partial^2 v}{\partial x^2}=-m\frac{\partial^2 v}{\partial t^2} \tag{11}$$

where p = the dynamic axial force; m = the mass of unit length; "–" represents the response direction.

If p is given by $p_0\sin wt$ (where w is the frequency of input force), the response can be expressed as follow:

$$v(x,t)=H_0\sin\overline{w}t\sin\tfrac{\pi x}{l}+e\sin\tfrac{\pi x}{l} \tag{12}$$

where H_0 is the function of $p,e,m,\overline{w},w,\dfrac{EI}{L}$.

Introducing expression (12) into eq. (11), we have

$$v(x,t)=\frac{e(1+\frac{\overline{w}^2}{w_1^{\,2}})}{1+\frac{p}{p_{cr}}+\frac{\overline{w}^2}{w_1^{\,2}}}\times\sin\tfrac{\pi x}{l} \tag{13}$$

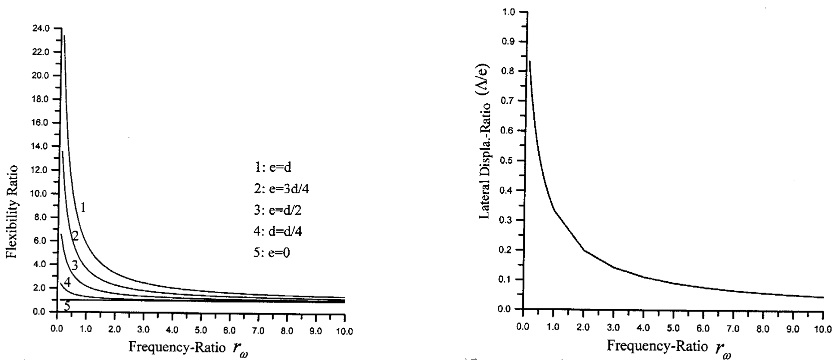
where w_1 = the first mode free vibration frequency of pipe.

Using (13), furthermore, the axial deformation δ and lateral deformation Δ can be expressed as

$$\delta=\frac{pl}{AE}[1+\tfrac{4e^2}{d^2}\times\frac{(1+\frac{p}{2p_{cr}}+\frac{\overline{w}^2}{w_1^{\,2}})}{(1+\frac{p}{p_{cr}}+\frac{\overline{w}^2}{w_1^{\,2}})^2}] \tag{14}$$

$$\Delta=-(\frac{e\frac{p}{p_{cr}}}{1+\frac{p}{p_{cr}}+\frac{\overline{w}^2}{w_1^{\,2}}})\sin\tfrac{\pi x}{l} \tag{15}$$

The deformation expressions obtained above are different from that derived from statically axial load. From eq. (13) we can see the response of pipeline not only has nonlinear relationship with dynamic axial force (p) but also has nonlinear relationship with frequency ratio $\dfrac{\overline{w}^2}{w_1^{\,2}}$. Fig. 8(a) and (b) are the calculation of flexibility ratio (axial flexibility of curved pipe/axial flexibility of straight pipe) as well as lateral displacement ratio ($\dfrac{\Delta}{e}$) for various frequency ratio. In this calculation the input dynamic axial force p is assumed to be $\dfrac{p_{cr}\sin\overline{w}t}{2}$. In these two figures, when the frequency of input dynamic axial force is much less than the natural frequency of pipeline (i.e low frequency ratio), the pipeline system has good flexibility. However, the lateral deformation of pipeline will be larger.



(a) Flexibility ratio(curved pipe/straight pipe) **(b) Lateral displacement ratio**

Figure 8: Flexibility ratio and lateral displacement ratio for various initial bow

ENERGY ABSORPTION AND MEMBER STRESS FOR SYSTEM USING PIPELINE WITH INITIAL BOW

Energy absorption

Under the action of axial force *p*, the critical energy [Young-Hong Hsei,1997] can be absorbed by a straight pipe is

$$U = \frac{p_{cr}{}^2 l}{2AE} \tag{16}$$

Also, before yielding the energy can be absorbed is

$$U = \frac{\sigma_y{}^2 l}{2E} \tag{17}$$

When a pipeline with initial bow is subjected to axial force, the first yielding point will occur at midpoint of the length and the energy absorbed by the pipeline is

$$U = \int_0^{\delta_y} p d\delta \tag{18}$$

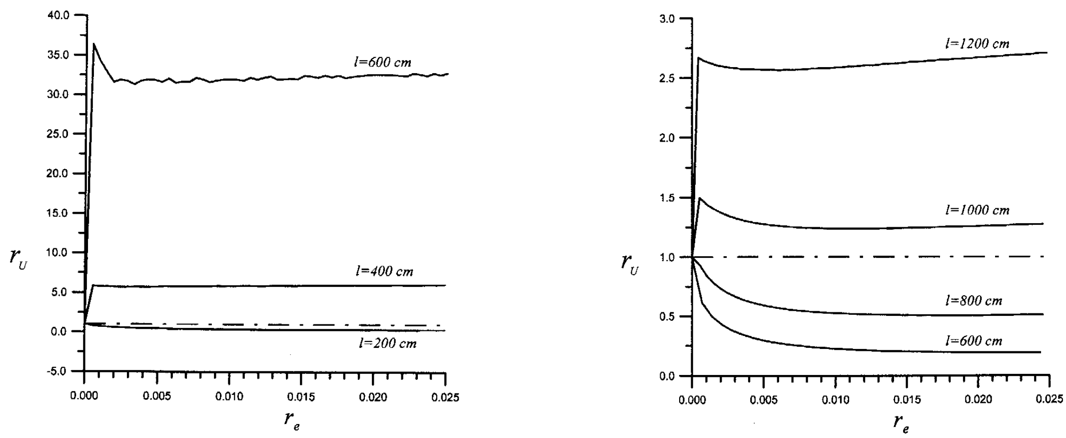
where δ_y = axial deformation of curved pipe under yielding axial force p_y; $d\delta$ can be derived from eq. (4).

After introducing the expression of δ_y and $d\delta$ into eq. (18), the equation will change to

$$U = \int_0^{p_y} \frac{pl}{AE}[1+\frac{4e^2}{d^2(1+\frac{p}{p_{cr}})^3}]dp \tag{19}$$

Using this expression we can evaluate the initial bow effect on the energy absorption of pipeline system. For investigating this effect, several pipes with different length have been analyzed. Fig. 9(a)~(b) show the energy absorption ratio for various bow ratio. In which the broken line is the reference line that the energy absorption ratio is equal to 1. From these two figures, it is observed that

1. A pipeline changes from straight to slight initial bow, the absorption energy increases rapidly. However, when the initial bow ratio reaches 0.002 the energy absorption approaches to a constant value (fig. 9(a)~(b)).
2. For constant length pipes(with initial bow), the pipeline with smaller diameter get better energy absorption (fig. 10).
3. For a single pipe with initial bow, the length/diameter ratio (*l*/D) is suggested to be greater than 44.5. If a pipe has this ratio, slight initial bow will help pipeline system to absorb large amount of input energy(fig. 11).



(a) D=60mm, t=3.4mm **(b) D=216mm, t=7.1mm**

Figure 9: Effect of bow ratio on energy absorption ratio for various pipeline length

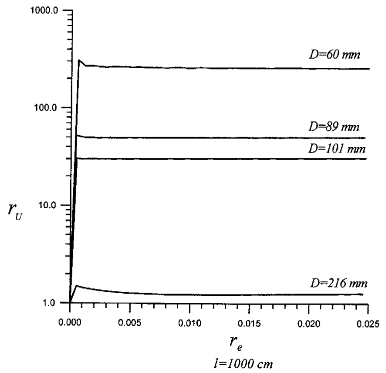


Figure 10: Effect of bow ratio on energy absorption ratio for various pipeline diameter

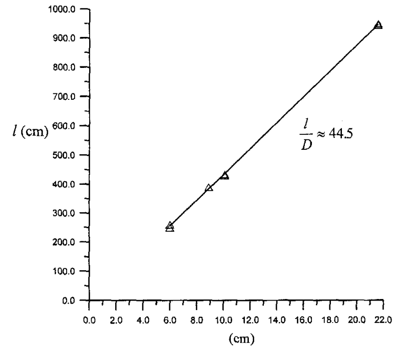


Figure 11: Length–Diameter relationship for pipeline with initial bow

Stress influence of pipeline with initial bow

Fig. 12 shows a simple pipeline system. In this system CD member is a pipe with initial bow. When point D is forced to move a displacement δ_D to the left, assuming the axial deformation of AB and BC are neglected, the axial force of CD can be derived from the equilibrium relationship of input energy and strain energy

$$P_{CD} = \frac{\delta_D}{\frac{1}{k'} + \frac{4L^3}{3EI}} \tag{20}$$

where k' = the axial rigidity of curved steel pipe.

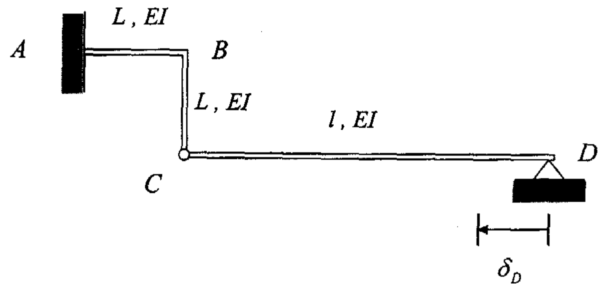


Figure 12: A simple pipeline system

The bending moment and shear of AB and BC at end of the members are

$$M_{AB} = M_{BA} = M_{BC} = \frac{3EI\delta_D L}{\frac{3EI}{k'} + 4L^3} \tag{21}$$

$$V_{BC} = \frac{3EI\delta_D}{\frac{3EI}{k'} + 4L^3} \tag{22}$$

Using eq. (20) and (21) the moment and shear of each member can be determined. When point D is forced to move, the bending moment and shear at member ends will decrease, and axial rigidity of the pipe will decrease too. Fig. 13 further shows the effect of stress reduction of pipeline with initial bow. The rate of moment reduction gets increased as the bow/diameter (*e*/*d*) ratio is getting larger. Also for a pipeline with longer length, the effect will be more visible.

The simple pipeline model shown in fig. 12 also can be used to discuss the energy absorption for pipeline system with initial bow. According to the basic structure theory, in fig. 12, when point D is subjected to axial force *p*, the force–displacement relationship can be expressed as

$$\delta_p = p \times \left(\frac{1}{k'} + \frac{4L^3}{3EI}\right) \tag{23}$$

and the strain energy stored in the system U can be obtained from the integration of $p-\delta_p$ curve shown in eq. (22).

$$U = \int p d\delta_p = \int \{(\frac{4pL^3}{3EI}) + \frac{pL}{AE}[1 + \frac{4e^2}{d^2(1+\frac{p}{p_{cr}})^3}]\} dp \tag{24}$$

Furthermore, using the calculated strain energy we can evaluate energy absorption ratio. Fig. 14 shows the energy absorption ratio for pipeline system with different initial bow and different length. It is evident that pipe with greater initial bow and longer length has a larger energy absorption.

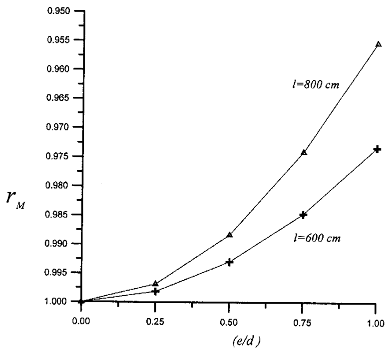


Figure 13: Effect of bow/diameter ratio on moment reduction

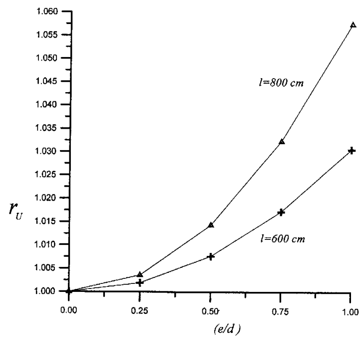


Figure 14: Effect of bow/diameter ratio on energy absorption ratio

CONCLUSIONS

According to the analyses and experiments proceeded in this paper, it is found that pipeline with initial bow will be able to improve the system flexibility and to reduce the member stress, when the system is under the longitudinal load or supporting movement caused by earthquake. In pipeline engineering such as water pipe, fuel gas pipe, oil pipe and telecommunication pipe, for reducing the earthquake damage, introducing the pipeline with initial bow is a way worth to be tried. However, for detail design, more investigation including experiment still need to be done.

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