EVALUATION OF STRUCTURAL COEFFICIENT BY DISPLACEMENT RESPONSE ESTIMATION USING THE EQUIVALENT LINEAR METHOD

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SUMMARY

This paper proposes a method for estimating a structure coefficient based on displacement response. First, formulas for estimating displacement response are evaluated using the equivalent linear method. Next, a method for determining a structure coefficient is proposed using the estimated displacement response. Finally, trends of the structure coefficient regarding the initial period, energy absorption ability, and allowable deformation are shown. They have the following tendencies.

1) Increase like a hyperbola having an upper limit as period ratio *TR* decreases.
2) Reduce like a straight line as equivalent viscous damping index β increases.
3) Reduce like a straight line or like an exponential function having an upper bound as allowable ductility factor μ increases.

PURPOSE

As the building regulation aims performance-based design, response displacement is perceived as a simple scale for evaluating earthquake resistance. The author studied the displacement response of structures during an earthquake. The design base shear strength based on estimated displacement response was reported [Shimazaki, 1988]. The distribution shape of shear coefficient for high-rise reinforced concrete structures within acceptable damage without deformation concentration was investigated [Shimazaki, 1992]. Based on these results, a design method oriented to the displacement response for a high-rise reinforced concrete frame building has been proposed [Shimazaki, 1996]. However, the design method based on displacement response for non high-rise buildings has difficulty because the constant displacement response rule does not usually hold true in practice.
Many countries use the structural coefficient to establish the design base shear coefficient. This value is a measure of the capacity of the structural system to absorb energy in the inelastic range through ductility and redundancy. It should be based on the estimation of the displacement response.

This paper proposes a method for determining the structural coefficient based on displacement response. First, formulas for estimating displacement response are evaluated for the bi-linear type design velocity response spectrum using the equivalent linear method. Next, a method for determining a value of the structure coefficient is proposed by using the estimated displacement response. Finally, trends of the values of the structure coefficient determined here regarding the initial period, energy absorption ability, and allowable deformation are shown.

2. DISPLACEMENT RESPONSE ESTIMATION USING THE EQUIVALENT LINEAR METHID

2.1 Equivalent Linear Method

The main characteristics of the equivalent linear method are,
1) Increase in deformation as the effective period increases,
2) Decrease in deformation as the equivalent viscous damping increases.

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It is said that the equivalent linear method can simulate nonlinear displacement response very well [Shibata, 1976; Moehle, 1984].

The effective period T_{ef} at maximum displacement response D_2 shown by the broken line in Figure 1 is given as a function of the ductility factor μ (maximum displacement/yield displacement). It is given by Equation (1) for an idealized elasto-plastic model.

$$T_{ef} = T_0 \cdot \sqrt{\mu} \qquad (1)$$

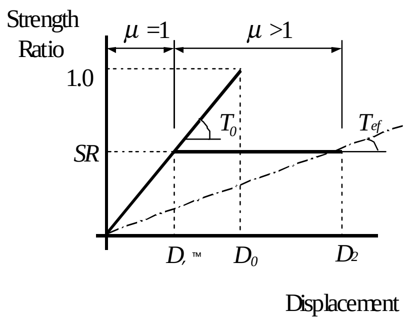


Figure 1 : Idealized elaso-plastic displacement response and effective period

The equivalent viscous damping factor h_{eq} for the system having the initial damping factor h_0 of 0.02 is given by Equation (2) for a reinforced concrete structure [Shibata 1976].

$$h_{eq} = 0.2(1 - \frac{1}{\sqrt{\mu}}) + 0.02 \qquad (2)$$

Changing of the response spectrum value by changing the damping factor is given by Equation (3) with the base-damping factor of 0.02 for the acceleration response spectrum [Shibata 1976].

$$\frac{S_a(h_{eq})}{S_a(0.02)} = \frac{8}{6 + 100 h_{eq}} \qquad (3)$$

The ratio of the displacement response spectrum of h_{eq} with h=0.02 is obtained by Equation (4) when Equation (3) applies to the displacement response spectrum.

$$\frac{S_D(h_{eq})}{S_D(0.02)} = \frac{0.4\sqrt{\mu}}{1.4\sqrt{\mu} - 1} \qquad (4)$$

2.2 Elasto-plastic Displacement Response of Reinforced Concrete Structure

The bilinear type design velocity response spectrum is assumed with bending at characteristic period T_g. The displacement response spectrum is shown in Figure 2. The elasto-plastic displacement response is divided into three regions by the equivalent period T_{ef}, the characteristic period T_g and the initial period T_0.

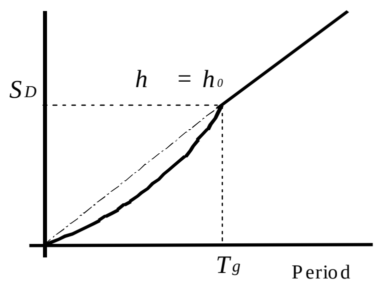


Figure 2 : Design displacement response spectrum

a) $T_0 > T_g, T_{ef} > T_g$

The displacement response spectrum is given as a straight line shown in Figure 3 (a). The displacement at T_{ef} increases linearly to D_1 as Equation (5).

$$D_1 = D_0 \cdot T_{ef} / T_0 = D_0 \cdot \sqrt{\mu} \qquad (5)$$

The displacement response decreases to D_2 as equivalent viscous damping increases.

$$D_2 = D_1 \cdot \frac{0.4\sqrt{\mu}}{1.4\sqrt{\mu}-1} = D_0 \cdot \frac{0.4 \cdot \mu}{1.4\sqrt{\mu}-1} \quad (6)$$

This is a function of the assumed ductility factor μ. D_2 must be equal to $D_y\mu$ according to Figure 1.
The yield deformation D_y can be defined as a function of strength ratio SR (yield strength/elastic response shear force) as shown in Figure 1.

$$D_y = D_0 \cdot SR \quad (7)$$

From Equation (6) and (7),

$$\mu = \frac{(0.4/SR+1)^2}{1.96} \quad (8)$$

The displacement ratio DR (maximum elasto-plastic displacement response/elastic displacement response) is obtained as follows.

$$DR = \frac{D_2}{D_0} = \frac{\mu D_y}{D_0} = \frac{(0.4+SR)^2}{1.96SR} \quad (9)$$

b•$T_0 < T_g$ •$T_{ef} < T_g$

The displacement response spectrum is a quadratic function of period as shown in Figure 3(b), so Equation (11) is obtained by using Equation (10) instead of using Equation (5).

$$D_1 = D_0 \left(T_{ef}/T_0\right)^2 = D_0 \cdot \mu \quad (10)$$

$$DR = \frac{SR}{(1.4-0.4/SR)^2} \quad (11)$$

Equation (11) becomes infinite and has no meaning in the region of SR<0.286.

c•$T_0 < T_g$ •$T_{ef} > T_g$

The relation between point D_0 and point D_3 on the hypothesis response displacement spectrum drawn by the broken line in Figure 3(c) is given by Equation (12) as a function of TR (initial period T_0/characteristics period T_g).

$$D_3/D_0 = 1/TR \quad (12)$$

$$DR = \frac{(0.4+SR)^2}{1.96SR} \cdot \frac{1}{TR} \quad (13)$$

At $T_0 < T_g$, DR is the minimum of Equations (11) and (13).

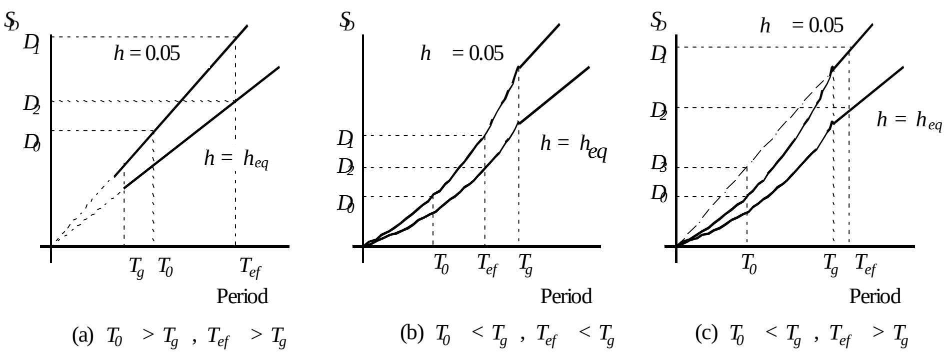


Figure 3 : Nonlinear displacement response using equivalent linear method

2.3 Comparison with Response Calculations

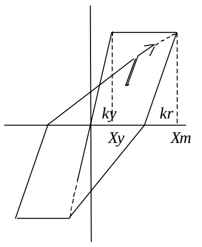
Response analyses were carried out to examine the accuracy of these relations. Ground motions used are shown in Table 1 with characteristic period T_g. The bilinear type hysteresis model as the reinforced concrete structure is shown in Figure 4. Numerical values of strength are changed in order from 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, to 0.05 times the shear strength defined as a dimensionless value with the unit mass system from the

smoothed acceleration response spectrum. Damping is assumed to be 0.02 initially, and proportional to the instantaneous stiffness.
The comparison with the proposed relation using Equations (9), (11), (13) and calculated values is shown in Figure 5 as the relation of strength ratio *SR* and displacement ratio *DR*. The proposed line is on the safe side except in the case of $SR<0.3$ with $T_0<T_g$. This is a meaningless range because the ductility factor becomes 8 or more.

The equation obtained here is good enough for practical use to estimate the nonlinear displacement response for reinforced concrete structures.

Table 1 : Ground motions used

Ground motion	Max. value of motion			Max. value of response spectra			
	Acc. cm/sec^2	Vel. cm/sec	Displ cm	Acc. cm/sec^2	Vel. cm/sec	Displ cm	T_g sec
El Centro NS	341.7	33.45	10.86	1209.8	109.67	36.27	0.57
El Centro EW	210.1	36.92	19.78	783.7	96.57	54.05	0.77
Taft NS	152.7	15.72	6.69	542.0	45.32	25.43	0.53
Taft EW	175.9	17.71	9.15	591.2	48.28	20.53	0.51
Tokyo 101 NS	74.0	7.63	4.38	201.8	22.63	6.57	0.70
Sendai 501 NS	57.5	3.46	1.94	226.8	10.52	3.02	0.29
Sendai 501 EW	47.5	3.82	2.14	215.2	13.61	3.31	0.40
Osaka 205 EW	25.0	5.08	4.14	124.2	13.35	7.41	0.68
Hachinohe NS	225.0	34.08	11.44	817.5	96.56	40.42	0.74
Hachinohe EW	182.9	35.81	13.26	803.2	119.11	47.93	0.93
Tho30-1FL NS	258.2	36.17	14.52	942.4	146.37	35.60	0.98
Tho30-1FL EW	202.6	27.57	9.11	955.7	82.12	33.23	0.54
Castaic EW	310.7	16.26	2.59	1014.3	57.56	9.08	0.36
Managua NS	317.5	29.48	6.66	1735.2	103.89	24.98	0.38
Los Angeles NS	249.9	27.27	12.65	874.8	106.10	55.75	0.76
Santa Barbara EW	128.4	18.79	5.24	344.3	62.86	20.35	1.15



$$Kr = \sqrt{Xy / Xm}$$

Figure 4 : Bilinear type hysteresis model as the reinforced concrete structure

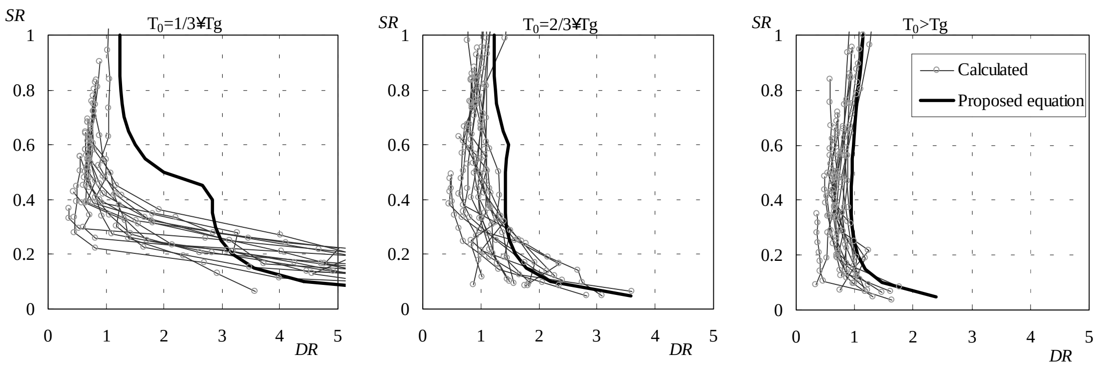


Figure 5 : Comparison with the proposed relation and numerical values

3. GENERALIZED NONLINEAR RESPONSE DISPLACEMENT ESTIMATION

3.1 Evaluation of Damping and Design Response Spectrum

The 5% damping spectrum is used as the design response spectrum. This is assumed to have the effect of soil-structure interaction damping.

Equation (14) is used for the equivalent damping instead of Equation (2).

$$h_{eq} = \beta(1 - \frac{1}{\sqrt{\mu}}) + h_0 \quad (14)$$

Here, h_0 is the initial damping factor and β is the viscous damping index. Calculated values of h_{eq} to μ with various β are shown in Figure 6.

Viscous damping index β is assumed as follows:

- 0.01 shear failure type reinforced concrete structure
- 0.1 frame type reinforced concrete structure with shear wall
- 0.15 frame type reinforced concrete structure with slipping of reinforcing bar at beam-column joint
- 0.2 frame type reinforced concrete structure
- 0.25 frame type steel structure

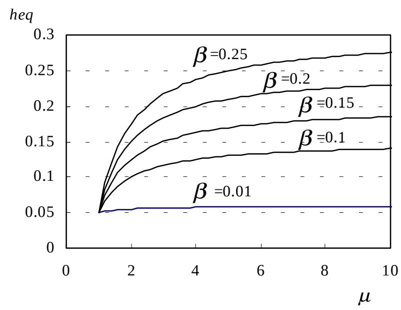


Figure 6 : Equivalent damping factor with ductility factor μ

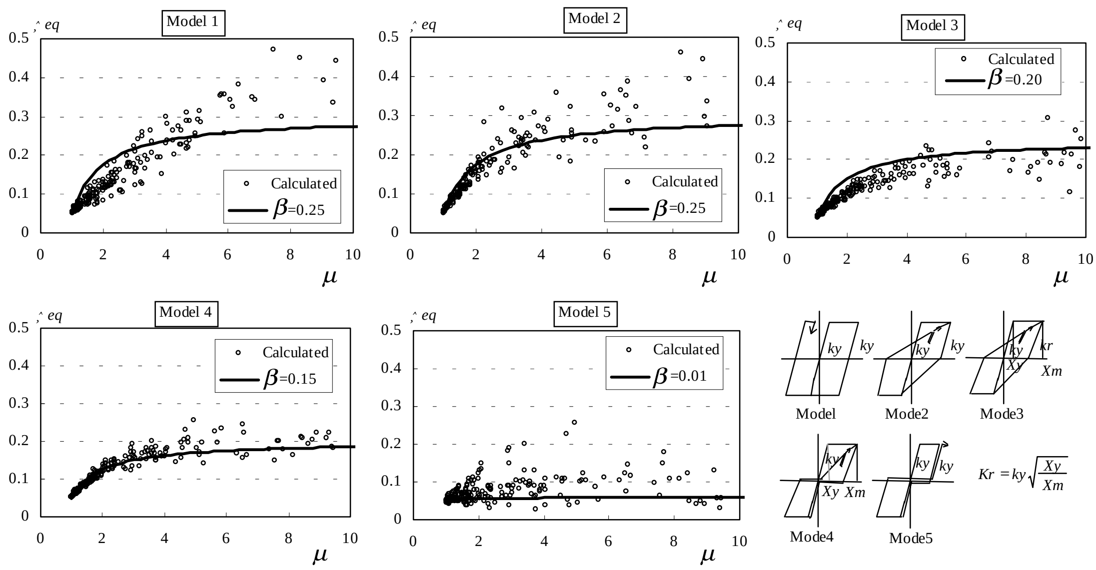


Figure 7 : Comparison of equivalent damping ratio

The relation between ductility factors and the equivalent damping factors defined by Takeda [Takeda 1976] is shown in Figure 7. The single-degree-of-freedom system is used for the numerical calculation with El Centro NS, Hachinohe EW, and Sendai501 ground motions. Five kinds of the bi-linear type hysteresis model shown in the figure are used with two types of second stiffness of 0.1% and 5% of the initial stiffness. The initial period was set as 1/3, 2/3, 1, 2, and 3 times T_g. The strength was set to 10 stages similarly previous section. Damping is assumed to be a function of instantaneous stiffness with an initial value of 5%. The proposed equivalent viscous damping ratio agrees well with the calculated results.

For the design displacement response spectrum with 5% initial damping, Equation (15) is used instead of Equation (3)[Inoue 1988].

$$\frac{S_D(h_{eq})}{S_D(0.05)} = \frac{2.25}{1.75 + 10h_{eq}} \quad (15)$$

Accordingly,

$$\frac{S_D(h_{eq})}{S_D(0.05)} = \frac{9\sqrt{\mu}}{(9+40\beta)\sqrt{\mu} - 40\beta} \quad (16)$$

3.2 Elasto-plastic Displacement Response Estimation by the Equivalent Linear Method

Using Equation (16) instead of Equation (4), the Equations (9) (11) and (13) become Equations (17) (18) (19).

a•$T_0 > T_g , T_{ef} > T_g$

$$DR = \frac{(9+40\beta SR)^2}{SR(9+40\beta)^2} \quad (17)$$

b•$T_0 < T_g$ •$T_{ef} < T_g$

$$DR = \frac{SR(40\beta SR)^2}{(9SR + 40\beta SR - 9)^2} \quad (18)$$

Equation (18) has nomeaning in the region of $SR < 9/(9+40\beta)$.

c•$T_0 < T_g$ •$T_{ef} > T_g$

$$DR = \frac{(9+40\beta SR)^2}{SR(9+40\beta)^2} \cdot \frac{1}{TR} \quad (19)$$

At $T_0 < T_g$, DR is the minimum of Equations (18) and (19).

3.3 Elasto-plastic Displacement Response

Figure 8 shows the relation between displacement response ratio *DR* and strength ratio *SR* for three dimensionless initial periods to the characteristic period T_g. The system with a long period shows few differences of the displacement response with differing the equivalent viscous damping.

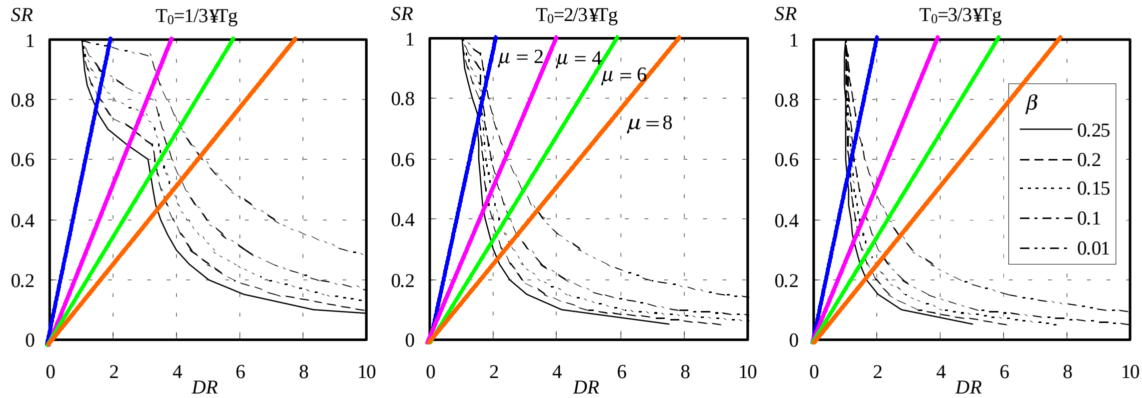


Figure 8 : Estimated displacement response

4. EVALUATION OF THE STRUCTURAL COEFFICIENT

4.1 Define Equation of the Structural Coefficient

The ductility factor is given by Equation (20) as a function of *DR* and *SR* for the idealized bi-linear type hysteresis model.

$$\mu = DR / SR \quad (20)$$

The relations of μ=2, 4, 6, 8 are drawn in Figure 8. The cross points are the values of structural coefficient *SC* (=*SR*) for the system with the deformation capability (allowable ductility factor), certain initial period and equivalent viscous damping.

The structural coefficient *SC* is given from Equations (17) (18) (19) and (20) as a function of the allowable ductility factor μ, equivalent viscous damping index β, and initial period ratio *TR*.

a•$T_0 > T_g, T_{ef} > T_g$

$$SC = \frac{360\beta + 9(9+40\beta)\sqrt{\mu}}{81\mu + 720\mu\beta - 1600\beta^2 + 1600\mu\beta^2} \quad (21)$$

b•$T_0 < T_g$ •$T_{ef} < T_g$

$$\mathrm{SC} = \frac{9\mu(9+40\beta) + 360\beta\sqrt{\mu}}{81\mu + 720\mu\beta - 1600\beta^2 + 1600\mu\beta^2} \quad (22)$$

c•$T_0 < T_g$ •$T_{ef} > T_g$

$$SC = \frac{360\beta + 9(9+40\beta)\sqrt{\mu TR}}{81\mu TR + 720\mu\beta TR - 1600\beta^2 + 1600\mu\beta^2 TR} \quad (23)$$

At $T_0 < T_g$, *SC* is the minimum of Equations (22) and (23). Although these equations are complicated, the calculation can be done by using a table calculation program, etc.

4.2 Structural Coefficient based on Response Displacement

Structural coefficient *SC* with parameters of the period ratio *TR*, ductility factor μ, and equivalent viscous damping index β are shown in Table 2 and Figures 9, 10, and 11.

The structural coefficient *SC* has the following trends for each parameter.

1) Increases like a hyperbola having an upper limit as the period ratio *TR* decreases.
2) Reduces like a straight line as equivalent viscous damping coefficient index β increases.
3) Reduces like a straight line or exponential function having an upper bound as allowable ductility factor μ increases.

Table 2 : Structural coefficient *Ds*

β	0.01				0.1				0.15				0.2				0.25			
TR μ	2	4	6	8	2	4	6	8	2	4	6	8	2	4	6	8	2	4	6	8
≧1	0.70	0.49	0.4	0.34	0.63	0.41	0.32	0.27	0.59	0.38	0.29	0.25	0.56	0.35	0.27	0.22	0.53	0.32	0.25	0.21
0.7	0.84	0.59	0.48	0.41	0.79	0.51	0.4	0.34	0.77	0.47	0.36	0.31	0.74	0.44	0.34	0.28	0.72	0.41	0.31	0.26
0.3	0.99	0.91	0.74	0.64	0.88	0.82	0.67	0.56	0.84	0.75	0.64	0.52	0.79	0.69	0.61	0.49	0.75	0.64	0.58	0.46

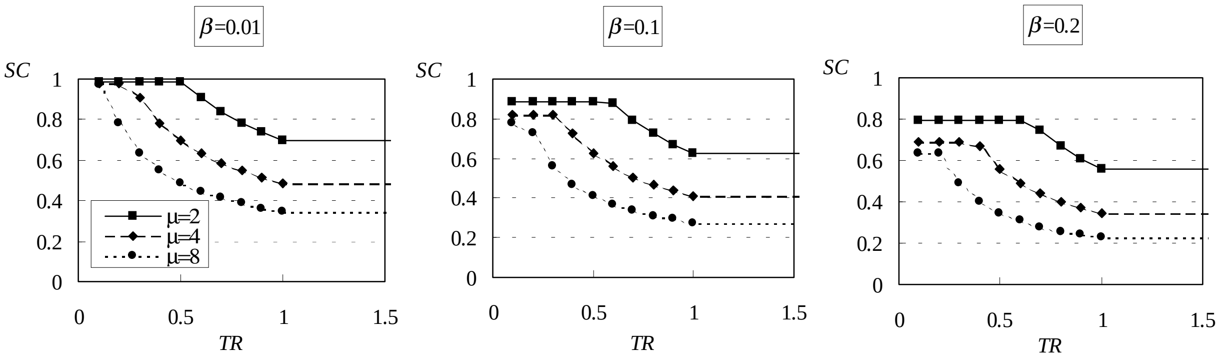


Figure 9 : The relation between *TR* and *SC*

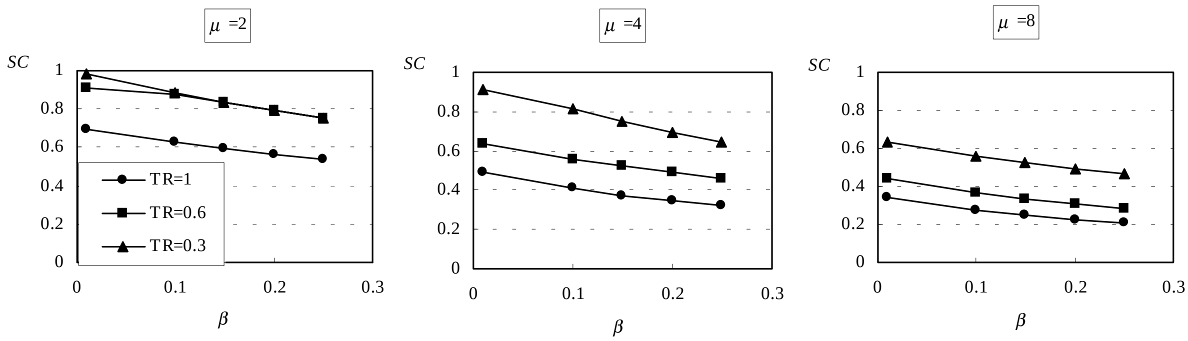


Figure 10 : The relation between β and *SC*

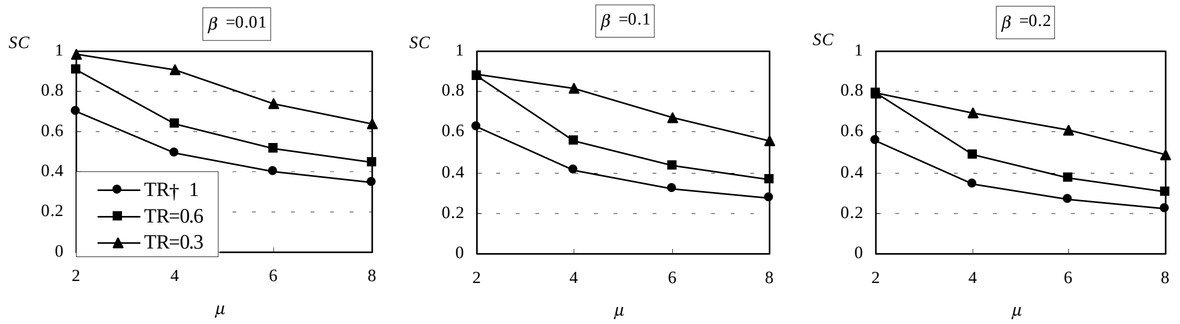


Figure 11 : The relation between μ and *SC*

The proposed *SC* is obtained for the idealized bi-linear hysteresis model. An existing structure usually can not be modeled as this type. This equation is applicable for *SC* using an equivalent bi-linear hysteresis model to define equal energy absolute ability.

5. CONCLUSION

This paper proposed a method for determining the structural coefficient for the estimated value of the response displacement using an equivalent linear method. The structural coefficient *SC* has the following trends.

1) Increases like a hyperbola having an upper limit as the period ratio *TR* decreases.
2) Reduces like a straight line as equivalent viscous damping coefficient index β increases.
3) Reduces like a straight line or exponential function having an upper bound as allowable ductility factor μ increases.

REFERENCES

Inoue, Y., et al. (1988), "A proposal for seismic design procedure of apartment houses including soil-structure interaction effect," *Proceedings of 9th WCEE,* VIII pp.365-370

Moehle, J.P. (1984), "Strong motion drift estimates for R/C structures," *ASCE,* ST, Vol.110, No.9, pp.1988-2001

Shibata, A. et al (1976), "Substitute structure method for seismic design in R/C," *ASCE*, ST, Vol.102, No.1, pp.1-18

Shimazaki, K. 1988), "Strong ground motion drift and base shear coefficient for R/C structures," *Proceedings of 9th WCEE*, V pp.165-170

Shimazaki, K. (1992), "Seismic coefficient distribution of high-rise reinforced concrete buildings," *Proceedings of 10th WCEE*, pp.4281-4286.

Shimazaki, K. (1996), "A primary design method oriented to the displacement response for high-rise reinforced concrete frame buildings subjected to earthquakes," *Proceedings of 11th WCEE*, Paper 630.