

## DYNAMIC CHARACTERISTICS OF PILE GROUPS UNDER VERTICAL VIBRATIONS

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### SUMMARY

To study the influence of pile-soil interaction on dynamic characteristics of pile groups, vertical vibration tests were conducted in a carefully designed small scale pile test facilities at the soil dynamics laboratory of IIT Madras, Chennai. Tests were conducted on 19 mm - model Aluminium single pile, 2 x 2 and 3 x 3 pile groups, for length to diameter ratios of 30, 40 and 50 and for spacing of 3d, 5d and 7d. The pile groups were subjected to vertical vibrations with smaller amplitude and high frequencies. The time histories of the displacement of the pile cap, the amplitude of the force transferred to the pile group and the phase difference between the applied force and displacement of the pile groups were measured at wide frequency ranges. The frequency dependent group stiffness and damping constants were evaluated directly from the test data.

The analysis of the test results indicate that the group stiffness increases with increase of frequency upto limiting frequency and then decreases. The damping constants were substantially high at the low frequencies and decreases with the increase of the frequency. The stiffness is found to be increasing and damping is found to be decreasing as the spacing between the piles decreases. The group stiffness evaluated as the vector sum of the individual pile stiffness obtained from single pile test data is found to be higher than the group stiffness obtained directly from the pile group tests. The group stiffness and damping either can be reduced by pile-soil-pile interaction or increased depending on the frequency and spacing between the piles.

A software was developed for calculation of dynamic group stiffness and damping constants based on Novak and Sheta approximate theory for calculation of single pile stiffness and damping constants with Dobry and Gazetas dynamic interaction factors. The computed results were compared with the experimental results. The estimated dynamic group stiffness and damping constants for 3 x 3 pile group are in good agreement with experimental results. The estimated stiffness and damping constants for 2 x 2 pile group are found to be overestimated practically for all the frequency ranges.

### INTRODUCTION

Determination of the pile group stiffness and damping is an important step in the analysis of pile-supported structures subjected to dynamic loading due to earthquakes, machinery, waves in marine environment etc. There are many approaches for solving the dynamic vertical response of a single pile. Novak (1991) gave an extended critical review of the more widely accepted methods of analysis. He showed that pile group response could not be deduced from single pile response without taking pile-soil-pile interaction into account. The dynamic characteristics of piles in a group may be quite different from those of a single pile. The dynamic response of pile groups by considering the interaction between the piles can be done by direct approach or superposition approach. In direct approach, piles in a group are considered to interact simultaneously (Wolf and Von Arx 1978, Banerjee et al 1987, Novak and Sheta 1982). In superposition approach, the interaction of only two piles is considered separately and used to formulate the stiffness matrix of the entire group. This method was originally proposed by Poulous (1968) for approximate static analysis of pile groups. He proposed static interaction factors for finding the static group efficiency of piles. Later, these interaction factors were used for finding the dynamic group efficiency by Kaynia and Kausel (1982), Dobry and Gazetas (1988), El-Marsafawi et al (1992). The above

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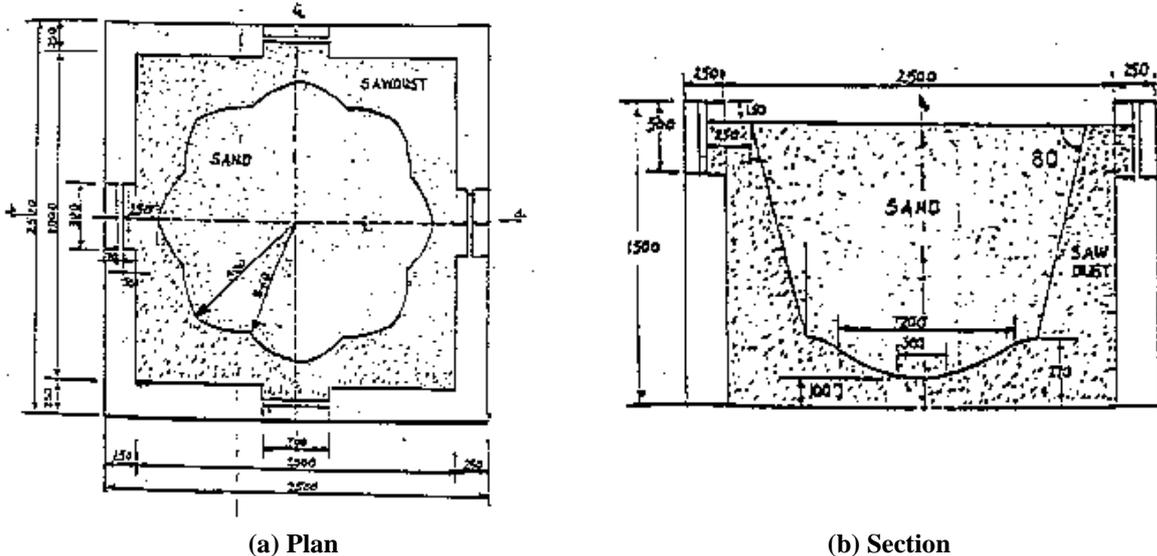
dynamic studies on pile groups have demonstrated that the dynamic response of pile group may differ substantially from their static response, and that pile group efficiency factor may attain a value greater or less than unity.

Dynamic Experiments on a large group of small closely spaced piles are conducted in the field by Novak and El-Sharnouby (1984). They concluded that the dynamic analytical techniques are able to predict the main features of dynamic behaviour of pile groups for small strains. El-Marsafawi et al (1984) conducted field experiments on two pile groups: one on model steel piles in non-cohesive soil and the other on full-scale concrete piles in clay. From the comparison of the experimental results with the theoretical predictions, they concluded that the linear theory gives a good estimate of the group stiffness but overestimates the damping. The results of vertical vibration tests carried on model steel piles by Srinivasulu et al (1996) indicate that a large difference between the experimental and the theoretical amplitudes at resonance is due to overestimation of damping by the theory. From the literature, it is clear that, many theories were developed for evaluation of dynamic stiffness and damping constants for single piles as well as for groups. But the experimental verification of these theories is very little especially for group of piles.

Therefore, the main objective of this work is to determine the variation of pile group stiffness and damping with frequency and to compare with the theoretical predictions.

**TESTING SYSTEM**

Tests were performed in a carefully designed small-scale pile test facility at the soil dynamics laboratory of IIT Madras. This facility involves a masonry tank of inner dimensions of 2m x 2m x 1.5m. Elastic Half Space is simulated in the tank to obtain minimum reflection and maximum absorption of the waves generated during the vertical vibration. Taking the cue from the literature (Stokoe & Woods 1972, Srinivasulu et al 1996), logarithmic arc spiral is taken as the boundary and it was fabricated with 80° inclination in the sectional view using 6 m diameter mild steel rods. This framework was wrapped around by geotextile to separate the soil and the absorption material. Saw dust is found to have maximum absorption characteristics among cork powder, sawdust and thermocool from the literature (Gazetas and Stokoe 1991). Hence, sawdust used as the absorption material. This was tightly packed between the masonry well and the framework. By this selection of the boundary profile and the absorption material, the elastic half space is simulated in the tank as shown in the Figure 1.



**Figure 1 : Elastic Half Space Simulation**

**SAND**

River sand is placed within the framework of tank using sand raising technique. In this method, the sand is allowed to drop through a pipe and is dispersed in the tank by a controlled height of free fall. The relative density of the sand is maintained constant all through the tank and is equal to 65%. The properties of the sand are given in Table 1. The dynamic properties of sand are obtained from in-situ cross-hole test and their values are given in Table 2.

**TABLE 1: PROPERTIES OF SAND**

Minimum Dry Density, $\gamma_{dmin}$	:15.10 kN/m <sup>3</sup>
Maximum Dry Density, $\gamma_{dmax}$	:17.72 kN/m <sup>3</sup>
Uniformity Coefficient, $C_U$	: 2.32
Coefficient of Curvature, $C_C$	: 1.03

**TABLE 2 : RESULTS OF CROSS HOLE TEST**

Depth	Shear Wave Velocity, $V_s$ (m/sec)	Dynamic Shear Modulus, $G_{max}$ (MPa)
+0.00	127.38	29.87
-0.15	135.14	33.62
-0.30	138.89	35.51
-0.45	142.45	37.36

**MODEL PILES AND PILE CAPS**

Aluminium model piles of 19 mm outside diameter with a thickness of 2 mm were used for conducting the vertical vibration tests. Tests were conducted on single piles and on 2 x 2 and 3 x 3 pile groups. Piles with length to diameter ratio (l/d) of 30, 40 and 50 were used. In the case of groups of pile, the pile spacing to diameter ratio (s/d) of 3, 5 and 7 were adopted. Pile caps made up of aluminium plate of size 130 mm X 130 mm x 20 mm in case of 2 x 2 groups, and of size 250 mm x 250 mm x 20 mm in case of 3 x 3 groups were used.

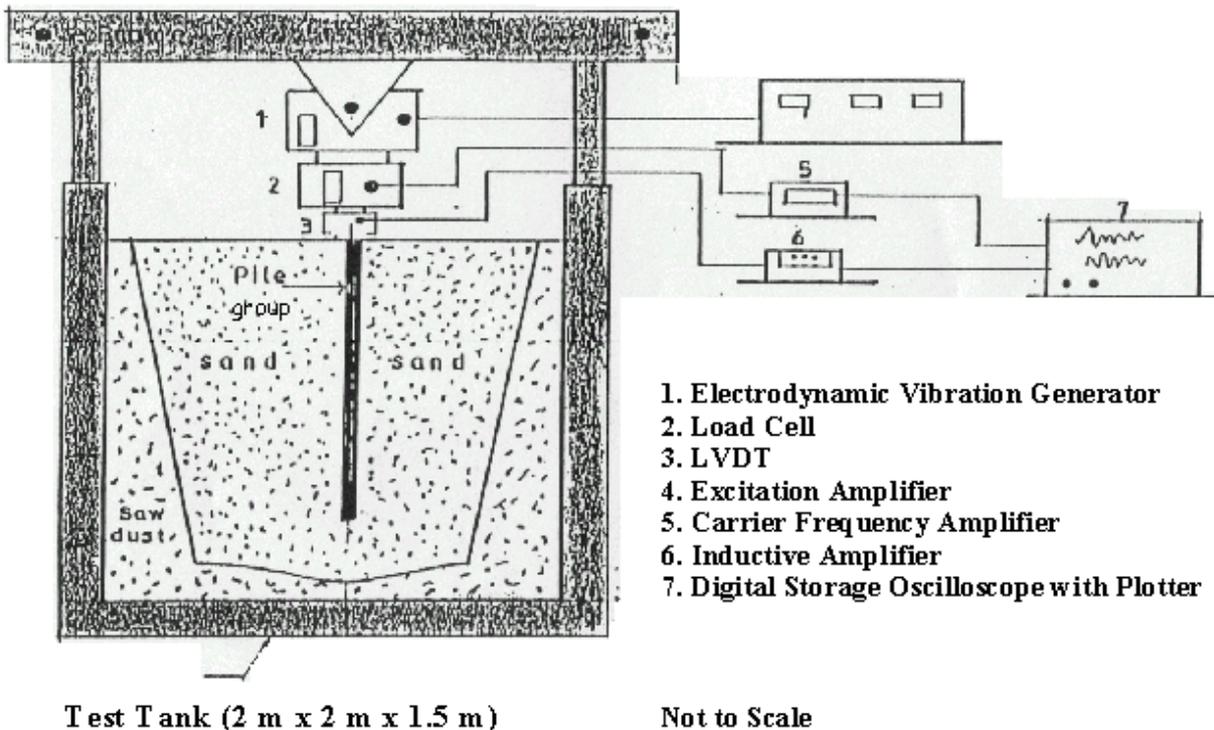
**TEST PROCEDURE**

Figure 2 shows the test set-up for conducting vertical vibration tests. Steady state sinusoidal vertical excitations with very small amplitudes (4N - 10N) but high frequencies (5Hz-250Hz) were given at the pile head level by means of an Electro dynamic exciter. *HBM* strain gauge load cell was connected between the exciter and the pile head to determine the load transferred to the pile head. LVDT of *HBM* type was fixed to the pile head to measure the displacement of the pile head. The time histories of the load transferred to the pile head and the pile head displacement and the time lag i.e. the phase difference between the load and displacement ( $\phi$ ) were observed and recorded through the *Gould* Digital storage oscilloscope. The frequency dependent stiffness constant ( $k_r$ ) and damping constant ( $k_i$ ) of the pile group directly evaluated from the test data using the following expressions as similar to adopted by Kim et al (1987)

$$K_r = (F_o / u) \cos \phi \quad k_i = (F_o / u) \sin \phi \quad (1)$$

where  $F_o$ ,  $u$  are the amplitudes of applied force and displacement respectively,  $\omega$  is the frequency of excitation in radians/sec.

Both single pile and pile groups were tested for three different excitation intensities of 4N, 8N and 10N.



**Figure 2 : Vertical Vibration Test Setup**

## TEST RESULTS AND DISCUSSIONS

### Dynamic Response Curves

Figure 3 shows the typical normalised response curves of 2 x 2 and 3 x 3 pile groups with length to diameter ratio ( $L/d$ ) of 30 and pile spacing to diameter ratio ( $s/d$ ) of 3. The normalised amplitude,  $A$  was calculated as,

$$A = \frac{u}{F_o} \quad (2)$$

in which  $u$  is the measured vertical displacement amplitude,  $F_o$  is the applied force amplitude. The occurrence of a distinguished single peak in the response curves indicate that the system behaves as a damped single degree of freedom system within the range of frequencies considered. The occurrence of resonance at higher frequency is due to the low lumped mass over the pile head. It can also be seen from Figure 3 that the normalised response curves for pile groups differ with each other particularly at frequencies closer and above resonant frequency. It may be due to the fact that, damping in this case not varies linearly with frequency.

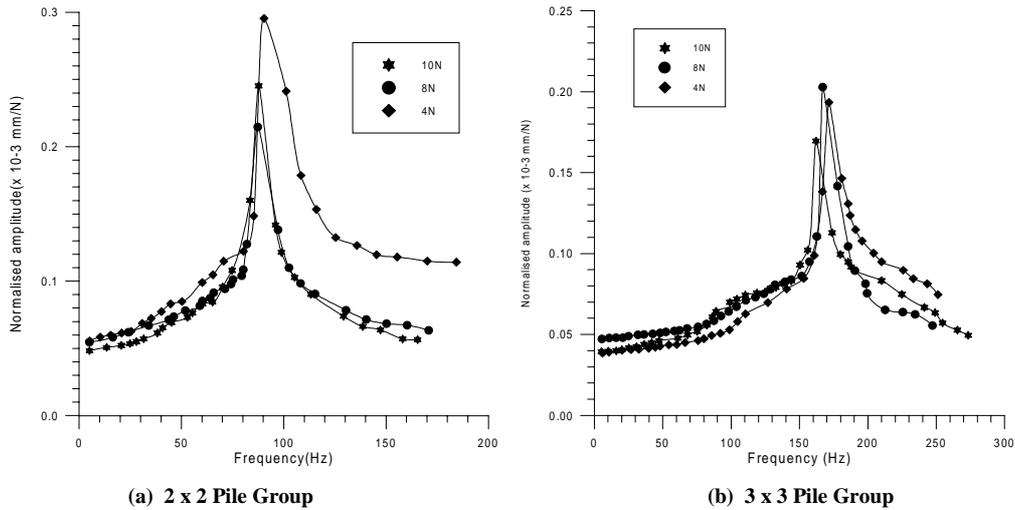


Figure 3 : Normalised Amplitude as a function of frequency ( $L/d = 30, s/d = 3$ )

### Dynamic Stiffness And Damping

The dynamic stiffness and damping constants of pile group are normalised with the static stiffness of the soil-pile system as follows :

$$\text{Normalized group stiffness} = \frac{\text{Dynamic group stiffness constant } (k_r)}{\text{No. of piles x static single pile stiffness } (k_s)} \quad (3)$$

$$\text{Normalized group damping} = \frac{\text{Group damping constant } (\bar{C})}{\text{No. of piles x static single pile stiffness } (k_s)} \quad (4)$$

$$\text{Where } \bar{C} = \frac{cV_s}{d}$$

$c$  = Equivalent viscous damping,  $V_s$  = Velocity of the shear wave,  $d$  = diameter of the pile.

The single pile static stiffness,  $k_s$  is evaluated using the following expression proposed by Makris et al (1996)

$$k_s = \frac{\frac{4}{1-\nu_s} + \frac{2\pi \tan(\mu L)}{\zeta} \frac{L}{r_o}}{1 + \frac{1}{\pi} \frac{G_s}{E_p} \frac{4}{1-\nu_s} \frac{\tan(\mu L)}{\mu L} \frac{L}{r_o}} G_s r_o \quad (5)$$

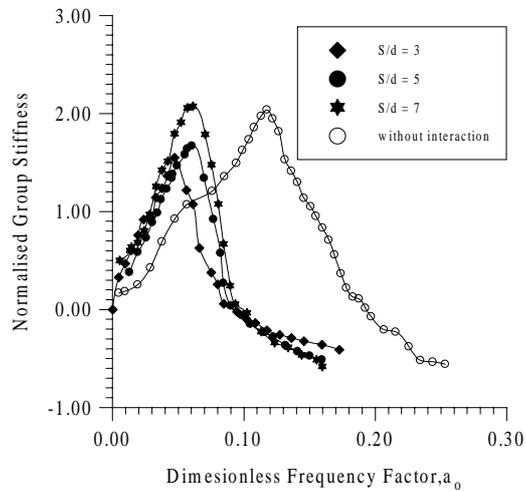
where,

$$\mu = \frac{\sqrt{2G_s/E_p}}{r_o}$$

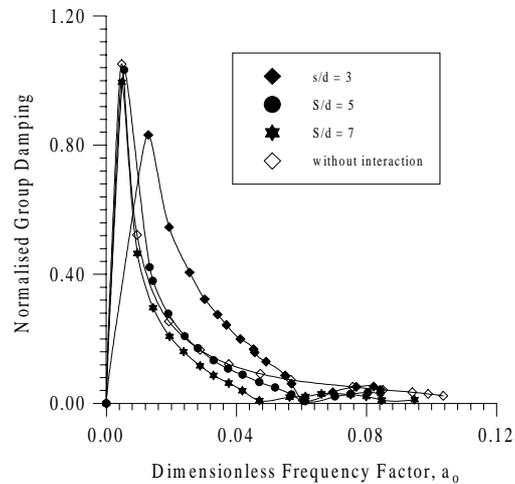
$G_s$  = Shear Modulus of the soil,  $E_p$  = Elasticity of the pile ( $=9 \times 10^4 \text{ N/mm}^2$ ),  
 $\nu_s$  = Poisson's Ratio of the soil ( $=0.25$ ),  $L$  = Length of the pile

### 2 x 2 pile Group

Figures 4 and 5 shows the normalized group stiffness and damping as a function of dimensionless frequency factor  $a_0$ , ( $=\omega d/2V_s$ ) for 2 x 2 pile group for various pile spacings. The normalized group stiffness and damping constants calculated as the vector sum of the individual pile stiffness and damping constants obtained from single pile tests (marked as without interaction) are also shown in Figures 4 and 5.



**Figure 4 : Normalised stiffness as a function of Dimensionless Frequency Factor (2 x 2 group, L/d = 30, Fo = 4 N)**



**Figure 5 : Normalised Damping Constant as a Function of Dimensionless Frequency Factor (2 x 2 group, L/d = 30, Fo = 4 N)**

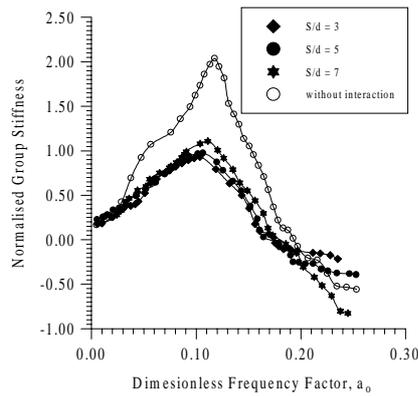
It can be observed from Figure 4, with the increase in the dimensionless frequency factor, the normalised stiffness increases, reaches a maximum value of above 2 at limiting dimensionless frequency factor and then decreases. It means that the group stiffness at limiting dimensionless frequency factor is about 2.0 times the sum of static stiffness of the individual piles. The normalised stiffness becomes negative at higher frequency factors. i.e the dynamic stiffness is lower than the static group stiffness at a frequency factor of 0.085-0.090. The negative values of stiffness may be due to the wave interference within the soil-pile system due to the vibration given on the system. Cylindrical waves generated from a pile travel with a certain phase and arrive at the neighbouring pile in exactly opposite phase, thereby inducing a negative displacement when compared to the displacement due to its pile's own load. Hence, the normalised stiffness is negative.

Figure 5 shows that at very low frequencies ( $a_0 < 0.05$ ), the group stiffness curve obtained from single pile test results coincides with experimental group stiffness curves but at wider range of frequencies ( $0.05 < a_0 < 0.25$ ) it significantly differs from the experimental curves. It indicates that the reduction of dynamic group stiffness is about 50% due to pile-soil-pile interaction at wider range of frequencies. The experimental group peak stiffness is lower than the estimated peak stiffness may be because, the resonance occurs at lower frequencies for pile groups than for single pile.

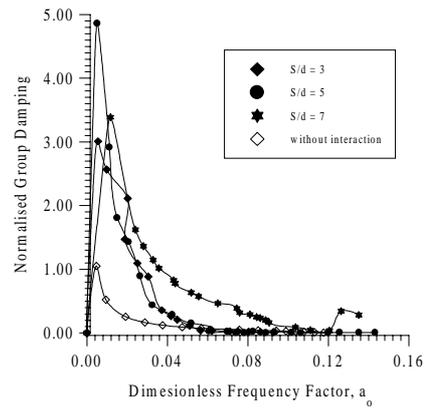
It is observed from Figure 5 that the damping is high at initial dimensionless frequency factors and then decreases with the increase in dimensionless frequency factor. However, group damping obtained directly from pile groups tests as well as from single pile tests.

### 3 x 3 Pile Group

Figures 6 and 7 shows the normalised group stiffness as a function of dimensionless frequency factor,  $a_0$  for 3 x 3 pile group. The pattern followed by these curves are same as in case of 2 x 2 group but the normalised peak stiffness values of 3 x 3 groups are higher than for 2 x 2 groups. From Figure 6, it is observed that the normalised stiffness is having a maximum value of about 0.9 at a frequency factor of 0.08 to 0.11. It means that the group stiffness at limiting dimensionless frequency factor is about 0.9 times the sum of static stiffness of the individual piles. The peak normalised stiffness for 3 x 3 group is less than 2 x 2 pile group. i.e., with the increase in the number of piles in a group, the group dynamic stiffness will not increase in proportional to the number of piles. In this case also, the normalised group stiffness becomes negative at higher frequency factors. i.e the dynamic stiffness is lower than the static group stiffness at a frequency factor of 0.15 - 0.25. The negative values of stiffness may be due to the wave interference within the soil-pile system due to the vibration given on the system. Figure 6 shows that the peak group stiffness estimated using single pile result is much higher than the experimental group stiffness.



**Figure 6 : Normalised stiffness as a function of Dimensionless Frequency Factor (3 x 3 group, L/d = 30, Fo = 4 N)**



**Figure 7 : Normalised Damping Constant as a Function of Dimensionless Frequency Factor (3 x 3 group, L/d = 30, Fo = 4 N)**

It is observed from Figure 7 that the damping constant is high at initial dimensionless frequency factors and then decreases with the increase in dimensionless frequency factor. The peak normalized damping is high for 3 x 3 pile group when compared with single pile and 2 x 2 pile group. The normalised damping constant obtained from the single pile result is lower than the experimental group damping for all the spacings considered.

## COMPARISON OF TEST RESULTS WITH NUMERICAL PREDICTIONS

### Development of Software

A program was developed in C++ to estimate the group stiffness and damping of pile groups by superposition approach. The dynamic characteristics of single pile were determined based on Novak and Sheta's (1988) approach. Soil stiffness matrix, pile stiffness matrix and pile mass matrix were evaluated separately and then assembled to get single pile stiffness and damping constants. The pile group effects were calculated using the dynamic interaction coefficients of only two piles at a time, based on simple one-dimensional physical approximation proposed by Dobry and Gazetas (1988).

The stiffness and damping constants evaluated by the experiments are compared with the estimated values obtained from software for both single pile and group of piles. Results are compared with and without considering the weak zone around the pile. The parameters assumed for the weak zone as per Novak and Sheta (1982) are as follows: weak zone thickness = 0.2 d, Shear Modulus Ratio = 0.1, Weak zone material Damping = 5%.

### Pile Group stiffness

Figure 8 shows the variation of the normalized stiffness with frequency factor at different force levels, for 2 x 2 and 3 x 3 pile groups. From the Figure it is observed that, a better prediction of the stiffness and damping of the first mode is achieved particularly for ascending side, but the entire response curves are not able to duplicated. The numerical prediction underestimates the group stiffness up to a frequency factor of 0.04 and it highly overestimates in case of 2 x 2 pile group. In case of 3 x 3 pile group, the numerical prediction without considering the weak zone underestimates the group stiffness. The theoretical solution evaluated considering with and without weak zone around the pile over estimates the peak stiffness.

### Pile group damping

Figure 9 shows the normalised damping as a function of dimensionless frequency factor for 2 x 2 group and 3 x 3 pile group. Figure 9(a) indicates that the group damping (considering with and without weak zone) highly overestimates the damping for 2 x 2 pile group. In case of 3 x 3 pile group, Figure 9(b) indicates the group damping estimated with and without weak zone around the pile is in good agreement with the test results.

## CONCLUSIONS

Dynamic experiments were carried on 2 x 2 and 3 x 3 groups with increasing excitation intensity for different frequencies, for different length to diameter ratios and for different spacings in order to establish the response curves. The frequency dependent stiffness and damping constants were evaluated directly from the test data. The stiffness and damping constants were also estimated based on Novak and Sheta solution for single pile with and

without weak zone around the pile and with Dobry and Gazetas dynamic interaction factors. The following conclusions were made based on experiments and theoretical results.

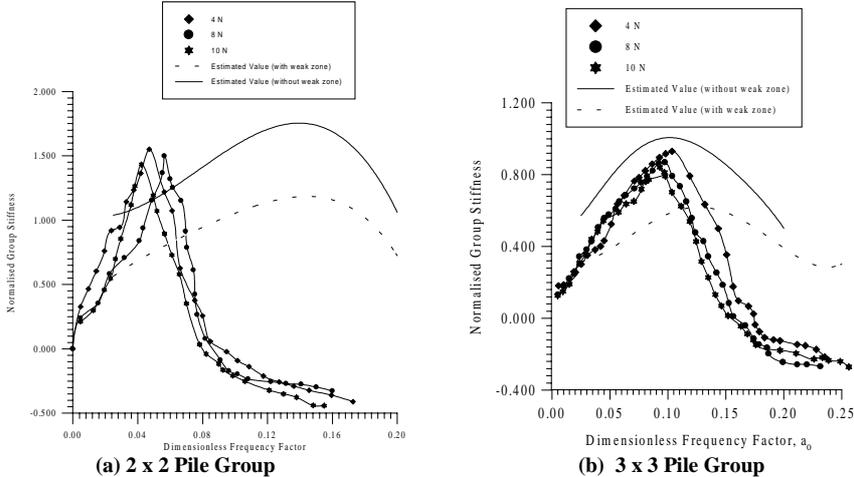


Figure 8 : Normalised Stiffness as a function of Dimensionless Frequency Factor (L/d = 30, s/d = 3)

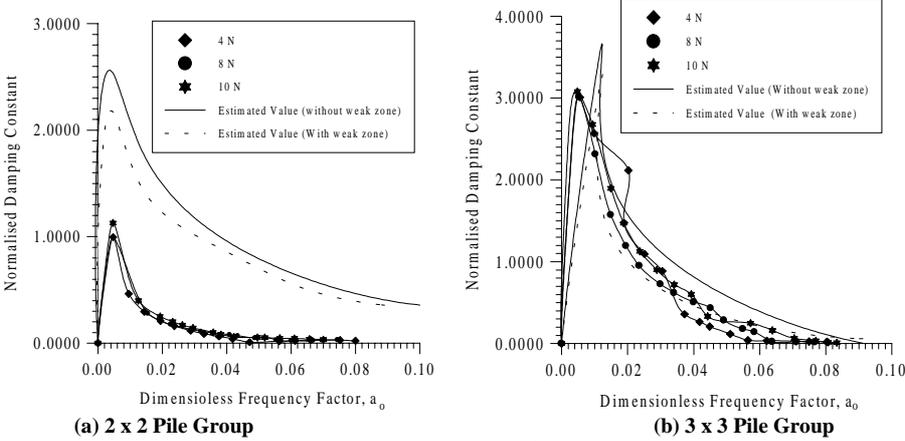


Figure 9 : Normalised damping as a function of Dimensionless Frequency Factor (L/d = 30, s/d = 3)

- At very low frequencies ( $a_0 < 0.05$ ), the group stiffness evaluated from experimental single pile stiffness without considering the interaction between the piles is practically the same as the experimental group stiffness while for higher range of frequencies ( $a_0 > 0.05$ ) is about 50% higher than the group stiffness determined from test results.
- The numerical prediction considering the weak zone around the pile highly underestimates the group stiffness for 2 x 2 pile group for the dimensionless frequency,  $a_0$  ranging from 0.047 to 0.14 and it predicts well for the other ranges of  $a_0$ .
- For 3 x 3 pile group, the estimated group stiffness considering weak zone around the pile is in good agreement with the experimental group stiffness for dimensionless frequency,  $a_0$  ranging from 0.005 to 0.140 and it is greater than 75% for  $a_0$  greater than 0.140.
- The group damping estimated with and without weak zone around the pile is in good agreement with the experimental group damping for 3 x 3 group and highly overestimates for 2 x 2 group.

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