RELIABILITY OF IDENTIFIED DYNAMIC SOIL PROPERTIES OF SUBSURFACE LAYERS IN GROUND BY VERTICAL ARRAY RECORDS

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SUMMARY

This paper describes the reliability of dynamic soil properties such as shear wave velocity and quality factor of subsurface layers that have been identified by using vertical array records of earthquake ground motions. It is assumed that horizontally layered ground is excited by vertically incident shear wave. Propagation law of errors is applied to estimate the variance of unknown properties. It is shown in numerical analysis that the reliability of estimated properties could be evaluated by the propagation low of errors.

INTRODUCTION

Strong ground motions are largely affected by the amplification effect of subsurface layers of the ground. Therefore, it is very important to estimate dynamic soil properties of subsurface layers in order to predict the characteristics of strong ground motion that influence the behavior of structures based on ground or lifeline facilities buried underground. The dynamic soil properties such as shear wave velocity for small amplitude of vibration can be estimated by borehole test *etc*. However, the values estimated by such method are rather different from those during strong earthquake ground motions.

In recent years, a lot of array observations of strong ground motions have been carried out, and several studies (Ohta,1975, Tsujihara *et al.*,1996, Sato *et al.*,1994, Annaka *et al.*,1994, Yoshida *et al.*,1995, *etc.*) have been done on identification of dynamic soil properties of subsurface ground using the vertical array records. However, few researchers have studied about the reliability of estimated values for unknown properties in identification. The influence of measurement noise on the accuracy of identified shear wave velocity and quality factor of each layer in horizontally layered subsurface ground have been studied by Tsujihara *et al*(1993). The method has been developed by Kurita *et al.*(1997) to estimate directly the dynamic soil properties through the sensitivity analysis using the vertical array records that were contaminated by noises, assuming that the noises are known.

In this study, we develop the theory to estimate the reliability of dynamic soil properties such as shear wave velocity and quality factor of subsurface layers identified by using vertical array records of the ground motions that may be contaminated by unknown noise during an earthquake.

2. FORMULATION

2.1 Multiple reflection theory

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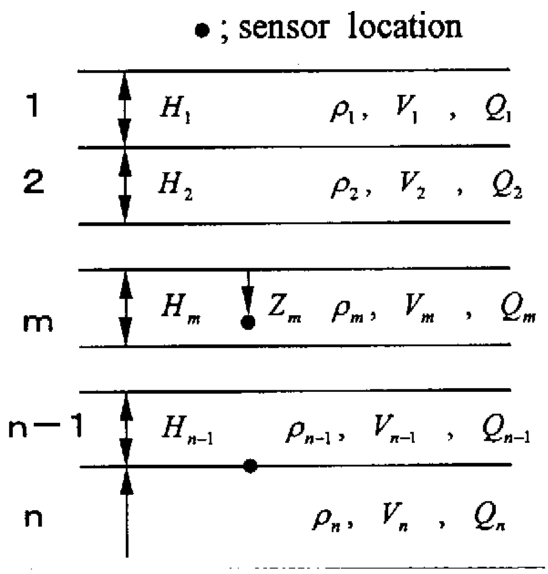

Figure 1 Horizontally layered ground model

The horizontally layered subsurface ground is assumed to be excited by vertical incident SH wave. Consider the identification of subsurface ground model as shown in Figure 1, in which H, ρ, V and Q denote the thickness, density, shear wave velocity and quality factor, respectively. The quality factor is assumed to be frequency-independent. The displacement and shear stress at the points p in the p-th layer and q in the q-th layer are represented by multiple reflection theory as follows (Haskell, 1960;Toki,1981).

$$\begin{Bmatrix} u_p \\ \tau_p \end{Bmatrix} = \left[R_p\right] \begin{Bmatrix} u_0 \\ 0 \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} u_q \\ \tau_q \end{Bmatrix} = \left[R_q\right] \begin{Bmatrix} u_0 \\ 0 \end{Bmatrix} \tag{2}$$

where u_p, u_q and u_0 denote the displacement at the points p, q and ground surface, respectively. τ_p and τ_q denote the shear stress at the points p and q, respectively. $[R_p]$ and $[R_q]$ are the 2×2 matrices obtained by

$$[R_p] = [T_p][S_{p-1}]\cdots[S_1] \tag{3}$$

$$[R_q] = [T_q][S_{q-1}]\cdots[S_1] \tag{4}$$

where $[S_m]$ is the 2×2 matrix representing the state of the m-th layer. The coefficients of the matrix are given by

$$\begin{cases} S_{m\cdot 11} = [\exp(ia_m\omega) + \exp(-ia_m\omega)]/2 \\ S_{m\cdot 12} = [\exp(ia_m\omega) - \exp(-ia_m\omega)]/(2ib_m\omega) \\ S_{m\cdot 21} = ib_m\omega[\exp(ia_m\omega) - \exp(-ia_m\omega)]/2 \\ S_{m\cdot 22} = S_{m\cdot 11} \end{cases} \tag{5}$$

in which $\omega(=2\pi f)$ is circular frquency, $i(=\sqrt{-1})$ is imaginary unit, and

$$a_m = H_m/(V_m\sqrt{1+i/Q_m}) \tag{6}$$

$$b_m = \rho_m V_m\sqrt{1+i/Q_m} \tag{7}$$

H_m, ρ_m, V_m and Q_m are the thickness, density, shear wave velocity and quality factor of the m-th layer, respectively. $[T_m]$ (m=p or q) in Equation (3) and (4) represents the similar matrix to $[S_m]$ but Z_m is used in place of H_m.

2.2 Identification of shear wave velocity and quality factor

Denoting Fourier spectra of the vertical array records at the points p and q(p<q) by $X_p(f)$ and $X_q(f)$, the amplitude of quasi transfer function between p and q can be obtained by

$$U_{pq}(f_j) = | X_p(f_j)/X_q(f_j) | \tag{8}$$

where f_j is the discrete frequency. On the other hand, theoretical transfer function can be derived from Equation (1) and (2) by

$$U_{pq}(f_j, \alpha) = | \gamma_p(f_j, \alpha)/\gamma_q(f_j, \alpha) | \tag{9}$$

where α denotes the unkown properties to be identified and $\gamma_m(f_j, \alpha)$ (m=p or q) denotes the (1,1) element of matrix $[R_m]$.

Then, identification problem of unknown properties such as shear wave velocity and quality factor of the layers upper of point q can be reduced to the problem of optimization, which is represented by

$$S(\alpha) = \sum_{j=1}^{N_f} \left\{ U_{pq}(f_j, \alpha) - U_{pq}(f_j) \right\}^2 \rightarrow \min \tag{10}$$

where N_f is the total number of discrete frequencies.

The objective function of Equation (10) can be replaced by the following equation, when the vertical array records are obtained at more than 2 points, namely, p, q ,$\cdots$, r(p<q$\cdots$<r),

$$S(\alpha) = \sum_{k=p,q,\cdots} \sum_{j=1}^{N_f} \left\{ U_{kr}(f_j, \alpha) - U_{kr}(f_j) \right\}^2 \rightarrow \min \tag{11}$$

2.3 Algorithm to evaluate accuracy of identified properties

In the followings, the standard deviations of the estimated values for unknown properties are derived assuming that $U_{pq}(f_j), j = 1,2,\cdots,N_f$ in Equation (8) can be observed.

Generally, the optimum estimates for unknown properties can be obtained by the following iterative manner.

$$\alpha_e = \alpha_0 + \hat{\alpha} \tag{12}$$

where α_e, α_0 and $\hat{\alpha}$ are optimum estimates, approximate values and correction for the unknowns, respectively. The correction is given by the least squares method.

$$\hat{\alpha} = -(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{L} \tag{13}$$

where $\mathbf{A}$ and $\mathbf{L}$ are the matrix of differential coefficients and vector of residuals, respectively. The subscript of 'T' denotes transposed operator. They are represented as follows.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N_f1} & a_{N_f2} & \cdots & a_{N_fN} \end{bmatrix} \tag{14}$$

$$a_{jk} = \partial U_{pq}(f_j;\alpha_c)/\partial \alpha_k \tag{15}$$

$$\mathbf{L} = \{l_1, l_2, \cdots, l_{N_f}\}^T \tag{16}$$

$$l_j = U_{pq}(f_j;\alpha_c) - U_{pq}(f_j) \tag{17}$$

where N is the total number of unknowns. Consider that $U_{pq}(f_j), j = 1,2,\cdots,N_f$ is contaminated by measurement error ε_j that consists of accidental noises and has the standard deviation σ without correlation.

$$\boldsymbol{\varepsilon} = \{\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_{N_f}\}^T \tag{18}$$

$$E\{\varepsilon_i\} = 0 \qquad (i = 1,2,\cdots,N_f) \tag{19}$$

$$E\{\varepsilon_i\varepsilon_j\} = \begin{cases} \sigma^2 & (i = j) \\ 0 & (i \neq j) \end{cases} \tag{20}$$

Denoting the error of $\hat{\alpha}$ by ε_α, the following equation can be derived from Equation (13).

$$\boldsymbol{\varepsilon}_\alpha = -(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\boldsymbol{\varepsilon} \tag{21}$$

Multiplying the transposition of ε_α to both sides of Equation (21), the following equation can be obtained.

$$\boldsymbol{\varepsilon}_\alpha\boldsymbol{\varepsilon}_\alpha^T = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\boldsymbol{\varepsilon}\,\boldsymbol{\varepsilon}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1} \tag{22}$$

The expectation of both sides can be shown using Equation (20) by

$$\begin{aligned} E\{\boldsymbol{\varepsilon}_\alpha\boldsymbol{\varepsilon}_\alpha^T\} &= (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T E\{\boldsymbol{\varepsilon}\,\boldsymbol{\varepsilon}^T\}\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1} \\ &= \sigma^2(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1} \\ &= \sigma^2(\mathbf{A}^T\mathbf{A})^{-1} \end{aligned} \tag{23}$$

where σ^2 is the variance of measurement error and its unbiased estimator m_0^2 is given by

$$m_0^2 = \frac{\sum_{j=1}^{N_f}\{U_{pq}(f_j;\alpha_c) - U_{pq}(f_j)\}^2}{N_f - N} \tag{24}$$

The left side of Equation (23) can be represented by

$$\begin{aligned} E\{\boldsymbol{\varepsilon}_\alpha\boldsymbol{\varepsilon}_\alpha^T\} &= \begin{bmatrix} E\{\varepsilon_{\alpha1}^2\} & E\{\varepsilon_{\alpha1}\varepsilon_{\alpha2}\} & \cdots & E\{\varepsilon_{\alpha1}\varepsilon_{\alpha N}\} \\ E\{\varepsilon_{\alpha2}\varepsilon_{\alpha1}\} & E\{\varepsilon_{\alpha2}^2\} & \cdots & E\{\varepsilon_{\alpha2}\varepsilon_{\alpha N}\} \\ \vdots & \vdots & \ddots & \vdots \\ E\{\varepsilon_{\alpha N}\varepsilon_{\alpha1}\} & E\{\varepsilon_{\alpha N}\varepsilon_{\alpha2}\} & \cdots & E\{\varepsilon_{\alpha N}\varepsilon_{\alpha N}\} \end{bmatrix} \\ &= \begin{bmatrix} \sigma_{\alpha1}^2 & \sigma_{\alpha1\alpha2} & \cdots & \sigma_{\alpha1\alpha N} \\ \sigma_{\alpha2\alpha1} & \sigma_{\alpha2}^2 & \cdots & \sigma_{\alpha2\alpha N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\alpha N\alpha1} & \sigma_{\alpha N\alpha2} & \cdots & \sigma_{\alpha N}^2 \end{bmatrix} \end{aligned} \tag{25}$$

Therefore, the standard deviation $\sigma_{\alpha i}$ of unknown property α_i can be obtained by

$$\sigma_{\alpha i} = m_0\sqrt{(\mathbf{A}^T\mathbf{A})_{ii}^{-1}} \tag{26}$$

where $(\mathbf{A}^T\mathbf{A})_{ii}^{-1}$ represents the (i,i) coefficient of matrix $(\mathbf{A}^T\mathbf{A})^{-1}$.

3. NUMERICAL ANALYSIS AND DISCUSSION

Monte Carlo Simulation has been carried out in order to examine the validity of the theory to estimate the accuracy of identified properties.

Figure 2 shows the ground model. The accelerometers are assumed to be installed at the depth of 20m and ground surface. The acceleration at ground surface is calculated by the multiple reflection theory of SH wave excited by the acceleration at the depth of 20m which is simulated as a band-limited white noise(band width is 0.1～10Hz and maximum amplitude is 50gal). Then different band-limited white noise(band width is 0.1～10Hz and maximum amplitude is 10gal) is added to the acceleration at ground surface. 1000 different sets of earthquake ground motions have been simulated in such manner. Figure 3 shows an example of simulated earthquake ground motions.

The properties identified using the simulated earthquake ground motions in Figure 3 are shown in Table 1, in which the initial value given to each unknown property is larger than the exact value by 5%. Figure 4 shows the amplitude of spectral ratio of simulated earthquake ground motions and the transfer functions calculated with initial values and estimated values for unknown properties. Table 2 shows the standard deviations of identified properties that are estimated by Equation (26) and their coefficients of variation. The coefficients of

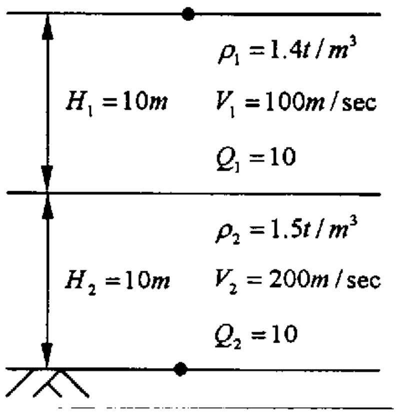

Figure 2 Analytical ground model

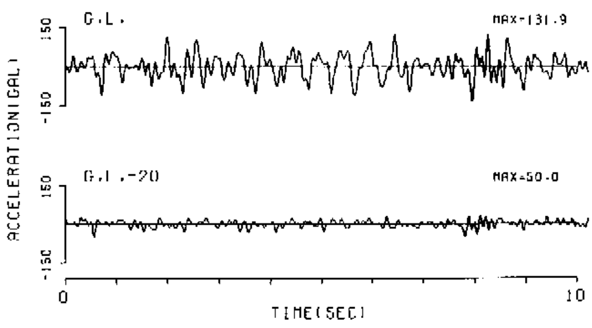

Figure 3 An example of simulated ground acceleration

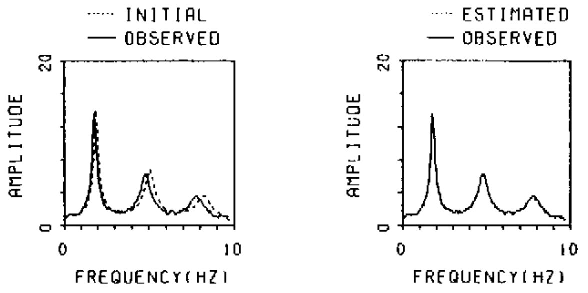

Figure 4 Spectral ratio of simulated ground motions and transfer functions of ground model

Table 1 Exact and estimated values for unknown properties

unknown property	exact value	estimated value
V_1	100.0	99.80
V_2	200.0	200.00
Q_1	10.0	10.74
Q_2	10.0	9.00

Table 2 Standard deviation and coefficient of variation for unknown properties

unknown property	standard deviation	coefficient of variation
V_1	0.269	0.00270
V_2	0.724	0.00362
Q_1	0.398	0.03706
Q_2	0.390	0.04333

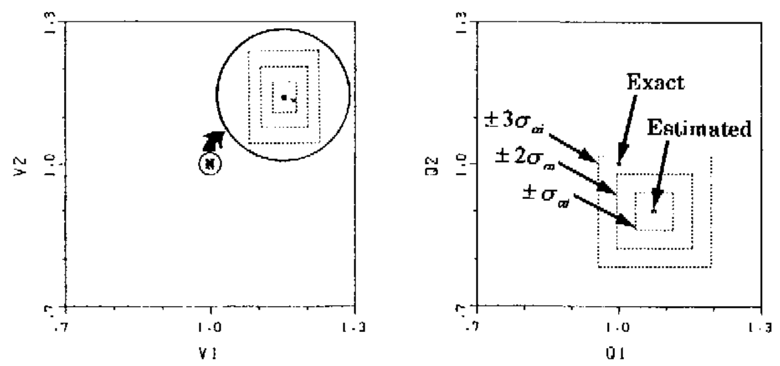

Figure 5 Estimated value and multiples of standard deviation for unknown property normalized by exact value

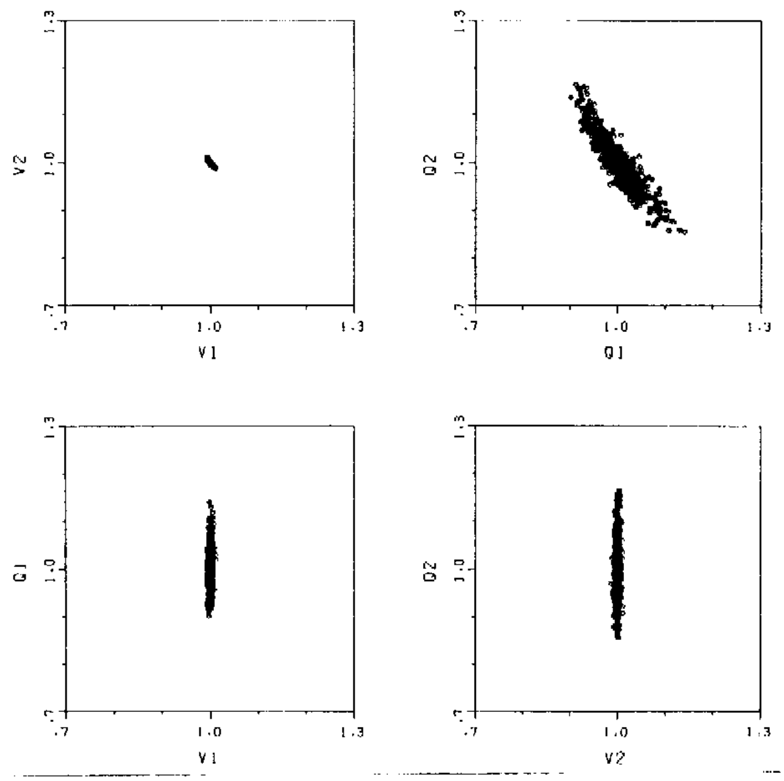

Figure 6 Estimated values for unknown properties using 1000 sets of simulated records

variation of quality factor are over 10 times as large as those of shear wave velocity. This fact means that accurate estimation of quality factor is more difficult than shear wave velocity. Figure 5 illustrates the estimated values for unknown properties and their standard deviations. They are normalized by the exact values. In this case, exact values for shear wave velocity are both within $\pm\sigma$, but those for quality factor are within $\pm 2\sigma$ and $\pm 3\sigma$ for Q_1 and Q_2 , respectively.

Figure 6 shows the dispersion of unknown properties identified by using 1000 sets of simulated earthquake ground motions. The estimated values for quality factor vary more widely than those for shear wave velocity. Table 3 shows the average of estimated values, standard deviations and coefficients of variation for unknown properties. Table 4 shows the total number of cases in which the exact value exist in between the estimated value $\pm\sigma$, $\pm 2\sigma$ and $\pm 3\sigma$, respectively. In every case of analyses, the ratio of $\varepsilon_{\alpha i}/\sigma_{\alpha i}$ (residual of estimated value and exact value for α_i / standard deviation calculated by Equation (26) for α_i) is calculated. Figure 7 shows the histograms of $\varepsilon_{\alpha i}/\sigma_{\alpha i}$. The theoretical curve of a standard normal distribution is also shown in

Table 3 The total number of cases in which the exact value for unknown properties exist in the range between estimated value ± the multiples of standard deviation

unknown property	estimated value ± the multiples of standard deviation		
	$\pm\sigma$	$\pm2\sigma$	$\pm3\sigma$
V_1	679	948	996
V_2	671	946	996
Q_1	694	944	998
Q_2	702	953	991

Table 4 Average of estimated values, standard deviations and coefficients of variation for unknown properties

unknown property	average value		
	estimated value	standard deviation	coefficient of variation
V_1	99.98	0.279	0.00279
V_2	200.05	0.728	0.00364
Q_1	10.04	0.362	0.03606
Q_2	9.98	0.488	0.04890

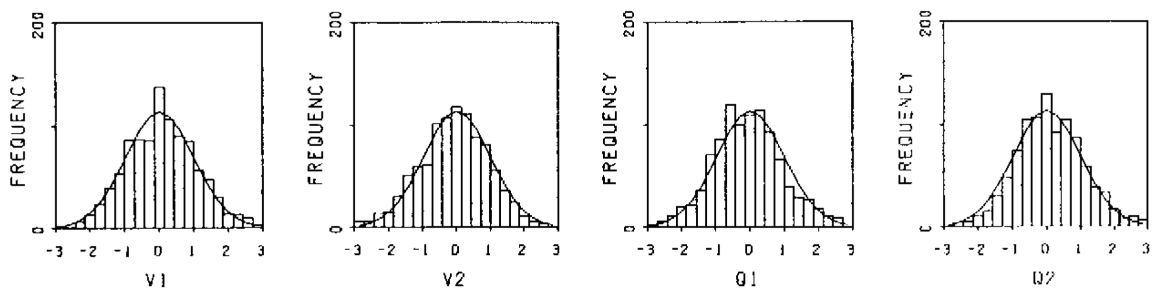

Figure 7 Histograms of estimation error for unknown properties normalized by exact values for unknown properties

Table 5 Value of χ_0^2

unknown property	exact value	estimated value
V_1	100.0	99.80
V_2	200.0	200.00
Q_1	10.0	10.74
Q_2	10.0	9.00

each graph. We carried out a test of goodness of fit by chai-square χ_0^2 with significant level of 1%. Table 5 shows the results of the test. Since all values of χ_0^2 are less than 37.57($=\chi^2_{0.01,20}$), the hypothesis that the histograms in Figure 7 obey the normal distribution can not be abandoned.

Thus, it has been demonstrated by the numerical analysis that the standard deviations of estimated values for unknown properties can be evaluated by Equation (26) when the amplitude of spectral ratio(Equation (10)) includes white noise. Therefore, the probability that the exact value exists in between the estimated value $\pm\sigma$, $\pm2\sigma$ and $\pm3\sigma$ is considered to be about 68.3%, 95.4% and 99.7%, respectively.

4. CONCLUSIONS

The theory to evaluate the accuracy of estimated values is formulated for identification of shear wave velocity and quality factor of subsurface ground using vertical array records of earthquake ground motions contaminated by unknown white noise. It has been demonstrated by Monte Calro Simulation that the present method is very effective.

The major results are as follows.

(1) The method has been presented that the reliability of identified properties can be evaluated by standard deviation for the estimated value, considering that white noise included in records comes out as the residual between the spectral ratio of vertical array records and the transfer function of the identified ground model.
(2) The coefficient of variation of quality factor is over 10 times as large as that of shear wave velocity, which indicates the difficulties of accurate identification of damping.
(3) The standard deviation calculated by the present method explains very well the relation between the exact values and estimated values for the identified properties.

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