0273

SEISMIC ANALYSIS OF FRAMES WITH SEMI-RIGID ECCENTRIC CONNECTIONS

Miodrag S SEKULOVIC[1], Ratko M SALATIC[2] And Rastislav S MANDIC[3]

SUMMARY

This paper deals with the effects of hysteretic and viscous damping in plane frames with semi-rigid connections. A flexible connection is modelled by rotational spring with non-linear moment rotation relationship using three-parameter power model. Viscous damping due to relative rotational velocity in connections is also introduced. The flexural complex stiffness matrix for a beam element with semi-rigid connections and viscous rotational dashpots is evaluated. The changes in element stiffness matrix introduced by eccentricity due to finite dimensions of beam-column assemblage have been also taken into account. Several examples are presented in order to show the influence of various parameters on hysteretic response and energy dissipation lumped in the connections of steel frames subjected to earthquake loading.

INTRODUCTION

Conventional analysis of frames is usually performed under assumption that a connection joining beam to column is either infinitely rigid or perfectly pinned. The experimental results show that the actual response of joints is far from the above idealisation. All connections transmit some moments and exhibit certain degree of flexibility. The concept of semi-rigid or flexible connections is recognized in several national codes for steel structures. The behavior of connection is described by non-linear moment-relative rotation relationship, which is usually determined by experiments. In structures subjected to dynamic loading the response is highly influenced by the behavior of connection (plastic yielding, local buckling and distortions). Appropriate hysteretic model is necessary in order to study this phenomenon. The area enclosed by hysteretic loops during the cycling is proportional to the energy dissipation and is direct measure of the hysteretic damping. However, apart from hysteretic damping some other forms of energy dissipation in a connection are present. They mainly result from friction between the elements forming the beam-column assemblage and may be modelled as viscous damping.

To study the process of energy dissipation in connection a stiffness matrix of a beam with semi-rigid and viscous connection has been derived. Independent hardening model [Chen and Saleb, 1982] has been used in order to model hysteretic response of connections. The influence of viscous damping dissipation in the connections has been also considered. An extensive parametric study has been performed in order to find out the influence of various parameters on the dynamic response due to earthquake loading.

A BEAM WITH SEMI-RIGID AND VISCOUS CONNECTIONS

Apart from non-linear rotational springs, rotational viscous dashpots are attached at beam ends (Fig.1). The total moment at each connection (i=1,2) are given in terms of relative rotation θ between beam end and column face (connection deformation) and relative angular velocity:

$$M_i(t) = k_i \theta(t) + c_i \dot{\theta}(t) \qquad i = 1',2' \tag{1}$$

[1] *Faculty of Civil Engineering, University of Belgrade, Yugoslavia*
[2] *Faculty of Civil Engineering, University of Belgrade, Yugoslavia*
[3] *Faculty of Civil Engineering, University of Belgrade, Yugoslavia*

where k_i and c_i are rotational spring stiffness and rotational viscous damping coefficients, while 'dot' denotes differentiation with respect to time. The tangent or secant form of the above relation may be written if non-linear springs and/or dashpots are considered. In the case of periodic response with circular frequency ω the following relation between the amplitudes may be developed:

$$M_{i(0)} = k_i^* \, \dot{\theta}_{i(0)} \qquad i = 1',2' \tag{2}$$

where complex stiffness of the connection is defined as the ratio between moment and relative rotation amplitudes:

$$k_i^* = k_i + j\,\omega c_i \qquad j = \sqrt{-1} \tag{3}$$

The relation between the amplitudes of element end force vector $\overline{\mathbf{R}}_0^{\,T} = \{\overline{T}_{1(0)}, \overline{M}_{1(0)}, \overline{T}_{2(0)}, \overline{M}_{2(0)}\}$ and the element end displacement vector can be expressed as:

$$\overline{\mathbf{R}}_0 = \mathbf{K_0}\,(\overline{\mathbf{q}}_0 - \boldsymbol{\theta}_0) \tag{4}$$

$$\text{where:} \quad \begin{aligned} \overline{\mathbf{q}}_0^{\,T} &= \left\{\overline{v}_{1(0)},\ \ \overline{\varphi}_{1(0)},\ \ \overline{v}_{2(0)},\ \ \overline{\varphi}_{2(0)}\right\}, \\ \boldsymbol{\theta}_0^{\,T} &= \left\{0,\ \ \theta_{1(0)},\ \ 0,\ \ \theta_{2(0)}\right\} \end{aligned} \tag{5}$$

are the vectors of displacement amplitudes and amplitudes of connection deformations of points 1' and 2' (Fig. 1), while **Ko** is classical beam element flexural stiffness matrix.

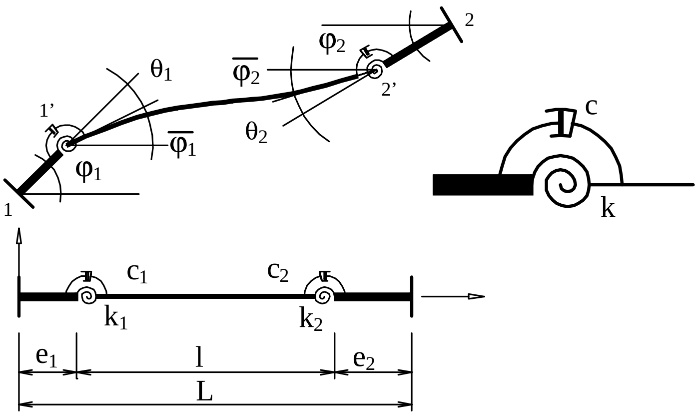

Figure 1: Deformation of a beam element

The finite width of frame nodal assemblages is modelled using an infinitely stiff short element. Assuming that nodal rotations are small we have:

$$\overline{\mathbf{q}}_0 = (\mathbf{I} + \mathbf{E})\,\mathbf{q}_0 \tag{6}$$

where **I** is identity matrix. The eccentricity matrix **E** and the vector of nodal (points 1 and 2 - Fig. 1) displacements amplitudes are:

$$\mathbf{E} = \begin{bmatrix} 0 & e_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -e_2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{q}_0^{\,T} = \left\{v_{1(0)},\ \varphi_{1(0)},\ v_{2(0)},\ \varphi_{2(0)}\right\} \tag{7}$$

From the above equations the following complex flexural stiffness in terms of nodal displacements is derived:

$$\mathbf{K}_{el}^{*} = \mathbf{K}^{*} + \mathbf{E}^{T}\mathbf{K} + \mathbf{E}\mathbf{K}^{*} + \mathbf{E}^{T}\mathbf{K}^{*}\mathbf{E} \tag{8}$$

Combining element bending stiffness with element axial stiffness, beam-column element stiffness matrix may be obtained. The stiffness matrix given with Eqn. (8) includes the influence of flexible and viscous connections that are lumped at distances e_1 and e_2 from nodal points. In the case when no eccentricities are present ($e_1 = e_2 = 0$), the evaluated stiffness matrix transforms to the complex matrix **K*** with the coefficients:

$$K_{11}^{*} = -K_{13}^{*} = K_{33}^{*} = \frac{12EI}{l^3}\frac{1+g_1^{*}+g_2^{*}}{\Delta^{*}} \qquad K_{14}^{*} = -K_{34}^{*} = \frac{6\,EI}{l^2}\frac{1+2g_1^{*}}{\Delta^{*}}$$

$$K_{12}^{*} = -K_{23}^{*} = \frac{6\,EI}{l^2}\frac{1+2g_2^{*}}{\Delta^{*}} \qquad K_{44}^{*} = \frac{6\,EI}{l}\frac{1+2\,g_1^{*}}{\Delta^{*}} \tag{9}$$

$$K_{22}^{*} = \frac{4\,EI}{l}\frac{1+3g_2^{*}}{\Delta^{*}} \qquad K_{24}^{*} = \frac{4\,EI}{l}\frac{1}{\Delta^{*}} \qquad K_{jk}^{*} = K_{kj}^{*}$$

The coefficients of matrix **K*** are the functions of the complex stiffness parameters:

$$g_i^{*} = \frac{EI}{l}\frac{1}{k_i + j\omega c_i} \qquad i = 1',2'$$
$$\Delta^{*} = 1+4\,g_1^{*}+4\,g_2^{*}+12\,g_1^{*}\,g_2^{*} \tag{10}$$

where EI and l are the flexural rigidity of a beam and beam length. Expanding the elements of complex stiffness matrix **K*** in Taylor series with respect to ω it follows:

$$\mathbf{K}^{*} = \mathbf{K} + j\,\omega\mathbf{C} \;+\; \text{(higher order terms)} \tag{11}$$

The matrices **K** and **C** are real and they have important physical meaning. The matrix **K** represents well known element stiffness matrix for a beam with flexible springs at both ends in the case when viscous dashpots are omitted. It can be obtained from Eqn. (9) substituting the complex stiffness parameters with the real stiffness parameters resulting from rotational springs:

$$g_i = \frac{EI}{l\,k_i} \qquad i = 1',2' \tag{12}$$

Note that if neither eccentricities nor springs and dashpots are present, the matrix (8) transforms to classical beam element flexural stiffness matrix **K**o. The matrix **C** is consistent element damping matrix. The elements of the matrix **C** are:

$$C_{11} = -C_{13} = C_{33} = \frac{36EI}{l^3}\frac{h_1+h_2+4g_1\,h_2+4g_1^2\,h_2+4g_2\,h_1+4g_2^2\,h_1}{\Delta^2}$$

$$C_{12} = -C_{23} = \frac{12EI}{l^2}\frac{2h_1+h_2+2g_1\,h_2+10g_2\,h_1+12g_2^2\,h_1}{\Delta^2}$$

$$C_{14} = -C_{34} = \frac{12EI}{l^2}\frac{h_1+2\,h_2+10g_1\,h_2+12\,g_1^2\,h_2+4\,g_2\,h_1}{\Delta^2} \tag{13}$$

$$C_{22} = \frac{4\,EI}{l}\frac{4\,h_1+h_2+24g_2\,h_1+36\,g_2^2\,h_1}{\Delta^2}$$

$$C_{24} = \frac{8EI}{l} \frac{h_1 + h_2 + 3\,g_1\,h_2 + 3\,g_2\,h_1}{\Delta^2}$$

$$C_{44} = \frac{4EI}{l} \frac{h_1 + 4h_2 + 24g_1\,h_2 + 36\,g_1^2\,h_2}{\Delta^2} \qquad\qquad C_{jk} = C_{kj}$$

where: $\quad h_i = \dfrac{c_i EI}{l\,k_i^2} \quad i = 1',2' \qquad \Delta = 1 + 4\,g_1 + 4\,g_2 + 12\,g_1\,g_2$

It is important to say that in the expansion of complex stiffness (11) the terms which are proportional to ω^2 are real so they obviously contribute to the consistent element mass matrix. However, these terms have been neglected as only the problems with lumped (nodal) masses have been considered. The influences of higher order terms in the Eqn. (11) are still under investigation and are not discussed in this paper.

SEMI-RIGID CONNECTION MODELING

Numerous experimental results have shown that the connection moment-rotation relationships are non-linear over the entire range of loading for almost all types of connections. To describe connection behavior, different mathematical models have been proposed. In this study, the three parameter power [Richard and Abbott, 1975] model is used to represent moment-rotation behavior of the connection under monotonic loading. The generalised form of this model is:

$$M = \frac{k_o\,\theta}{\left[1 + \left(\dfrac{\theta}{\theta_0}\right)^n\right]^{\frac{1}{n}}} \tag{14}$$

where M_u is ultimate moment capacity, n shape parameter, k_o initial connection stiffness and $\theta_o = M_u / k_o$ reference plastic rotation. The shape of equation (14) for the two types of connection (Double Web Angle - DWA, Top and Seat angle with Double Web Angle - TSDWA) is shown in Fig. 2a. The details of these connection can be seen in reference [Chen, Goto and Liew, 1996].

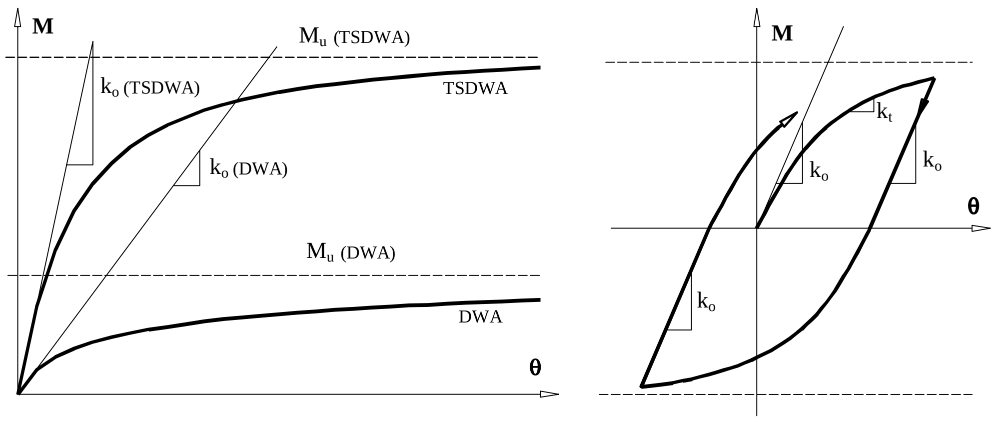

Figure 2: Three parameter power model (left). Independent hardening model (right)

The independent hardening model was adopted to express connection behavior under cyclic loading. Cyclic moment-rotation relation based on this model is schematically shown in Fig. 2b. The skeleton curve used in the model was obtained from three parameter power model.

SOLVING EQUATIONS OF MOTION

The equations of motion of a frame subjected to base excitation can be written in the following form:

$$\hat{\mathbf{M}}\,\Delta\ddot{\mathbf{U}} + \hat{\mathbf{C}}\,\Delta\dot{\mathbf{U}} + \hat{\mathbf{K}}\,\Delta\mathbf{U} = -\hat{\mathbf{M}}\,\Delta\ddot{\mathbf{u}}_g \qquad (15)$$

where $\hat{\mathbf{M}}$ is diagonal system mass matrix, $\hat{\mathbf{C}}$ system damping matrix and system $\hat{\mathbf{K}}$ secant stiffness matrix, $\Delta\ddot{\mathbf{U}}$, $\Delta\dot{\mathbf{U}}$ and $\Delta\mathbf{U}$ are increments of accelerations, velocities and displacements relative to base. System matrices are established from element matrices in usual way. The equations of motions are integrated using well known Newmark average acceleration method. Within each time step iterative algorithm based on evaluating secant stiffness matrix has been performed to resolve unbalanced forces to zero.

PARAMETRIC STUDY AND CONCLUSIONS

The parametric study was performed with an example of simple portal steel frame (Fig.3). The frame was subjected to Montenegro earthquake 1979 (N-S component, maximum ground acceleration 4.29m/sˆ2) lasting for four seconds. The system response was analysed during twelve seconds.

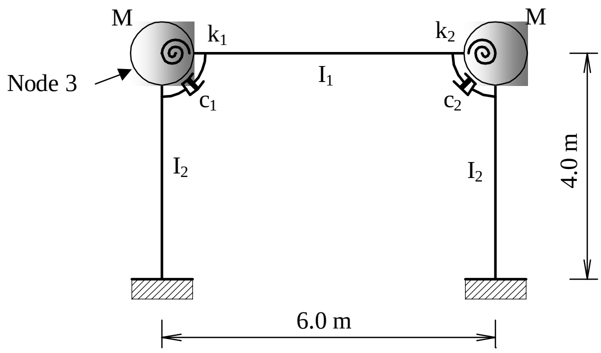

Figure 3: Geometry of semi-rigid simple portal frame

Two types of non-linear semi-rigid connections (DWA and TSDWA) have been analysed. Linear spring with initial stiffness of TSDWA model has also been considered. The viscous damping coefficient has been varied between 0 and 290 kNm s/rad.

It is obvious, that in problems with non-linear semi-rigid connections, damping results from both hysteretic and viscous phenomena. In frames with flexible connections (connection type DWA), the influence of hysteretic damping on amplitude decay is more significant than in a rigid frame (Fig. 5). It is recognized from Fig. 4, that in models where viscous damping in connections is introduced, the level of residual deformation is increased. Figures 6 and 7 show how hysteretic loops converge to different residual deformation in frames with and without viscous dashpots in joints where TSDWA connection type is used. As far as we know, no reliable experimental data concerning viscous dumping in semi-rigid connections are available. In order to predict frame response and residual deformation with more accuracy, some tests have to be conducted to obtain real values of viscous damping coefficients for different connection types.

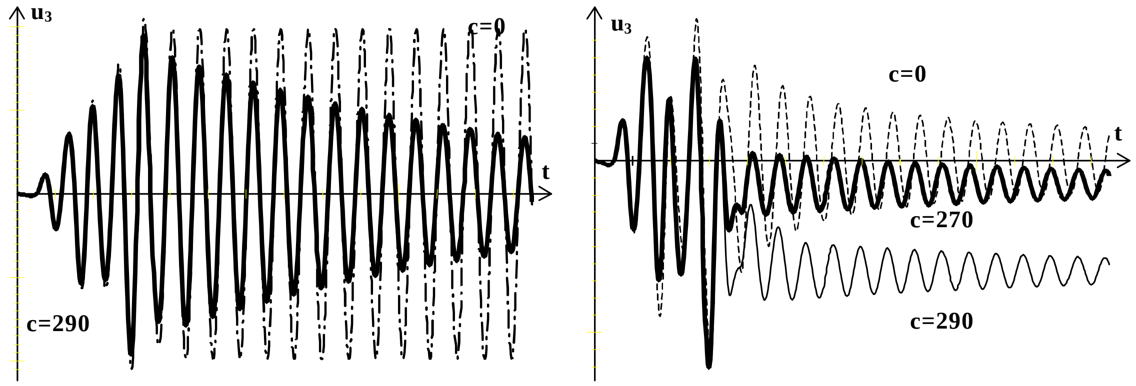

Figure 4: Horizontal displacement of node 3. Connection type: linear with initial stiffness of TSDWA (left) Nonlinear TSDWA (right)

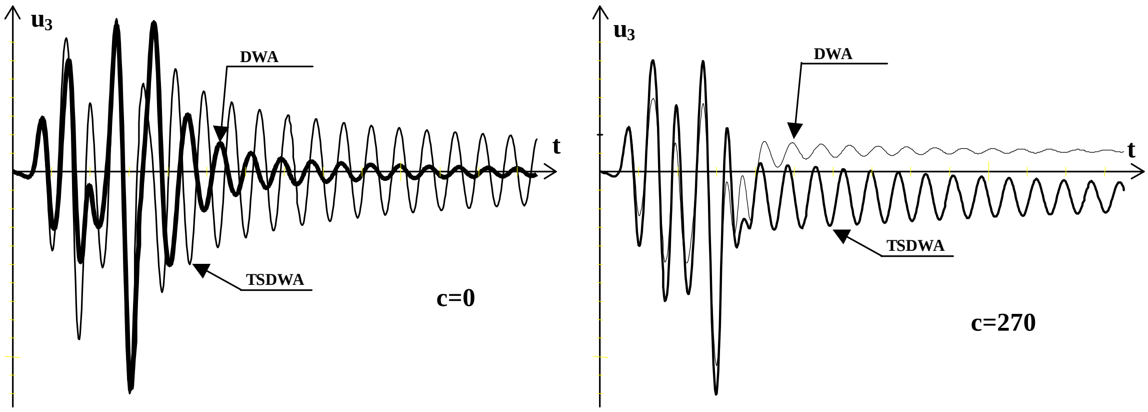

Figure 5: Horizontal displacement of node 3 in the case of nonlinear spring for two types of connections

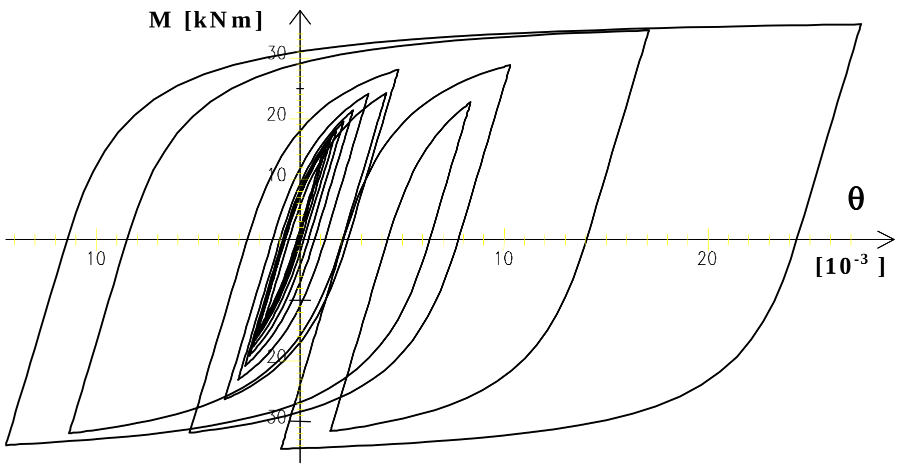

Figure 6: Hysteretic loops for spring moment in node 3, connection type TSDWA, c = 0

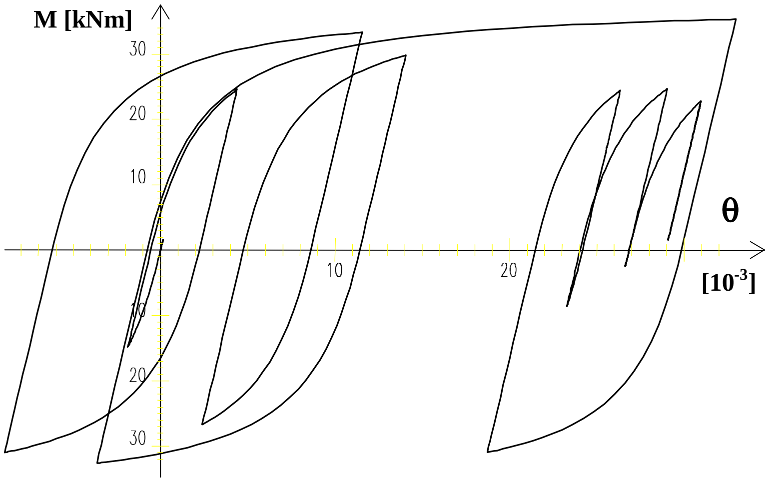

Figure 7: Hysteretic loops for spring moment in node 3, connection type TSDWA, c =290 kNm s/rad

REFERENCES

Ang, K.M.and.Morris, G.A (1984), "Analysis of three-dimensional frame with flexible beam-column connection", Canadian Journal of Civil Engineering, 11, pp 241-254.

Chan, S.L and Ho, G.W.M. (1991), "Nonlinear vibration analysis of steel frames with semirigid connections", ASCE Journal of Structural Engineering, 120, pp1075-1087.

Chen, W.F., Goto, Y. and Richard Liew, J.Y. (1996), *Stability design of semi-rigid frames,* John Wiley & Sons Inc. New York.

Chen, W.F. and Lui, E.M. (1987), "Effect of joint flexibility on the behavior of steel frames", Computers and Structures, 26, pp 719-732.

Chen, W.F. and Saleeb, A.F. (1982),"Uniaxial behavior and modeling in plasticity", Structural Engineering Report No. CE-STR-82-32, School of Civil Engineering Purdue University.

Dhillon, B.S.and Abdel-Majid, S. (1990), "Interactive analysis and design of flexibility connected frames", Computers and Structures, 36, pp189-202.

Jones, S.W., Kirby, P.A. and Nethercot ,D.A. (1983), "The analysis of frames with semi-rigid connections" – a state-of-the-art report, Journal of Constructional Steel Research, 3, pp 2-13.

Kawashima, S. and Fujimoto, T. (1984), "Vibration analysis of frames with semi-rigid connections", Computers and Structures, 19, pp 85-92.

Richard, R.M and Abbott, B.J. (1975), "Versatile elastic-plastic stress-strain formula", ASCE Journal of Engineering Mechanics Division, 101 (EM4), pp 511-515.

Shi, G.and Atlyri, S.N. (1989), "Static and dynamic analysis of space frames with non-linear flexible connections" International Journal of Numerical Methods in Engineering, 28, pp 2635-2650.

Suarez, L.E., Singh, M.P. and Matheu, E.E. (1996), "Seismic response of structural frameworks with flexible connections", Computers and Structures, 58, pp 27-41.