A PERFORMANCE-BASED OPTIMAL DESIGN METHODOLOGY INCORPORATING MULTIPLE CRITERIA

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SUMMARY

A general framework is presented for optimal design based on multiple design criteria which is suitable for performance-based design of structural systems operating in an uncertain dynamic environment. Reliability-based design criteria are used to maintain user-specified levels of structural safety by properly taking into account the uncertainties in the seismic loads that a structure may experience during its lifetime, as well as modeling uncertainties. Code-based requirements are easily incorporated into the optimal design. The methodology is demonstrated with a simple example involving the design of a three-story steel-frame building for which the ground motion uncertainty is characterized by a probabilistic response spectrum developed from a standard seismic hazard analysis.

INTRODUCTION

The decision-making process in the design of civil engineering systems requires the selection of the most promising choice for the design from a large set of possible alternatives, based on an evaluation using specified criteria reflecting the acceptability of a design. Such criteria usually include costs, structural engineering criteria, client preferences, social, political, legal and economic considerations, and liabilities from uncertain risks arising, for example, from construction practice and environmental loads such as earthquakes and strong winds. Many of these design criteria are conflicting. The development of a computer-aided optimal design decision process should allow the designer to trade off these criteria in a controlled manner during the design optimization. A software package CODA has been developed to assist the design decision process by using methodologies such as object-oriented programming, multi-criteria decision theory, stochastic optimization (including genetic algorithms) and reliability integral approximations [Beck et al., 1996].

In order to be able to trade off lifetime reliability of performance and construction cost for a design, the uncertainties in the structural response due to the uncertainties in the loads exciting the structure must be considered. These uncertainties, particularly for seismic loads, can be very influential factors in the design decisions. In addition, the uncertainties arising from modeling assumptions and simplifications may significantly influence the calculation of the seismic risk. A methodology that incorporates both load and modeling uncertainties in the optimal design and analysis framework is presented here and is illustrated with an application to a simple frame structure. For demonstration purposes, the four performance levels suggested in SEAOC's Vision 2000 [SEAOC, 1995] are used.

OPTIMAL DESIGN METHODOLOGY

The design decision-making process is an iterative procedure where a preliminary design is cycled through stages of analysis, evaluation and revision to achieve a design which is optimum in some chosen sense. In the

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proposed methodology, a formal treatment of these three design stages is made so that the decision-making process can be implemented in software to aid the designer in selecting an optimal design. The methodology handles the key aspects of decision making in a design process in a consistent and rational way. Also, uncertainties related with the structural design are incorporated into the design process in a quantitative and explicit manner.

An important ingredient in the proposed optimal design methodology is the introduction of preference functions $\mu_i(q_i)$ to implement the design criteria in a 'soft' form during the evaluation process, where q_i is a performance parameter such as construction cost, peak lifetime interstory drift ratio or lifetime earthquake losses. For the i -th design criterion, the preference of a particular design $\underline{\theta}$ is evaluated through a measure $\mu_i(q_i(\underline{\theta}))$ of the performance parameter $q_i(\underline{\theta})$, where values of μ_i range from 0 to 1. A larger value of μ_i implies that the designer prefers the corresponding design more, as judged by the i -th design criterion. A preference function can also be viewed as giving a measure of the degree of satisfaction of each design criterion based on the calculated performance parameter values for a given design. For example, the extreme values $\mu_i(q_i(\underline{\theta})) = 0$ or $\mu_i(q_i(\underline{\theta})) = 1$ imply that the current design given by $\underline{\theta}$ is totally unacceptable or perfectly satisfactory, respectively. As an illustration, a preference function for the interstory drift ratio risk is given in Figure 1(d) where it is implied that an interstory drift ratio risk lower than F_l is perfectly acceptable while interstory drift ratio risk higher than F_u is completely unacceptable. Intermediate values express the degree to which the user feels the design gives 'acceptable performance'.

The overall evaluation measure of the design specified by $\mu(\underline{\theta})$ is built up from the individual measures $\mu_i(q_i(\underline{\theta}))$, $i = 1,\dots,N_c$, of the N_c design criteria through a preference aggregation rule which must satisfy certain axioms of consistency [Beck et al., 1999]. A preference aggregation rule is simply a functional relationship between the overall design evaluation measure μ and the individual preference values μ_i for all of the design criteria. Some of the preference functions μ_i may correspond to design parameter constraints in a 'soft' form and, therefore, these μ_i s will depend directly on the design parameter values. Soft forms of the design criteria using preference functions allow the design criteria to be traded off against each other when determining the optimal design. This is a very important aspect for properly obtaining multi-criteria optimal design where conflicting criteria exist.

The selected preference aggregation rule is the multiplicative trade-off strategy given by:

$$\mu(\underline{\theta}) = [\mu_1(\underline{\theta})]^{m_1}[\mu_2(\underline{\theta})]^{m_2}\cdots[\mu_{N_c}(\underline{\theta})]^{m_{N_c}} \quad (1)$$

where $m_i = w_i / \sum_{j=1}^{N_c} w_j$, $i = 1,\dots,N_c$, and w_i is a positive importance weight assigned to the i -th design criterion which can be used to control the trade-off process. The user can give more influence to some design criteria than others by assigning larger values to their importance weights. The choice of the values for these weights is subjective, but the user can gain experience in their selection in any design problem by investigating the influence that different values for the weights have on the final optimal design and on the corresponding preference values for each design criterion. The importance weights w_i can be also viewed from another perspective: since there is no natural scale for preferences over all the diverse criteria, there is a need to be able to independently control their influence during the trade-off which occurs in the optimization process. If the w_i 's are all equal, the trade-off is governed by the inherent sensitivity of each μ_i with respect to $\underline{\theta}$.

To address the lifetime economic effects of seismic risk, economic factors such as the construction cost and the 'lifetime risk' of the structure may be included in the preference functions. For example, the lifetime risk may be quantified through the peak interstory drift since this is highly correlated with the costs of structural and nonstructural repair after an earthquake. It may also be addressed directly by the expected lifetime earthquake losses [Ang et al., 1996], which can be better communicated to the building owner.

To obtain the optimal design for given importance weights, the preference $\mu(\underline{\theta})$ is maximized numerically with respect to $\underline{\theta}$. For design parameters defined on a continuous space, the adaptive random search method has been used in our work [Masri, 1980]. It should be noted, however, that many design parameters are defined on a discrete space. For example, steel I-beam sections are available commercially in only a discrete set of sizes. For this case, a genetic algorithm may be used for the optimization of $\mu(\underline{\theta})$ [Chan, 1997].

TREATMENT OF UNCERTAINTIES

The structural response quantities of interest will be uncertain because of modeling errors and the unknown loads that the completed structure will experience over its lifetime. The performance of the structure is usually judged in this case by the lifetime system reliability, or equivalently, the failure probability, that is, the probability of unacceptable performance over the lifetime of the structure.

The first step in developing an expression for the failure probability given an earthquake event, designated by $P(F|\underline{\theta})$ for a design corresponding to $\underline{\theta}$, is to characterize the seismic hazard at the construction site by a set of ground motion parameters $\underline{\alpha}$ (for example, peak ground acceleration, response spectrum ordinates, duration of motion, frequency content, etc.). For most probabilistic hazard models in use, these parameters depend, through appropriate 'attenuation' relationships, on a set of uncertain 'seismicity' variables, designated by a vector $\underline{\phi}$, accounting for the uncertain regional seismic environment. For example, $\underline{\phi}$ may include variables such as earthquake magnitude, fault dimensions, source parameters, epicentral distance, propagation path properties and local site conditions. The uncertain values of $\underline{\phi}$ are described by a probability density function $p(\underline{\phi})$. For example, $p(\underline{\phi})$ might be chosen to model the probability of occurrence of an earthquake of a given magnitude and the probability of fault rupture at a specific location on a fault.

The required attenuation relationships are often derived by an empirical fit to the observed data. There is uncertainty associated with these attenuation models, even when $\underline{\phi}$ is known, which is reflected by the scatter of the analyzed data about the mean or median model predictions. Therefore, the attenuation relationship should give a probabilistic description $p(\underline{\alpha}|\underline{\phi})$ of the ground motion parameters $\underline{\alpha}$ given the seismicity parameters $\underline{\phi}$.

Knowing the ground motion parameters $\underline{\alpha}$ for a site does not completely specify the structural excitation. Furthermore, the structural model parameters $\underline{\psi}$ corresponding to a particular design $\underline{\theta}$ are not known accurately. To establish $P(F|\underline{\theta})$ from the different sources of uncertainties, $P(F|\underline{\theta},\underline{\psi},\underline{\alpha})$, the conditional probability of failure for the given design $\underline{\theta}$, structural model $\underline{\psi}$ and ground motion parameters $\underline{\alpha}$, may first be set up. This can be obtained by a fragility analysis. The uncertainties in the seismic environment and ground motion modeling are then combined using the total probability theorem to determine the failure probability given that an earthquake has occurred and given a structural model:

$$P(F|\underline{\theta},\underline{\psi}) = \int P(F|\underline{\theta},\underline{\psi},\underline{\alpha})\,p(\underline{\alpha}|\underline{\phi})\,p(\underline{\phi})\;d\underline{\alpha}\;d\underline{\phi} \qquad (2)$$

Finally, the effects of uncertainty in the structural model parameters are included to yield the probability of failure given that an earthquake has occurred:

$$P(F|\underline{\theta}) = \int P(F|\underline{\theta},\underline{\psi})\,p(\underline{\psi}|\underline{\theta})\,d\underline{\psi} \qquad (3)$$

where $p(\underline{\psi}|\underline{\theta})$ is the conditional probability density for the structural parameters $\underline{\psi}$ given the design parameters $\underline{\theta}$ which is defined by the chosen probability model.

The failure probability over the lifetime of the structure is computed using an occurrence model for earthquake events. Assuming that the occurrences of earthquake events follow a Poisson arrival process, the probability that a performance requirement is not satisfied during the lifetime t years of the structure, is given by:

$$P(F_{life}|\underline{\theta}) = 1 - \exp[-\nu\;P(F|\underline{\theta})\;t] \qquad (4)$$

where ν is the expected number of earthquake events per annum in the region around the site.

Reliability Computations

It is reasonable to numerically evaluate the multi-dimensional integral in (2) or (3) only if the dimension is low. Otherwise, efficient asymptotic methods [Breitung, 1989; Papadimitriou et al., 1997] or importance sampling simulation methods [Schueller and Stix, 1987] can be used.

Consider first the case for which the conditional failure probability $P(F|\underline{\theta},\underline{\psi},\underline{\alpha})$ is a smooth function of $\underline{\psi}$ and $\underline{\alpha}$. An asymptotic approximation for the reliability integrals can be obtained which uses the value and Hessian matrix of the logarithm of the integrand at the 'design points' where the integrand is maximized [Papadimitroiu et al., 1997; Au et al., 1999]. The optimization method developed by Yang and Beck (1998) can be used to search and locate the maxima. When $\underline{\psi}$ and $\underline{\alpha}$ completely determine whether the structure has failed or not, $P(F|\underline{\theta},\underline{\psi},\underline{\alpha})$ assumes a value of either 0 or 1. In this case, the reliability integral can be evaluated approximately using available second order reliability methods [e.g., Der Kiureghian et al., 1987; Breitung, 1989; Polidori et al., 1999].

Importance sampling methods may also be used to obtain an estimate for the failure probability. These methods also serve as a check against the accuracy of the asymptotic estimates. These methods are generally applicable for both cases of $P(F|\underline{\theta},\underline{\psi},\underline{\alpha})$ discussed above. The efficiency of these methods depend critically on the choice of the sampling density. When information about the design point(s) is available and the integrand is characterized by a multi-modal function, the sampling density may be chosen as a mixture distribution centered among the design point(s) [Au et al., 1999]. It should be noted, however, that there are many cases where the search for design point(s) is difficult or the problem is not characterized by design point(s). In such cases, an efficient adaptive importance sampling method developed by Au and Beck (1999) may be used. In this method, a Markov chain is generated whose states are asymptotically distributed according to the optimal importance sampling density and, therefore, samples are simulated that populate the important region of the integrand. The sampling density is then constructed as a mixture distribution among the Markov chain samples. The method is efficient, robust and applicable for a variety of topologies of the integrand.

ILLUSTRATIVE EXAMPLE

Structural Model and Design Criteria

The methodology is demonstrated by applying it to the design of a three-story, single-bay frame [Beck et al., 1999]. The frame members are taken as steel I-beams with the length of the floor beams fixed at 6.1 m and the height of the story columns fixed at 3.05 m. The design parameters $\underline{\theta}$ are the member flange width B and web depth D in cm for the beams and columns, that is, $\underline{\theta} = (B_{beam}, D_{beam}, B_{col}, D_{col})$. The optimization is made over a continuous space of $\underline{\theta}$. The possible range of values for design parameters are shown in Figures 1(a), (b). The flange and web plate thicknesses are held fixed at 0.625 cm. Gravity loads are taken as 2.87×10^{-3} MPa and 2.39×10^{-3} MPa for the dead and live loads, respectively, for each floor and the roof. An out-of-plane tributary width of 2.54 m is used for the gravity load calculations.

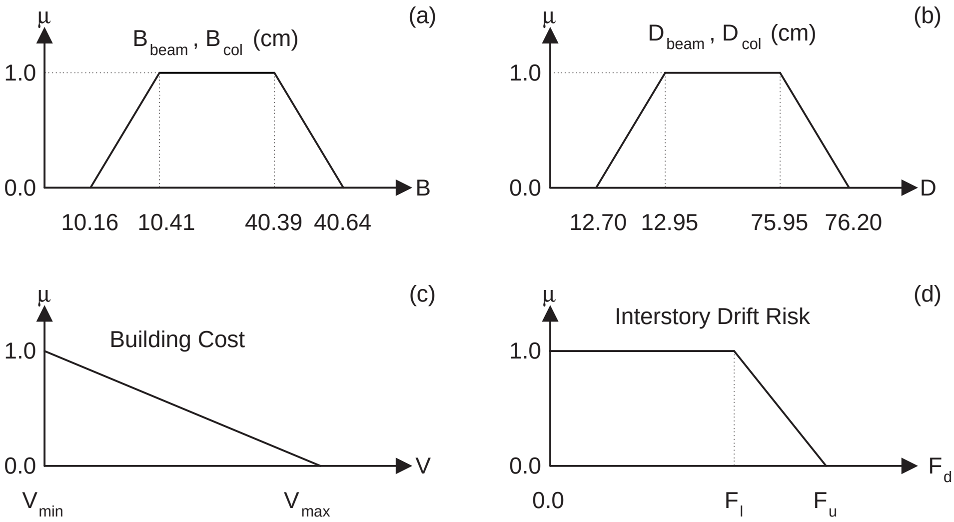

Figure 1: Design criteria and corresponding preference functions

The objective is to determine $\underline{\theta}$ so that the frame design is optimized according to design criteria involving the following performance parameters: flange width, web depth, building cost and probability of unacceptable peak lifetime interstory drift (drift risk). The corresponding preference functions are shown in Figure 1. The same importance weight is assigned to each design criterion in the aggregation of preference values in (1).

For this example, the building cost C is expressed simply as the sum of a construction (or fabrication) cost C_{con} and a material cost, that is, $C(\underline{\theta}) = C_{con} + c_s\, V(\underline{\theta})$, where c_s is the material cost per unit steel volume and V is the volume of steel used in the design. The variation in the construction costs for structural members of different sizes is assumed negligibly small, so that C_{con} is essentially independent of $\underline{\theta}$. The preference function can then be expressed in terms of a normalized performance parameter:

$$q_{\mathrm{cost}} = (C - C_{\min}) \,/\, (C_{\max} - C_{\min}) = (V - V_{\max}) \,/\, (V_{\max} - V_{\min}) \tag{5}$$

where $V_{\max} = 362{,}810\ \mathrm{cm}^3$ and $V_{\max} = 73{,}742\ \mathrm{cm}^3$ are the steel volumes corresponding to the maximum and minimum allowable member section sizes prescribed by the geometric constraints. The preference function for the building cost can therefore be expressed in terms of the steel volume $V(\underline{\theta})$ for a design given by $\underline{\theta}$. As shown in Figure 1(c), a linearly decreasing function is used to specify the preference values for the building cost in terms of the steel volume, with $\mu = 1$ at the minimum allowable volume and $\mu = 0$ at the maximum allowable volume. In the tables of results presented later, the building cost is reported as the volume of steel, V.

Only one mode of 'failure' is considered here. 'Failure' occurs if the peak interstory drift ratio risk F_d exceeds a specified level over the lifetime of the structure. Failure modes corresponding to strength requirements can also be considered, although it has been found that they do not control the design of the present structure. In the numerical results, four pairs of peak interstory drift ratios and associated acceptable risks over the lifetime of the structure (taken to be 50 years) are considered which correspond to the performance levels suggested in Vision 2000 [SEAOC, 1995]: (0.2%, 68.5%), (0.5%, 50%), (1.5%, 10%) and (2.5%, 5.13%), for fully operational, operational, life-safe and near collapse states, respectively.

Probabilistic Seismic Hazard Model

In the probabilistic seismic hazard model considered, ground motion is characterized by the pseudo-velocity response spectrum $S_v(T,\zeta)$ where T is the period and ζ is the damping ratio of a single degree-of-freedom linear oscillator. The attenuation formula proposed by Boore et al. (1993, 1994) is used to model $S_v(T,\zeta)$ in terms of earthquake magnitude and epicentral distance:

$$\log_{10}[S_v(T,\zeta)] = \log_{10}[\hat{S}_v(T,\zeta)] + \varepsilon(T,\zeta) \tag{6}$$

where

$$\log_{10}[\hat{S}_v(T,\zeta)] = \hat{b}_1 + \hat{b}_2(M-6) + \hat{b}_3(M-6)^2 + \hat{b}_4 r + \hat{b}_5 \log_{10} r + \hat{b}_6 G_b + \hat{b}_7 G_c \tag{7}$$

Here $r = \sqrt{R^2 + h^2}$, where R is the epicentral distance and h is a fictitious event depth determined by the regression analysis; G_b and G_c are soil type parameters which take a value 0 or 1 depending on the soil classification at the site. The best estimates of the parameters $\hat{b}_i \equiv \hat{b}_i(T,\zeta)$ appearing in the model for $\hat{S}_v(T,\zeta)$ have been determined by Boore et al. (1993, 1994) by regression analysis of a large database of accelerograms. The function $\varepsilon(T,\zeta)$ in (6) represents the uncertain model error in the actual spectral amplitudes $S_v(T,\zeta)$ compared with the estimated amplitudes $\hat{S}_v(T,\zeta)$ from the model. It is assumed to have a Gaussian distribution over the range of periods analyzed, with zero mean and variance given in Boore et al. (1994).

Only the epicentral distance R and the earthquake magnitude M are considered to be uncertain in this study. The probability distributions for these seismicity parameters are derived by assuming a simple seismicity model as follows. The earthquake sources are point sources located in a circular area with a radius of $R_{\max}$ centered at the site where the building is located. It is assumed that an earthquake is equally likely to occur at any point inside this circular source region, so the probability $p(R)dR$ is simply the ratio of the area of a strip of width dR located R distance away from the center to the area of the circle with radius $R_{\max}$, yielding the probability density function $p(R) = 2R / R_{\max}^2$. The probability density function $p(M)$ for the earthquake magnitude based on a truncated Gutenberg-Richter relationship is

$$p(M) = b' e^{-b'M} / (e^{-b'M_{\min}} - e^{-b'M_{\max}}) \tag{8}$$

where $M_{\min}$ and $M_{\max}$ are the lower and upper regional bounds for the earthquake magnitude, and $b' = b \log_e 10$. The expected number of events per annum falling into the magnitude range considered is $\nu = 10^{a-bM_{\min}} - 10^{a-bM_{\max}}$. The following data are used for the parameters of the seismicity model: $R_{\max} = 50\,\text{km}$, $M_{\min} = 5.0$, $M_{\max} = 7.7$, $b = 1.0$, $a = 5.0$. These parameters give $\nu = 1.0$ for the seismicity rate per annum. The epicentral distance and the earthquake magnitude are assumed to be independent.

Reliability Computations

The uncertain parameter set $\underline{\phi}$ for the ground motion model describing $S_v(T,\zeta)$ consists of the magnitude M and the epicentral distance R , so the probability density function corresponding to $p(\underline{\alpha}|\underline{\phi})$ in the general theory described earlier is $p(S_v|M,R)$, where it has been assumed that only the fundamental mode contributes significantly to the displacement response. (Design examples with multi-mode analysis can be found in Beck et al., 1999). The fundamental modal damping ratio is assumed to be $\zeta_1 = 5\%$. The interstory drift of the i -th floor, d_i , is related to S_v by $d_i = \beta_{i1} S_v(T_1, \zeta_1)$ where β_{i1} is the effective fundamental modal participation factor for the i -th floor. Assuming that the occurrences of earthquake events follow a Poisson arrival process, the drift risk $P(F_{life}|\underline{\theta})$ over the lifetime t of the structure is computed from (4) where

$$P(F|\underline{\theta}) = P(\bigcup_{i=1}^{n}\{d_i > d_{allow}\}|\underline{\theta}) = \int P(\bigcup_{i=1}^{n}\{d_i > d_{allow}\}|\underline{\theta}, S_v) p(S_v|M,R) p(M,R)\; dS_v\; dM\; dR \tag{9}$$

is the failure probability given the occurrence of an event. Uncertainties in the structural model parameters are assumed negligible. The resulting integral is in the form of the classical system reliability problem for components connected in series: if any of the components fails, that is, if any $d_i(S_v, \underline{\theta}) > d_{allow}$, the system is considered failed. It is evaluated using the asymptotic method described in Polidori et al. (1999).

Numerical Results

As mentioned earlier, for the multiple performance levels, the following interstory drift ratio and corresponding failure probability (in other words, exceedance probability or risk) over the lifetime $t = 50$ yr, are used: 0.2% with 68.5% risk, 0.5% with 50% risk, 1.5% with 10% risk, and 2.5% with 5.13% risk. These risk levels correspond to the F_l in Figure 1(d) (while F_u is taken as $1.01F_l$). It should be noted that in this example all reliability calculations are based on linear dynamics since a response spectrum approach is used, but for higher performance levels (life-safe and near collapse), nonlinear behavior would be expected. Nonlinear structural models can be treated within the framework presented here by simulation of the drift using a suite of accelerograms scaled to a specified value of S_v at the fundamental elastic period. A log-normal distribution is then fitted to the simulated drift values to compute $P(d_{\max} > d_{allow}|\underline{\theta}, S_v)$ [e.g., Shome et al., 1998].

Table 1: Optimal design for $\underline{\theta} = (B_{beam}, D_{beam}, B_{col}, D_{col})$

	Freq./Fully Oper.		Occas./Oper.		Rare/Life-Safe		V. Rare/Near Coll.	
Criteria	Value	μ_i	Value	μ_i	Value	μ_i	Value	μ_i
B_{beam} (cm)	10.41	1.00	10.41	1.00	10.41	1.00	10.41	1.00
D_{beam} (cm)	73.66	1.00	52.10	1.00	38.80	1.00	31.67	1.00
B_{col} (cm)	10.41	1.00	10.41	1.00	10.41	1.00	10.41	1.00
D_{col} (cm)	70.39	1.00	38.65	1.00	28.43	1.00	23.40	1.00
Vol (cm^3)	212,714	0.519	150,806	0.733	123,503	0.828	109,369	0.877
F_d	0.685	1.00	0.500	1.00	0.100	1.00	0.051	1.00
Overall μ		0.897		0.950		0.969		0.978
Period	0.304s		0.537s		0.770s		0.959s	

The results are presented in Table 1. To be able to see the effect of each performance level, one drift performance level at a time is considered. The results imply that if all Vision 2000 levels were simultaneously considered, the optimal design of the structure would be governed by the 0.2% interstory drift ratio with a corresponding 68.5% risk over the lifetime. This case is equivalent to the requirement of being 'fully operational after frequent events' for 'basic facilities' [SEAOC, 1995]. Notice that there is twice as much steel used in the optimal design for the fully-operational criterion than for the near-collapse criterion. If this occurred during an actual structural design, the design engineer would no doubt have to justify the considerable increase in costs to the owner. One possible approach is to demonstrate that the decrease in the expected lifetime earthquake losses outweighs the increase in building costs when the fully-operational criterion controls the design.

In Table 1, the optimal flange widths for beams and columns B_{beam} and B_{col} are always 10.41cm, which is the smallest size that still gives $\mu = 1.0$ in the preference function shown in Figure 1(a). This occurs because it is more cost-effective to provide the necessary bending stiffness by increasing the web depths D_{beam} and D_{col} rather than the flange widths B_{beam} and B_{col}. However, if the flange widths are reduced below 10.41cm, the reduction in the preference in Figure 1(a) outweighs the improvement in the cost preference in Figure 1(c).

CONCLUSIONS

The proposed optimal design methodology provides a rational basis for incorporating seismic load and modeling uncertainties in the design process and to make reliability-based optimal design decisions that meet specified multiple performance criteria. Code-based requirements are also easily incorporated into the process [Beck et al., 1999]. This new framework is well-suited for performance-based design of structures under uncertainty. Although the optimal design framework has been demonstrated for a special class of ground motion and structural models, it is very flexible and more sophisticated models can easily be treated. For example, advances in seismic hazard models can easily be incorporated into the framework. Also, an inelastic finite element program can be included along with reliability approximations for nonlinear systems to more realistically treat large deformations. The general framework presented here for multi-criteria optimal design under risk is potentially applicable to a wide range of engineering systems, including buildings, bridges, offshore structures, equipment and piping systems.

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