0340

ADAPTIVE RESPONSE CONTROL AND ASSOCIATE SYSTEM MONITORING ON INTERACTIVE SOIL-STRUCTURE CONSTRUCTIONS DURING SEISMIC EXITATIONS

Kensuke BABA[1], Naoto YABUSHITA[2], Nobuhiro KASUMOTO[3], Ryo HASEGAWA[4] And Yutaka INOUE[5]

SUMMARY

The model reference adaptive manipulation is applied to the response control systems as well as to the monitoring ones of system compositions, through the Lyapunov procedure of asymptotic stability. The manipulated plants are set on the soil-structure interacting constructions under earthquake excitations, which are accompanied with the time delay behavior because of the seismic waves radiating for a far ground. According to the numerical analysis, the control systems are well worked inside the interactive situations, while the monitoring ones are likely done for the exciting spectra rich enough.

INTRODUCTION

It is a hard task to mitigate the seismic responses of structural constructions built on a soil ground, because of the unpredictable excitations and the complicate soil-structure interactions.

The present study is concerned with establishing the active control systems to reduce the response processes for violent earthquakes, and in addition, ensuring their associate monitoring systems to identify the dynamic characteristics of the interactive compositions. The practical manipulation is required to contain the time variable gains extracted from the feedforward circulations as well as the feedback ones, in consideration of the temporary inputs and the transient outputs. It is moreover necessary for the active working to be settled in the dynamic stability approved over the total systems including the subject plants and the control means.

According to the Lyapunov theorem of asymptotic stability, the model reference adaptive algorithm is applicable to such manipulation problems as bring the interactive phenomena in company with the time delay manner. The virtual responses of the reference models are straightly corresponding to the objective states targetted by the actual responses of the supervised plants. The reduction of the state differences is merely achieved after the adaptive arrangement between the reference models and the subject plants. The elimination of the system tolerances is not always accomplished, which is unnecessary for the control systems giving concentrative attention to the response processes, but essential for the monitoring ones observing the transit processes of the composite formation. It is available to diminish the system tolerances simultaneously with the state differences, when the spectral properties are sufficiently plentiful in the exciting contents.

[1] *Architectural Engineering, Osaka University, Osaka, Japan. E-mail kbaba@arch.eng.osaka-u.ac.jp*
[2] *Takenaka Corporation, Osaka, Japan. E-mail yabushita.naoto@takenaka.co.jp*
[3] *Shimizu Corporation, Tokyo, Japan. E-mail kasumoto@sek.shimz.co.jp*
[4] *Architectural Engineering, Osaka University, Osaka, Japan.*
[5] *Architectural Engineering, Osaka University, Osaka, Japan.*

2. SOIL-STRUCTURE INTERACTION SYSTEMS

The response processes are represented for the upper-structures with base foundations in the following equations of motion, which are evaluated separately from those of the surrounding soil ground,

$$\sum_{m=0}^{2} L_m x^{(m)}(t) = g(t) + W w(t) + U u(t) \tag{2-1}$$

The motion vector $x(t)$ is measured relative to the seismic disturbances $w(t)$. The manipulation forces $u(t)$ are worked through the feedback and feedforward circulations. In addition, the successive forces $g(t)$ are developed on the base foundations as the reaction efforts against the separated ground motions. The interactive forces are remarked by the convolution integrals on the foundation-ground interface, inclusive of the motion vector and the impedance matrix $G(t)$ in the time domain,

$$g(t) = -G(t) * x(t) \tag{2-2}$$

The impedance matrix is usually obtained as the frequency function on the complex plane and often provided in the numerical manner. To simulate the numeral data on the analytical construction, the following series expansion is prepared with the real coefficients G_{mn} on the base of the physical causality,

$$\tilde{G}(\omega) = \sum_{m=0}^{M} \sum_{n=0}^{N} G_{mn} (i\omega)^m \exp(-i\omega t_n) \ ,$$

$$G(t) = \sum_{m=0}^{M} \sum_{n=0}^{N} G_{mn} \delta^{(m)}(t - t_n) \qquad : t_n = n\pi/\omega_L$$

in which the sampling delay times t_n are determined by the cutoff frequency ω_L to avoid the distorted phenomena of aliasing. The frequency function is transformed into the chain of pulses along the time axis, which is expressed by the Dirac's delta functions and their modified ones with the differenctial and behindhand operations. The delaying manner inside the impedances is physically related to the wave radiation for the far infinity of the soil ground. The equations of motion are rearranged with the state descriptions to have the advantage over formulating the manipulative role,

$$z_P^{(1)}(t) = A_P z_P(t) + f_P(t) + D_P w(t) + B u(t) \tag{2-3}$$

The elements of the controlled plants are explained into the detail of the state vector $z_P(t)$, the system matrix A_P, the disturbing one D_P, the driving one B and the successive vector $f_P(t)$ containing the delay times as follows,

$$z_P(t) = \begin{bmatrix} x^{(1)}(t) \\ x(t) \end{bmatrix} , \quad A_P = \begin{bmatrix} -A_2^{-1} A_1 & -A_2^{-1} A_0 \\ I & 0 \end{bmatrix} ,$$

$$D_P(t) = \begin{bmatrix} A_2^{-1} W \\ 0 \end{bmatrix} , \quad B(t) = \begin{bmatrix} A_2^{-1} U \\ 0 \end{bmatrix} , \qquad : m = 0, 1, 2$$

$$A_m = L_m + G_{m0} \quad , \quad f_P(t) = -\sum_{m=0}^{1} \sum_{n=1}^{N} F_{mn} z_P^{(m)}(t - t_n) \qquad : M = 2$$

in which the upper bound M=2 among the series expansion is associate with the number of differential times inside the equations of motion.

3. REFERENCE MODELS

To confirm the ideal states preferred by the controlled plants, the reference models are settled asymptotically stable in the next state equation,

$$z_M^{(1)}(t) = A_M z_M(t) + D_M E w(t) \tag{3-1}$$

which are disturbed by the same excitations $w(t)$ as the ones attacking on the subject plants. The exciting amplitudes are modified to insert the constant matrix **E** between the input arrangements. The system matrix $\mathbf{A}_M$ is made of the damping one $\mathbf{R}_1$ to be symmetric and positive definite as well as done of the stiffness one $\mathbf{R}_0$ and the inertia one $\mathbf{R}_2$ to keep the asymptotic stability inside the reference models.

$$\mathbf{A}_M = \begin{bmatrix} -\mathbf{R}_2^{-1}\mathbf{R}_1 & -\mathbf{R}_2^{-1}\mathbf{R}_0 \\ \mathbf{I} & \mathbf{0} \end{bmatrix} ,$$

$$\mathbf{R}_0 = \mathbf{R}_0^T > 0 \quad , \quad \mathbf{R}_1 = \mathbf{R}_1^T > 0 \quad , \quad \mathbf{R}_2 = \mathbf{R}_2^T > 0$$

The dynamic characteristics of asymptotic stability are also widespread over the whole of the manipulation systems, while the symmetric and positive definite matrix **P** is requested extractive from the next Lyapunov equation,

$$\mathbf{A}_m^T \mathbf{P} + \mathbf{P}\mathbf{A}_m = -\mathbf{Q} \qquad : \mathbf{Q} = \mathbf{Q}^T > 0 \tag{3-2}$$

which stands on the reference matrix $\mathbf{A}_M$ of asymptotic stability and the inhomogeneous term **Q** of symmetry and positive definiteness. The unique matrix **P** is generally given in the numerical manner, however, it is possible to obtain the solution in the analytical style when the reference systems have the proportional damping, in which the closed form of the matrix **P** is useful for making the efficient rule on the adaptive operations. The damping matrix $\mathbf{R}_1$ is, accordingly, composed of the following series expansion with the stiffness $\mathbf{R}_0$ and the inertia $\mathbf{R}_2$ in the reference systems having N degrees of freedom,

$$\mathbf{R}_1 = \mathbf{R}_0 \sum_{n=0}^{L-1} a_n (\mathbf{R}_0^{-1}\mathbf{R}_2)^n \qquad : N \geq L \geq 1 \tag{3-3}$$

The scalar expansion coefficients a_n are determined by the next simultaneous equation, in company with the natural frequencies ω_n of the undamped systems and the associate modal properties of the equivalent damping ratios h_n,

$$\sum_{m=1}^{L} \omega_n^{1-2m} a_m = 2 h_n \qquad : L \geq m, n \geq 1$$

With the enforcing term **Q** composed of only the stiffness matrices $\mathbf{R}_0$ along its diagonal, the analytical solution **P** is obtained in the following compact form,

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{12}^T & \mathbf{P}_{22} \end{bmatrix} : \quad \mathbf{Q} = 2\begin{bmatrix} q_0\mathbf{R}_0 & \\ & q_2\mathbf{R}_0 \end{bmatrix} \quad , \; q_0 > 0 \;,\; q_2 > 0 \tag{3-4}$$

$$\mathbf{P}_{11} = (q_0\mathbf{R}_0 + q_2\mathbf{R}_2) \sum_{n=0}^{L-1} \frac{1}{a_n} (\mathbf{R}_2^{-1}\mathbf{R}_0)^{n-1} \quad ,$$

$$\mathbf{P}_{12} = q_2 \mathbf{R}_2 \quad ,$$

$$\mathbf{P}_{22} = \mathbf{P}_{11}\mathbf{R}_2^{-1}\mathbf{R}_0 + q_2\mathbf{R}_2 \sum_{k=0}^{L-1}\sum_{m=0}^{L-1}\sum_{n=0}^{L-1} \frac{a_k a_m}{a_n} (\mathbf{R}_2^{-1}\mathbf{R}_0)^{n-m-k+1}$$

The diagonal element $\mathbf{P}_{22}$ is shown a little difficult with three times of summation, which is dropped out of the controlling algorithm, whereas necessary for the monitoring one.

4. ADAPTIVE CONTROL ALGORITHM

The plant systems are excited not only by the natural disturbances $\mathbf{w}(t)$ but also by the artificial forces $\mathbf{u}(t)$ generated around the feedback and feedforward loops,

$$\mathbf{u}(t) = \mathbf{K}(t)\eta(t) \tag{4-1}$$

in which the components of the time variable gain $\mathbf{K}(t)$ are associate with those of the extended state vector $\eta(t)$, namely, the response state $\mathbf{z}_P(t)$, its modified state behindhand or differential and moreover the exciting disturbances $\mathbf{w}(t)$, where any component is thrown out of the state vector.

$$K(t)=[K_B(t)\ \ K_{11}(t)\ \ K_{10}(t)\cdots K_{N1}(t)\ \ K_{N0}(t)\ \ K_F(t)]\ ,$$

$$\eta^T(t)=[z_P^T(t)\ \ z_P^{(1)T}(t-t_1)\ \ z_P^T(t-t_1)\cdots z_P^{(1)T}(t-t_N)\ \ z_P^T(t-t_N)\ \ w^T(t)]$$

The manipulation forces are worked to reduce the state differences $e(t)$ between the reference systems and the plant ones, so that the state discrepancies correspond to the next tolerance equation,

$$e^{(1)}(t)=A_M e(t)+\Phi(t)\eta(t)\ , \tag{4-2}$$

$$e(t)=z_M(t)-z_P(t)$$

The system difference matrix $\Phi(t)$ is left to include the variable gains, which is divided into the part $\Phi_1(t)$ in relation with the integral operation to keep the system stability and the other $\Phi_2(t)$ with the proportional working to improve the system adaptability.

$$\Phi(t)=\Phi_1(t)+\Phi_2(t)$$

$$=[A_M-A_P-BK_B(t)\cdots F_{mn}-BK_{mn}(t)\cdots D_M E-D_P-BK_P(t)]$$

Both the parts of the system difference are associate with the variable matrix $\Psi(t)$ conformed to the non-linear structures of the state tolerances and the extended states,

$$\Phi_1^{(1)}(t)=\Psi(t)\Gamma_1 \qquad : \Gamma_1=\Gamma_1^T>0$$

$$\Phi_2(t)=\Psi(t)\Gamma_2 \qquad : \Gamma_2=\Gamma_2\geq 0\ , \tag{4-3}$$

$$\Psi(t)=-Pe(t)\eta^T(t) \qquad : P=P^T>0$$

The symmetric supplement matrix Γ_1 is positive definite, another supplement Γ_2 is relaxed positive semidefinite and the other supplement P is picked out of the Lyapunov equation (3-2) mentioned ahead. The scalar candidate $V(t)$ is prepared for the Lyapunov function in the quadratic form of the state differences and the system tolerances, which is designed to secure the whole of the adaptive systems against the unstable situation,

$$V(t)=e^T(t)Pe(t)+tr[\Phi_1(t)\Gamma_1^{-1}\Phi_1^T(t)] \tag{4-4}$$

$$V^{(1)}(t)=-e^T(t)Qe(t)-2tr[\Psi(t)\Gamma_2\Psi^T(t)]$$

The time derivative of the candidate is stated in the negative value of the quadratic form, under the conditions of constructing the system tolerances in Eq.(4-3) and at once holding the existence of the Lyapunov equation (3-2). In other words, the candidate is led to the objective function. The adaptive rule is established to clarify the gain matrix $K(t)$ with the derivative of the system tolerances bringing the variable matrix $\Psi(t)$,

$$\Phi^{(1)}(t)=-BK^{(1)}(t)=\Psi(t)\Gamma_1+\Psi^{(1)}(t)\Gamma_2 \tag{4-5}$$

in which the integral working supplemented by the matrix Γ_1 is satisfied with the minimum requirement of the asymptotic stability and the proportional one by the matrix Γ_2 is reserved for improving the manipulative operations.

5. ADAPTIVE MONITORING ALGORITHM

It is sometimes happened that the system monitoring examination is necessary to identify the subject plants, for instance, when the physical properties are uncertain or forced halfway into the change of the system characteristics. It is situated here the coefficient matrices A_P and D_P are unknown in the next state equation of the interactive constructions, and indeed the expanded factors are done in the successive term $f_P(t)$,

$$z_P^{(1)}(t)=A_P z_P(t)+f_P(t)+D_P w(t) \tag{5-1}$$

The monitoring systems are, on the other side, set to have the known coefficients A_M asymptotically stable and to go through the manipulation forces $u_M(t)$ associate with the variable gain matrix $K_M(t)$ and the extended state vector $\eta(t)$ in the on-line manner,

$$z_M^{(1)}(t)=A_M z_M(t)+u_M(t)\ , \tag{5-2}$$

$$u_M(t)=K_M(t)\eta(t)\ ,$$

$$K_M(t)=[K_B(t)-A_M\ \ K_{11}(t)\ \ K_{10}(t)\cdots K_{N1}(t)\ \ K_{N0}(t)\ \ K_F(t)]$$

The adaptive monitoring efforts are related to the adaptive control operations by exchanging the function of the subject plants for the one of the reference models. When the state differences $e(t)$ are taken same as done in the control systems, the tolerance equation and the system discrepancies $\Phi_M(t)$ are given similar to those in the previous definition,

$$e^{(1)}(t)=A_M e(t)+\Phi_M(t)\eta(t) ,$$

$$\Phi_M(t)=[K_B(t)-A_P \cdots K_{mn}(t)-F_{mn} \cdots K_P(t)-D_P]$$

The adaptive rule is also described with the derivative of the gain matrix $K_M(t)$ in the next equation, which is identical to Eq.(4-5) but merely reverse to the previous sign after exchanging the plant function and the reference one,

$$K_M^{(1)}(t)=\Psi(t)\Gamma_1+\Psi^{(1)}(t)\Gamma_2 \qquad : \Gamma_1>0 , \Gamma_2\geq 0 \tag{5-3}$$

By the way, it is saved for the present monitoring algorithm that the state differences are asymptotically reduced, therefore the total systems run for standstill portions and the system tolerances $\Phi_M(t)$ do not always come to the vanishing points.

$$\Phi_M(t)\eta(t)\to 0 \qquad : e(t)\to 0$$

For the system differences to disappear and for the gain matrices to approach toward the plant coefficients, the spectral characteristics must be sufficiently rich in the excitations $w(t)$. In accordance, it is said that the exciting conditions are more strictly applied to the monitoring arrangement than to the control one.

6. NUMERICAL STUDIES

The numerical investigation is carried out in the fundamental soil-structure interaction systems, to research the facilities of the adaptive management for the active control or the system monitoring, as shown in Fig.1. The interactive constructions are forced to move laterally and rotationally under the horizontal acceleration $\alpha_G(t)$ of seismic excitations. The upper structures of the height H have the concentrated mass m_1, the shearing stiffness k and the inner damping coefficient c. The base foundations are thin and square of the length 2b, the mass m_0 and the inertia radius r_0 around the lateral axis. The elastic soil ground is tightly contact with the foundations and spread over a half space, which is remarked by the Poisson's ratio ν=0.25, the mass density ρ and the shearing modulus μ or the shearing velocity v_G. The impedance function contains the horizontal component expanded with the coefficients k^H_{mn} on the soil-foundation interface, as well as the rotational component with the coefficients k^R_{mn}. It is available to adopt the upper bound N=1 in the sampling expansions, when the soil ground is disposed homogeneous and extended boundless. The reference models are characterized asymptotically stable with the proportional damping and clearly distinctive with the fundamental frequency ω_M and its associate damping ratio ζ_M. The lateral responses $x_n(t)$ and the rotational one $\phi(t)$ are given on the upper structures (n=1) and on the base foundations (n=0), parallel to the lateral manipulations $u_n(t)$ and the rotating one $u_R(t)$. The physical parameters and the physical components are arranged in the following dimensionless form with the head script (—),

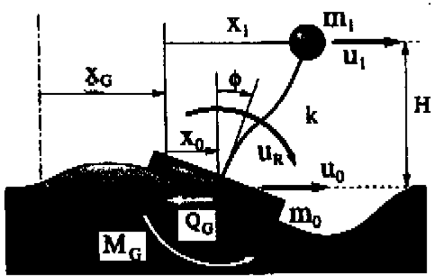


Fig. 1 Soil - structures interaction systems

$$\bar{x}_n(t)=x_n(t)/x_G , \quad \bar{\phi}(t)=\phi(t)H/x_G ,$$

$$\bar{u}_n(t)=u_n(t)/(\omega_G^2 m_G x_G) , \quad \bar{u}_R(t)=u_R(t)/(2b\omega_G^2 m_G x_G) ,$$

$$\bar{k}^H_{mn}=\omega_G^m k^H_{mn}/(\mu b) , \quad \bar{k}^R_{mn}=3\omega_G^m k^R_{mn}/(\mu b^3) ,$$

$$\bar{t}=t\omega_G , \quad \bar{\omega}_L=\omega_L/\omega_G=2 , \quad \bar{\omega}_M=\omega_M/\omega_G ,$$

$$\bar{H}=H/b=1\ ,\ \bar{r}_0=r_0/b=1/\sqrt{3}\ ,\ \zeta=c/(2\sqrt{m_1/k})\ ,$$

$$\bar{m}_n=m_n/m_G=0.4\ ,\ \sqrt{m_1/k}/\omega_G=1\ ,\ \gamma_{mF}/\gamma_{mB}=0.5\ ,$$

$$\omega_G=v_G/b\ ,\ m_G=8\rho b^3\ ,\ \alpha_G(t)_{max}=\omega_G{}^2x_G\ ,$$

$$\Gamma_m=\begin{bmatrix}\gamma_{mB}\mathbf{I} & \\ & \gamma_{mF}\mathbf{I}\end{bmatrix}\ ,\ \mathbf{E}=\varepsilon\mathbf{1} \qquad :\ m=0,1,2\ ,\ n=0,1$$

The interactive compositions may be strictly uncertain, because of the complicate soil materials and moreover the changeable surrounding settlement around the soil-foundation interface. Therefore, the delay arrangement is intentionally dropped out of the gain components $\mathbf{K}_{mn}(t)$, to examine both the controlling facilities and the monitoring ones in the present analysis. Fig.2 covers the response processes under the artificial manipulations as well as the lateral NS components of the 1940 El Centro Earthquake. The controlled response records are stood on a good level in comparison with the uncontrolled ones, which are saved from the irrational affect of missing the behindhand gains owing to the robust organization.

As shown in Fig.3 and on Table 1, the monitoring records are also nearly favorable for the preliminary contents. The rotating terms are, however, left out of the presets in spite that the main part of the earthquake is repeatedly excited. The peculiar appearances of the rotations are related to the physical surroundings that the delay components are superior among the rotating motions in case of the impedance function defined on the square base foundations.

7. CONCLUDING REMARKS

The following remarks are brought out of the present study:

(A) It is principally available that the soil-structure interaction systems are definitely formulated in the time domain, when the frequency impedance function is simulated on the series expansion after the causal conditions and transformed into the analytical form.

(B) The model reference adaptive manipulations are applicable not only to the on-line control systems but to the system monitoring ones, even though set up on the interactive constructions.

(C) The manipulative facilities are graded up when the Lyapunov matrix equation is analytically solved through the reference models having the proportional damping characteristics.

(D) The adaptive control formations are robust in want of the successive gains for the time behind responses. On the other hand, the monitoring formations are not always complete by taking the associate gains free of the delaying manners.

8. REFERENCES

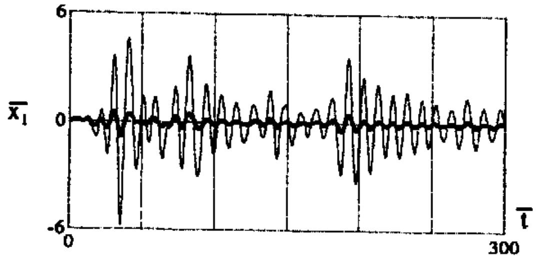
Baba, K., K. Park and Y. Inoue (1994) Dynamical analysis of soil-structure interaction systems under earthquake excitations. *Proc. 9th Japan Earthq. Engrg. Symp.*, Vol.1, pp.1147-1152.

Baba, K., N. Yabushita, N. Kasumoto, H. Murakami and Y. Inoue (1998) Model reference adaptive control on soil-structure interaction systems under earthquake disturbances. *Proc. 2nd World Conf. Struct. Contr.*, Vol.2, pp.1549-1558.

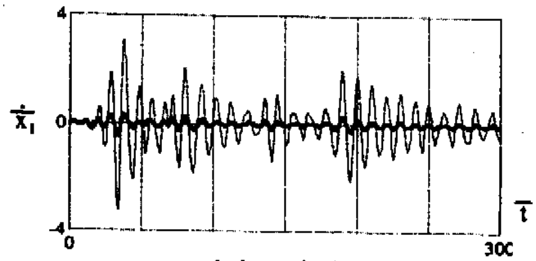
Baba, K., N. Yabushita, H. Murakami and Y. Inoue (1998) Model reference adaptive control on soil-structure interaction systems for earthquake disturbances. *Theor. Appl. Mech.*, Vol.47, pp.273-281.

Ghanem, R. and M. Bujakov (1994) Adaptive control of non-linear dynamical systems with uncertainties. *Proc. 1st World Conf. Struct. Contr.*, Vol.2, pp.TA4/23-TA4/32.

Murakami, H., N. Yabushita, K. Baba and Y. Inoue (1998) Seismic behavior of soil-structure interaction systems under model reference adaptive control. *Proc. 10th Earthq. Engrg. Symp.*, Vol.3, pp.2947-2952.

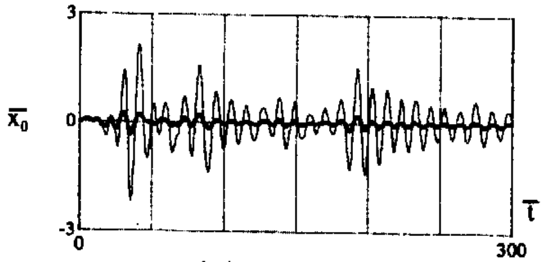


relative displacement

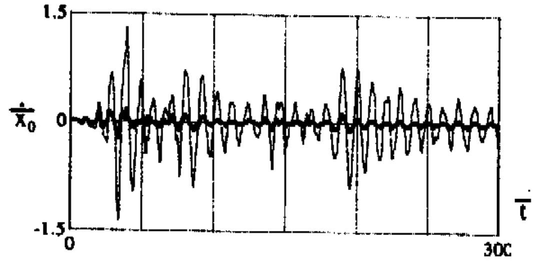


relative velocity

horizontal translation at upper mass

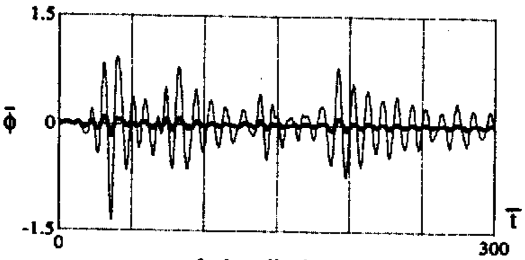


relative displacement

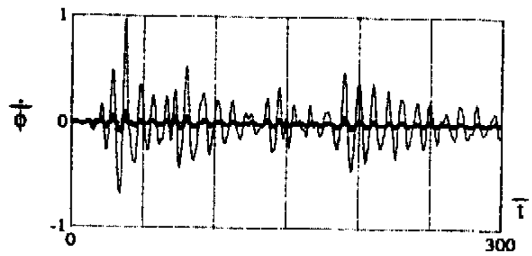


relative velocity

horizontal translation at foundation

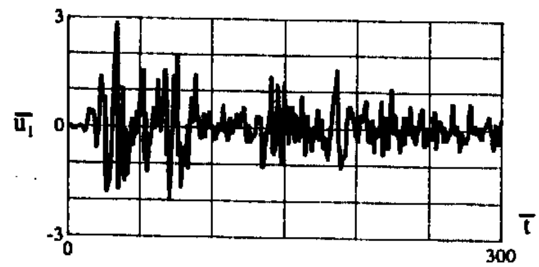


relative displacement

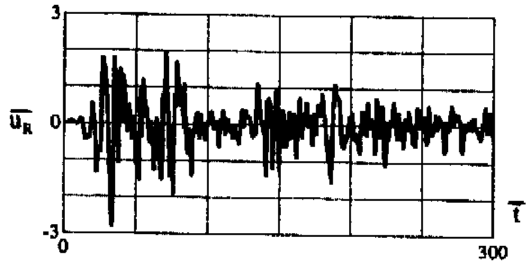


relative velocity

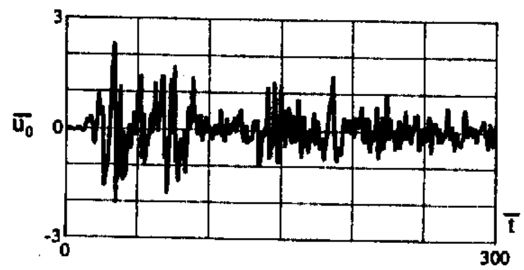
rotation at foundation



horizontal control force at upper mass



rotational control force at foundation



horizontal control force at foundation

Fig. 2 Response processes and control forces

$\gamma_{1g} = 1.0$, $\gamma_{2g} = 1.5$, $q_0 = q_2 = 1.0$, $\varepsilon = 0.2$

$\overline{\omega}_M = 0.6079$, $\zeta_{M1} = 0.2$, $\zeta = 0.0$

——— uncontrolled

——— controlled

Table 1 System compositions of interactive plants after monitoring operation, in reference to the presets.

$\gamma_{1B}=1.0$, $\gamma_{2B}=0.1$, $q_0=q_2=10000$, $\varepsilon=0.5$, $\zeta=0.03$, $\mathbf{A}_P=\begin{bmatrix} -\mathbf{A}_1 & -\mathbf{A}_0 \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$, $\mathbf{B}_P=\begin{bmatrix} -\mathbf{b} \\ \mathbf{0} \end{bmatrix}$

	$\mathbf{A}_0$	$\mathbf{A}_1$	$\mathbf{b}$
preset	1.0000 −1.0000 −1.0000	0.0600 −0.0600 −0.0600	1.0000
	−0.8966 2.0370 0.8966	−0.0538 0.6704 0.0538	0.8966
	−2.9997 2.9997 5.5367	−0.1800 0.1800 4.3182	0.0000
monitoring at $\bar{t}$ = 30,000			
Model (A)	0.9911 −0.9911 −0.9911	0.0632 −0.0632 −0.0632	1.0000
	−0.6148 1.4787 0.6148	−0.1342 0.6856 0.1342	0.8135
	−0.1579 0.1579 0.7822	−0.2326 0.2326 0.6716	0.1094
Model (B)	0.9961 −0.9961 −0.9961	0.0539 −0.0539 −0.0539	1.0000
	−0.8997 2.0369 0.8997	−0.0550 0.6746 0.0550	0.8983
	−1.0852 1.0852 3.3004	−0.3810 0.3810 1.2239	0.0075
Model (C)	0.9963 −0.9963 −0.9963	0.0595 −0.0595 −0.0595	1.0000
	−0.8990 2.0451 0.8990	−0.0523 0.6722 0.0523	0.8948
	−1.0893 1.0893 3.3312	−0.3762 0.3762 1.2163	0.0073

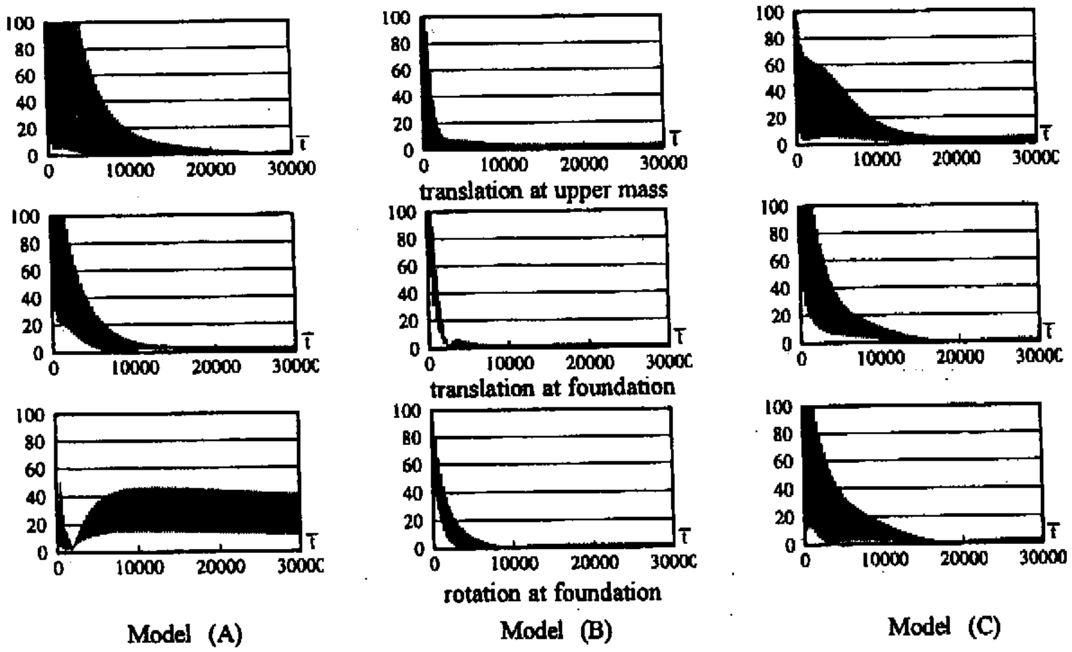


Fig. 3 Displacement difference processes relative to plant responses

Reference Model (A) : shearing type having triangle constraint of the first mode, uniform distribution of masses, $\bar{\varpi}_{M1}=0.6117$ and $\zeta_{M1}=0.09$.

Reference Model (B) : rocking type having damping properties proportional to stiffnesses, $\bar{\varpi}_{M1}=0.6036$ and $\zeta_{M1}=0.09$.

Reference Model (C) : rocking type having Rayleigh damping properties, $\bar{\varpi}_{M1}=0.6036$, $\bar{\varpi}_{M2}=1.2374$, $\zeta_{M1}=0.0972$ and $\zeta_{M2}=0.1757$.