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EFFECT OF IMPACT VIBRATION ABSORBER WITH HYSTERESIS DAMPING TO EARTHQUAKE EXCITATION

Shigeru AOKI[1] And TakeshI WATANABE[2]

SUMMARY

An analytical method is proposed for the random response of the primary mass with an impact damper having hysteresis damping. The impact damper is one of the dynamic vibration absorbers in which motion of auxiliary mass is limited by motion limiting stop or placed inside a container. In actual collision phenomena, energy loss for an impact and duration of collision are not negligible small. The energy loss and duration of collision can be considered by introducing hysteresis loop characteristics. In this paper, considering above mentioned points, an analytical model with hysteresis damping is introduced. The mean square response of the primary mass is obtained from moment equations introducing equivalent linearization method. As earthquake excitations, nonstationary filtered white noises, product of envelope function with respect to time and stationary filtered white noise, are used. Using the proposed method, the random response of the primary mass with the impact damper having hysteresis damping are compared with those of the primary mass with the impact damper having no hysteresis damping. It is concluded that the impact damper having hysteresis damping is more effective for reducing the vibration of primary mass subjected to earthquake excitations.

INTRODUCTION

For reducing the vibrations of structures and mechanical equipment subjected to earthquake excitations, some types of the dynamic vibration absorber are used(Kawazoe et. al. 1998, Reed et. al. 1998). An impact damper is one of the dynamic vibration absorbers in which motion of auxiliary mass is limited by motion limiting stop or placed inside a container. Many studies on impact damper have been carried out (Masri 1972, Masri and Ibrahim 1973, Davies 1980, Soong and Dargush 1997). The response of the system with impact characteristics which are motion-limiting constraints or clearance is of great importance for several engineering applications (Moon 1983, Moon and Shaw 1985). In some papers, an analytical model with energy loss for an impact represented by the coefficient of restitution is used(Aidanpaa and Gupta 1993). It is assumed that duration of collision is negligible small in comparison with the whole period of its vibration. However, in some conditions, duration of collision is not negligible small. The results from the model taking the duration of collision into account coincide more closely than those from the model neglecting it, with the results from experiment(Watanabe 1989). In other papers, a model with bilinear restoring force-deformation relation is used where stiffness increases during collision(Lin 1991). In this model, energy loss for an impact is not considered. Hence, before the behavior of physical system can be examined analytically, it is necessary to establish an appropriate model for the system. Generally speaking, the model is based on the result of experimental investigations of actual structures.

The authors proposed that the energy loss and duration of collision can be modelled by assuming hysteresis

loop characteristics in the relation between restoring force and penetration (Watanabe 1984, Aoki and Watanabe 1996).

[1] *Department of Mechanical Engineering, Tokyo Metropolitan College of Technology, Tokyo, Japan Email:aoki@tokyo-tmct.ac.jp*
[2] *Faculty of Education and Human Sciences, Yamanashi University, Kofu, Japan Fax:+81-55-220-8791*

In this work, too, they assume that the collision phenomena can be modeled by hysteresis loop characteristics, an analytical model with hysteresis damping is introduced and an analytical method is proposed for random response of the primary mass with an impact damper considering energy loss and duration of collision.

Earthquake excitations are nonstationary random processes. The mean square response is obtained for response of such systems. In this paper, mean square response of the primary mass is obtained from moment equations introducing equivalent linearization method. As earthquake excitations, nonstationary filtered white noises, product of envelope function with respect to time and stationary filtered white noise, are used. The maximum response of the primary mass is also obtained from mean square response.

Using the proposed method, some numerical results concerning the random response of the primary mass with an impact damper having hysteresis damping are obtained and are compared with those of the mass having no hysteresis damping.

2. ANALYTICAL METHOD

An analytical model consists of a primary mass m,P and auxiliary mass m,Q as shown in Fig.1. This model is usually used for analysis of impact damper. In order to consider the energy loss for an impact and duration of collision, the relation between force of restitution and penetration is assumed to be represented by hysteresis loop characteristics as shown in Fig.2. The equations of motion are

$$\left.\begin{aligned} &m_1\ddot{x}_1+c_1(\dot{x}_1-\dot{y})+k_1(x_1-y)+c_2(\dot{x}_1-\dot{x}_2)+k_2(x_1-x_2)-f(x_{21},\dot{x}_{21})=0\\ &m_2\ddot{x}_2+c_2(\dot{x}_2-\dot{x}_1)+k_2(x_2-x_1)+f(x_{21},\dot{x}_{21})=0\end{aligned}\right\} \qquad (1)$$

where m_1 and m_2 are the mass of the primary mass and that of the damper, respectively. c_1 and c_2 are the damping coefficient. k_1 and k_2 are the spring constants. x_1 and x_2 are the absolute displacement. $f(x_{21},\dot{x}_{21})$ is the force of restitution and is defined by piecewise-linear characteristics shown in Fig.2.

$f(x_{21},\dot{x}_{21})$ is assumed to be given as following equation using equivalent damping coefficient c_{eq} and equivalent stiffness k_{eq}.

$$f(x_{21},\dot{x}_{21})=c_{eq}(\dot{x}_2-\dot{x}_1)+k_{eq}(x_2-x_1) \qquad (2)$$

As input acceleration $\ddot{y}$, nonstationary filtered white noise is given as the following equations.

$$\left.\begin{aligned} &\ddot{z}_g+2\zeta_g\omega_g\dot{z}_g+\omega_g^2 z_g=-\ddot{y}_g\\ &\ddot{y}=I(t)(\ddot{z}_g+\ddot{y}_g)\end{aligned}\right\} \qquad (3)$$

where ζ_g is the damping ratio of the ground model and ω_g is the natural circular frequency of the ground model. $I(t)$ is envelope function which represents nonstationary characteristics of amplitude. In this paper, two types of envelope functions as shown in Fig.3(a) and 3(b) are used. $\ddot{y}_g$ is input acceleration of the base rock and is given as stationary white noise.

Substituting Eq.(2) into Eq.(1), equations of motion are written as:

$$\left.\begin{aligned} &m_1\ddot{x}_1+c_1(\dot{x}_1-\dot{y})+k_1(x_1-y)+(c_2+c_{eq})(\dot{x}_1-\dot{x}_2)+k_{eq}(x_1-x_2)=0\\ &m_2\ddot{x}_2+(c_2+c_{eq})(\dot{x}_2-\dot{x}_1)+k_{eq}(x_2-x_1)=0\end{aligned}\right\} \qquad (4)$$

Eqs.(3) and (4) can be written as follows:

$$\left.\begin{aligned} &\ddot{z}_2=2\zeta_1\omega_1\dot{z}_1+\omega_1^2 z_1-(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})(1+\gamma)\dot{z}_2-\omega_{eq}^2(1+\gamma)z_2\\ &\ddot{z}_1=-2\zeta_1\omega_1\dot{z}_1-\omega_1^2 z_1+(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})\gamma\dot{z}_2+\omega_{eq}^2\gamma z_2-I(t)(2\zeta_g\omega_g\dot{z}_g+\omega_g^2 z_g)\\ &\ddot{z}_g=-2\zeta_g\omega_g\dot{z}_g-\omega_g^2 z_g-\ddot{y}_g\end{aligned}\right\} \qquad (5)$$

where $\zeta_1(=c_1/2\sqrt{m_1k_1})$ and $\zeta_2(=c_2/2\sqrt{m_2k_2})$ are the damping ratio of the primary mass and that of the impact damper. $\omega_1(=\sqrt{k_1/m_1})$ and $\omega_2(=\sqrt{k_2/m_2})$ are the natural circular frequency. $\gamma(m_2$

$/m_1)$ is the mass ratio of the damper to the primary mass. $z_1\,(=x_1-y)$ and $z_2\,(=x_2-x_1)$ are the relative displacement. $\zeta_{eq}\,(=c_{eq}/2\sqrt{m_{eq}k_{eq}})$ and $\omega_{eq}\,(=\sqrt{k_{eq}/m_{eq}})$ are the equivalent damping ratio and the equivalent natural circular frequency of the damper.

The moment equations of the second moments are given as follows (Roberts and Spanos 1990):

$$\dot{\mathbf{V}}=\mathbf{G}\mathbf{V}^T+\mathbf{V}\mathbf{G}^T+\mathbf{D} \tag{6}$$

where

$$\mathbf{G}=\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -a_2 & a_4 & -a_1 & a_3 & a_7 I(t) & a_8 I(t) \\ a_2 & -a_6 & a_1 & -a_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -a_8 & -a_7 \end{bmatrix} \tag{7}$$

where $a_1=2\zeta_1\omega_1$, $a_2=\omega_1^2$, $a_3=(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})\gamma$, $a_4=\omega_{eq}^2\gamma$, $a_5=(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})(1+\gamma)$, $a_6=\omega_{eq}^2(1+\gamma)$, $a_7=2\zeta_g\omega_g I(t)$, $a_8=\omega_g^2$.

$$\mathbf{V}=\begin{bmatrix} \sigma_{z1}^2 & \kappa_{z1z2} & \kappa_{z1\dot{z}1} & \kappa_{z1\dot{z}2} & \kappa_{z1zg} & \kappa_{z1\dot{z}g} \\ \kappa_{z1z2} & \sigma_{z2}^2 & \kappa_{\dot{z}1z2} & \kappa_{z2\dot{z}2} & \kappa_{z2zg} & \kappa_{z2\dot{z}g} \\ \kappa_{z1\dot{z}1} & \kappa_{\dot{z}1z2} & \sigma_{\dot{z}1}^2 & \kappa_{\dot{z}1\dot{z}2} & \kappa_{\dot{z}1zg} & \kappa_{\dot{z}1\dot{z}g} \\ \kappa_{z1\dot{z}2} & \kappa_{z2\dot{z}2} & \kappa_{\dot{z}1\dot{z}2} & \sigma_{\dot{z}2}^2 & \kappa_{\dot{z}2zg} & \kappa_{\dot{z}2\dot{z}g} \\ \kappa_{z1zg} & \kappa_{z2zg} & \kappa_{\dot{z}1zg} & \kappa_{\dot{z}2zg} & \sigma_{zg}^2 & \kappa_{zg\dot{z}g} \\ \kappa_{z1\dot{z}g} & \kappa_{z2\dot{z}g} & \kappa_{\dot{z}1\dot{z}g} & \kappa_{\dot{z}2\dot{z}g} & \kappa_{zg\dot{z}g} & \sigma_{\dot{z}g}^2 \end{bmatrix} \tag{8}$$

$$\mathbf{D}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\pi S_0 \end{bmatrix} \tag{9}$$

where S_0 is power spectral density of white noise, input excitation of the base rock. From Eq. (6) using Eqs. (7) - (9), the following moment equations are obtained.

$$\frac{\partial \sigma_{z1}^2}{\partial t}=2\kappa_{z1\dot{z}1}$$

$$\frac{\partial \kappa_{z1z2}}{\partial t}=\kappa_{\dot{z}1z2}+\kappa_{z1\dot{z}2}$$

$$\frac{\partial \kappa_{z1\dot{z}1}}{\partial t}=\sigma_{\dot{z}1}^2-\omega_1^2\sigma_{z1}^2+\omega_{eq}^2\gamma\kappa_{z1z2}-2\zeta_1\omega_1\kappa_{z1\dot{z}1}$$
$$+(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})\gamma\kappa_{z1\dot{z}2}+\omega_g^2 I(t)\kappa_{z1zg}+2\zeta_g\omega_g I(t)\kappa_{z1\dot{z}g}$$

$$\frac{\partial \kappa_{z1\dot{z}2}}{\partial t}=\kappa_{\dot{z}1\dot{z}2}+\omega_1^2\sigma_{z1}^2-\omega_{eq}^2(1+\gamma)\kappa_{z1z2}+2\zeta_1\omega_1\kappa_{z1\dot{z}1}$$
$$-(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})(1+\gamma)\kappa_{z1\dot{z}2}$$

$$\frac{\partial \kappa_{z1zg}}{\partial t}=\kappa_{\dot{z}1zg}+\kappa_{z1\dot{z}g}$$

$$\frac{\partial \kappa_{z1\dot{z}g}}{\partial t}=\kappa_{\dot{z}1\dot{z}g}-\omega_g^2\kappa_{z1zg}-2\zeta_g\omega_g\kappa_{z1\dot{z}g}$$

$$\frac{\partial \sigma_{z2}^2}{\partial t}=2\kappa_{z2\dot{z}2}$$

$$\frac{\partial \kappa_{\dot{z}1z2}}{\partial t}=\kappa_{\dot{z}1\dot{z}2}-\omega_1^2\kappa_{z1z2}+\omega_{eq}^2\gamma\sigma_{z2}^2-2\zeta_1\omega_1\kappa_{\dot{z}1z2}$$
$$+(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})\gamma\kappa_{z2\dot{z}2}+\omega_g^2 I(t)\kappa_{z2zg}+2\zeta_g\omega_g I(t)\kappa_{z2\dot{z}g}$$

$$\frac{\partial \kappa_{z2\dot{z}2}}{\partial t}=\sigma_{\dot{z}2}^2+\omega_1^2\kappa_{z1z2}-\omega_{eq}^2(1+\gamma)\sigma_{z2}^2+2\zeta_1\omega_1\kappa_{\dot{z}1z2}$$
$$-(2\zeta_2\omega_2+2\zeta_{eq}\omega_{eq})(1+\gamma)\kappa_{z2\dot{z}2}$$

$$
\begin{aligned}
\frac{\partial \kappa_{z2zg}}{\partial t} &= \kappa_{\dot{z}2zg} + \kappa_{z2\dot{z}g} \\
\frac{\partial \kappa_{z2\dot{z}g}}{\partial t} &= \kappa_{\dot{z}2\dot{z}g} - \omega_g^2 \kappa_{z2zg} - 2\zeta_g \omega_g \kappa_{z2\dot{z}g} \\
\frac{\partial \sigma_{\dot{z}1}^2}{\partial t} &= 2\{-\omega_1^2 \kappa_{z1\dot{z}1} + \omega_{eq}^2 \gamma \kappa_{\dot{z}1z2} - 2\zeta_1 \omega_1 \sigma_{\dot{z}2}^2 \\
&\quad + (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) \gamma \kappa_{\dot{z}1\dot{z}2} + \omega_g^2 I(t) \kappa_{\dot{z}1zg} + 2\zeta_g \omega_g I(t) \kappa_{\dot{z}1\dot{z}g}\} \\
\frac{\partial \kappa_{\dot{z}1\dot{z}2}}{\partial t} &= -\omega_1^2 \kappa_{z1\dot{z}2} + \omega_{eq}^2 \gamma \kappa_{z2\dot{z}2} - 2\zeta_1 \omega_1 \kappa_{\dot{z}1\dot{z}2} \\
&\quad + (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) \gamma \sigma_{\dot{z}2}^2 + \omega_g^2 I(t) \kappa_{\dot{z}2zg} + 2\zeta_g \omega_g I(t) \kappa_{\dot{z}2\dot{z}g} \\
&\quad + \omega_1^2 \kappa_{z1\dot{z}1} - \omega_{eq}^2 (1+\gamma) \kappa_{\dot{z}1z2} + 2\zeta_1 \omega_1 \sigma_{\dot{z}1}^2 \\
&\quad - (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) (1+\gamma) \kappa_{\dot{z}1\dot{z}2} \\
\frac{\partial \kappa_{\dot{z}1zg}}{\partial t} &= -\omega_1^2 \kappa_{z1zg} + \omega_{eq}^2 \gamma \kappa_{z2zg} - 2\zeta_1 \omega_1 \kappa_{\dot{z}1zg} \\
&\quad + (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) \gamma \kappa_{\dot{z}2zg} + \omega_g^2 I(t) \sigma_{zg}^2 + 2\zeta_g \omega_g I(t) \kappa_{zg\dot{z}g} \\
&\quad + \kappa_{\dot{z}1\dot{z}g} \\
\frac{\partial \kappa_{\dot{z}1\dot{z}g}}{\partial t} &= -\omega_1^2 \kappa_{z1\dot{z}g} + \omega_{eq}^2 \gamma \kappa_{z2\dot{z}g} - 2\zeta_1 \omega_1 \kappa_{\dot{z}1\dot{z}g} \\
&\quad + (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) \gamma \kappa_{\dot{z}2\dot{z}g} + \omega_g^2 I(t) \kappa_{zg\dot{z}g} + 2\zeta_g \omega_g I(t) \sigma_{\dot{z}g}^2 \\
&\quad - \omega_g^2 \kappa_{\dot{z}1zg} - 2\zeta_g \omega_g \kappa_{\dot{z}1\dot{z}g} \\
\frac{\partial \sigma_{\dot{z}2}^2}{\partial t} &= 2\{\omega_1^2 \kappa_{z1\dot{z}2} - \omega_{eq}^2 (1+\gamma) \kappa_{z2\dot{z}2} + 2\zeta_1 \omega_1 \kappa_{\dot{z}1\dot{z}2} \\
&\quad - (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) (1+\gamma) \sigma_{\dot{z}2}^2\} \\
\frac{\partial \kappa_{\dot{z}2zg}}{\partial t} &= \omega_1^2 \kappa_{z1zg} - \omega_{eq}^2 (1+\gamma) \kappa_{z2zg} + 2\zeta_1 \omega_1 \kappa_{\dot{z}1zg} \\
&\quad - (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) (1+\gamma) \kappa_{\dot{z}2zg} + \kappa_{z2\dot{z}g} \\
\frac{\partial \kappa_{\dot{z}2\dot{z}g}}{\partial t} &= \omega_1^2 \kappa_{z1\dot{z}g} - \omega_{eq}^2 (1+\gamma) \kappa_{z2\dot{z}g} + 2\zeta_1 \omega_1 \kappa_{\dot{z}1\dot{z}g} \\
&\quad - (2\zeta_2\omega_2 + 2\zeta_{eq}\omega_{eq}) (1+\gamma) \kappa_{\dot{z}2\dot{z}g} - \omega_g^2 \kappa_{\dot{z}2zg} - 2\zeta_g \omega_g \kappa_{\dot{z}2\dot{z}g} \\
\frac{\partial \sigma_{zg}^2}{\partial t} &= 2\kappa_{zg\dot{z}g} \\
\frac{\partial \kappa_{zg\dot{z}g}}{\partial t} &= \sigma_{\dot{z}g}^2 - \omega_g^2 \sigma_{zg}^2 - 2\zeta_g \omega_g \kappa_{zg\dot{z}g} \\
\frac{\partial \sigma_{\dot{z}g}^2}{\partial t} &= 2(-\omega_g^2 \kappa_{zg\dot{z}g} - 2\zeta_g \omega_g \sigma_{\dot{z}g}^2) + 2\pi S_0
\end{aligned} \tag{10}
$$

3. EQUIVALENT LINEARIZATION METHOD

In this paper, the equivalent damping ratio ζ_{eq} and the equivalent natural circular frequency ω_{eq} are approximately obtained by stationary random vibration theory since the effect of impact damper is great at the main shock.

When the system is subjected to harmonic excitation, dissipated energy during one cycle is

$$E_N' = 2\frac{1}{2} k_2 (Z_2 - e_0)(Z_{23} - e_0) \tag{11}$$

where Z_2 is the amplitude of response. And

$$Z_{23} = e_0 + (1 - K_1/K_2)(Z_2 - e_0) \tag{12}$$

Then,

$$E_N' = K_1 (1 - K_1/K_2)(Z_2 - e_0)^2 \tag{13}$$

It is assumed that the response is narrow band random process and the probability distribution function of Z is given by the Rayleigh distribution as follows:

$$p(Z) = \frac{Z_2}{\sigma_{z2}^2} \exp\left(-\frac{Z_2^2}{2\sigma_{z2}^2}\right) \tag{14}$$

where σ_{z2}^2 is variance of relative displacement of z_2. The expected value of En' is obtained as:

$$\begin{aligned} E_N &= \int_0^{\infty} E_N' p(Z_2)\, dZ_2 \\ &= \int_0^{\infty} K_1 (1-K_1/K_2)(Z_2-e_0) \frac{Z_2}{\sigma_{z2}^2} \exp\left(-\frac{Z_2^2}{2\sigma_{z2}^2}\right) dZ_2 \\ &= K_1(1-K_1/K_2)\left[2\sigma_{z2}^2 \exp(-y_0^2)(y_0^2+1)\right. \\ &\quad \left. -2\sqrt{2}\sigma_{z2} e_0 \{y_0 \exp(-y_0^2) + \sqrt{\pi}\,\mathrm{erfc}(y_0)/2\} + e_0^2 \exp(-y_0^2)\right] \end{aligned} \tag{15}$$

where

$$\left.\begin{aligned} y_0 &= \frac{e_0}{\sqrt{2}\sigma_{z2}} \\ \mathrm{erfc}(y_0) &= 1 - \frac{2}{\sqrt{\pi}} \int_0^{y_0} \exp(-y_0)\, dy \end{aligned}\right\} \tag{16}$$

When the system is subjected to harmonic excitation, dissipated energy by the damper with the equivalent damping coefficient is given as:

$$E_T' = \pi c_{eq} \omega_2 Z_2^2 \tag{17}$$

When the probability distribution function of Z_2 is represented by the Rayleigh distribution, the expected value of E_T' is obtained as:

$$E_T = 2\sigma_{z2}^2 \pi \omega_2 c_{eq} \tag{18}$$

Since E_N is equal to E_T, c_{eq} is given as:

$$c_{eq} = \frac{E_N}{2\sigma_{z2}^2 \pi \omega_2} \tag{19}$$

Equivalent stiffness k_{eq} is approximated as shown in Fig.4. When the system is subjected to harmonic excitation, equivalent stiffness k_{eq}' is given as:

$$k_{eq}' = \begin{cases} \{k_2 e_0 + (k_2+K_1)(Z_2-e_0)\}/Z_2 & : Z_2 \geqq e_0 \\ k_2 & : Z_2 \leqq e_0 \end{cases} \tag{20}$$

The expected value of k_{eq}' is obtained as:

$$\begin{aligned} k_{eq} &= \int_0^{\infty} k_{eq}' p(Z_2)\, dZ_2 \\ &= \int_0^{e_0} k_2 \frac{Z_2}{\sigma_{z2}^2} \exp\left(-\frac{Z_2^2}{2\sigma_{z2}^2}\right) dZ_2 \\ &\quad + \int_{e_0}^{\infty} \{k_2 e_0 + (k_2+K_1)(Z_2-e_0)\} \frac{Z_2}{\sigma_{z2}^2} \exp\left(-\frac{Z_2^2}{2\sigma_{z2}^2}\right) dZ_2 \\ &= k_2 + K_1 \exp(-y_0^2) - K_1 \sqrt{\pi} y_0\, \mathrm{erfc}(y_0) \end{aligned} \tag{21}$$

And,

$$\left.\begin{aligned} 2\zeta_{eq}\omega_{eq} &= \omega_2^2 K_1/k_2 (1-K_1/K_2)\left[2\sigma_{z2}^2 \exp(-y_0^2)(y_0^2+1)\right. \\ &\quad \left. -2\sqrt{2}\sigma_{z2} e_0 \{y_0 \exp(-y_0^2) + \sqrt{\pi}\,\mathrm{erfc}(y_0)/2\} + e_0^2 \exp(-y_0^2)\right] \\ \omega_{eq}^2 &= \omega_2^2 + \omega_2^2 \{K_1/k_2 \exp(-y_0^2) - K_1/k_2 \sqrt{\pi} y_0\, \mathrm{erfc}(y_0)\} \end{aligned}\right\} \tag{22}$$

4. NUMERICAL EXAMPLES

The maximum value of mean square response of excitation acceleration $\sigma_{\ddot{z}g}$ is given as:

$$\sigma_{\ddot{z}g} = \sqrt{\frac{\pi}{2}\frac{\omega_g(1+4\zeta_g^{\,2})}{\zeta_g} S_0} \tag{23}$$

The mean square response of primary mass $\sigma_{z1}^{\,2}$ and that of damper $\sigma_{z2}^{\,2}$ are obtained by Eq. (10) using Eqs. (21) and (22). The maximum values of relative displacement of primary mass z_{1max} and z_{2max} normalized by the maximum value of excitation acceleration are defined as follows (Tajimi 1960):

$$z_{1max} = \sigma_{z1}/\sigma_{\ddot{z}g}$$
$$z_{2max} = \sigma_{z2}/\sigma_{\ddot{z}g} \tag{24}$$

Gap size e_0 is determined by using the maximum value of relative displacement of the linear damper without collision z_{2lmax} as follows:

$$e_0 = d^* z_{2lmax} \tag{25}$$

Fig.5 shows the mean square response of the primary mass $\sigma_{z1}^{\,2}$ for $\gamma = 0.1$, $\zeta_1 = 0.01$, $T_1 = 1.0$s, $\zeta_2 = 0.01$, $T_2 = 1.0$s, $\zeta_g = 0.4$ and $T_g = 1.0$s. And, $T_1 = 2\pi/\omega_1$, $T_2 = 2\pi/\omega_2$, $T_g = 2\pi/(\sqrt{1-\zeta_g^{\,2}}\,\omega_g)$. The impact damper gives the same effect of reduction of the maximum response as elastic damper. $\sigma_{z1}^{\,2}$ of the mass with impact damper decreases earlier than that with elastic damper.

Table 1 and Table 2 show z_{1max} and z_{2max} for some values of mass ratio γ and nonlinear parameters K_1/k_2 and K_2/k_2. From these tables, the maximum displacement of the primary mass is reduced when the impact damper is used. The effect of reduction is almost same as elastic damper. Comparing with the maximum response of the elastic damper, the maximum response of the impact damper having hysteresis damping is significantly reduced. The impact damper gives the same effect of reduction of the maximum response as elastic damper without the large response of the impact damper itself.

5. CONCLUSIONS

Considering energy loss and duration of collision, an analytical method is proposed for the random response of the primary mass with the impact damper having hysteresis damping. The mean square response and the maximum response of the primary mass are obtained from moment equations introducing equivalent linearization method. Using the proposed method, the random response of the primary mass with the impact damper having hysteresis damping are compared with those of the mass with the impact damper having no hysteresis damping. It is concluded that the impact damper having hyseresis damping is more effective for reducing the vibration of primary mass subjected to earthquake excitations.

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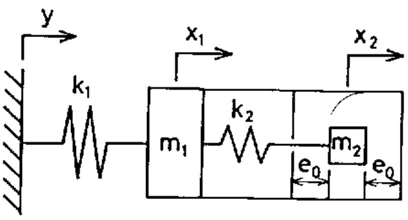


Fig.1: Analytical model

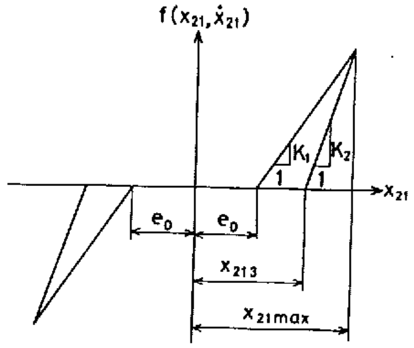
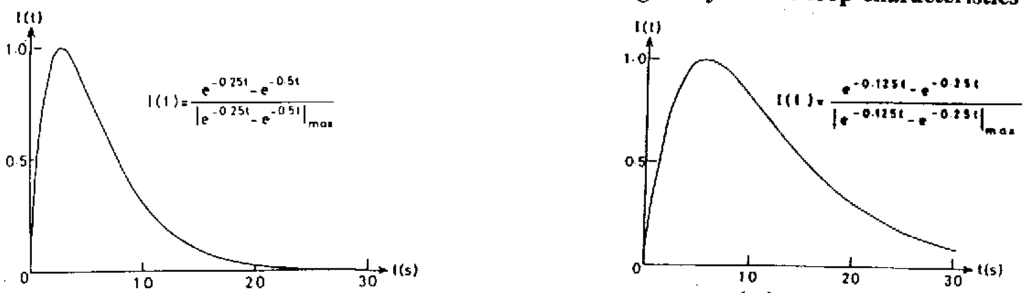


Fig.2: Hysteresis loop characteristics



(a) Type I

(b) Type II

Fig.3: Envelope functions

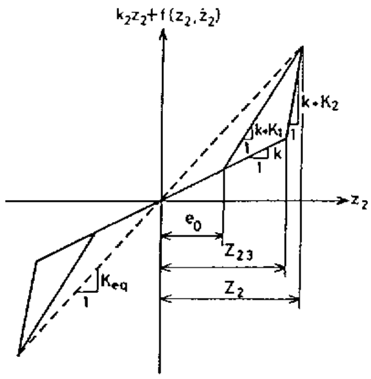


Fig.4: Equivalent stiffness

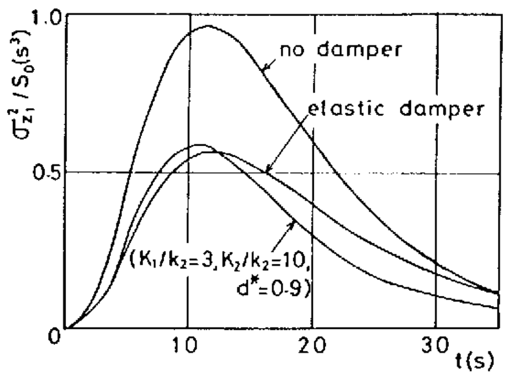


Fig.5: Mean square response of primary mass
(γ=0.1, ζ_1=0.01, T_1=1.0s, ζ_2=0.01, T_2=1.0s, ζ_g=0.4, T_g=1.0s)

Table 1: The maximum value of relative displacement of primary mass and damper (s^2)
(ζ_1 =0.01, T_1 =1.0s, ζ_2 =0.01, T_2 =1.0s, ζ_g =0.4, T_g =1.0s, Type I)

γ	K_1/k_2	K_2/k_2	d*	z_{1max}	z_{2max}
	no damper			0.120	
0.10	elastic damper			0.091	0.272
	3	10	0.9	0.096	0.175
			0.8	0.097	0.164
			0.7	0.098	0.152
			0.6	0.100	0.138
			0.5	0.103	0.125
	10	30	0.9	0.100	0.142
			0.8	0.102	0.131
			0.7	0.104	0.119
			0.6	0.107	0.107
			0.5	0.109	0.094
0.25	elastic damper			0.100	0.167
	3	10	0.9	0.104	0.111
			0.8	0.105	0.105
			0.7	0.106	0.098
			0.6	0.108	0.091
			0.5	0.110	0.083
	10	30	0.9	0.108	0.091
			0.8	0.110	0.085
			0.7	0.112	0.078
			0.6	0.114	0.070
			0.5	0.117	0.062

Table 2: The maximum value of relative displacement of primary mass and damper (s^2)
(ζ_1 =0.01, T_1 =1.0s, ζ_2 =0.01, T_2 =1.0s, ζ_g =0.4, T_g =1.0s, Type II)

γ	K_1/k_2	K_2/k_2	d*	z_{1max}	z_{2max}
	no damper			0.147	
0.10	elastic damper			0.113	0.328
	3	10	0.9	0.114	0.214
			0.8	0.116	0.193
			0.7	0.118	0.180
			0.6	0.120	0.162
			0.5	0.123	0.149
	10	30	0.9	0.120	0.169
			0.8	0.122	0.156
			0.7	0.125	0.143
			0.6	0.128	0.128
			0.5	0.131	0.114
0.25	elastic damper			0.126	0.207
	3	10	0.9	0.126	0.136
			0.8	0.127	0.128
			0.7	0.128	0.120
			0.6	0.130	0.111
			0.5	0.132	0.101
	10	30	0.9	0.130	0.112
			0.8	0.132	0.104
			0.7	0.134	0.095
			0.6	0.137	0.086
			0.5	0.140	0.076