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AN INTERMITTENTLY PROPAGATING SHEAR-DISLOCATION SEISMIC SOURCE MODEL

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SUMMARY

A simplified three-dimensional seismic source model is developed in this paper. Employing the methods of self-similar potentials and rotational superposition, it has been possible to find the analytical solution for the problem of an elliptical shear dislocation that propagates on a plane in a layered elastic half-space steadily or intermittently. The induced ground motions are shown to change considerably if the intermittent spreading is included in the dislocation model. As expected, a layered structure of the medium influences the character of the surface response significantly. The results suggest that intermittent faulting processes as well as local geology must be taken into account in the development of a reasonably earthquake source model.

INTRODUCTION

In evaluation of earthquake-resistant structures, it is a common practice to perform the dynamic analysis and find the response of the structures subjected to certain prescribed strong ground motions. The design is then evaluated and modified when necessary. Among the approaches that have been used to create a set of input ground motions for seismic analysis of structures is the development of a seismic source model. For shallow-focus earthquakes, it has been shown that the shear-dislocation models give results in good agreement with field geodetic measurements [Trifunac and Udwadia 1974].

Many studies on the modelling of fault-rupture processes using kinematic dislocation models have been carried out since the 1960s. Maruyama [1963], Aki [1968], and Haskell [1969] solved the problems with simple kinematic dislocation sources in an unbounded homogeneous medium. In 1964, Burridge and Knopoff developed a theory, which shows the equivalence of a double-couple point source to an impulsive dislocation. Since then, the response of a medium due to a double-couple point source that includes the effects of a free surface or layered crustal structures has been calculated by several authors using various techniques [Helmberger 1968; Kennett 1974; Swanger and Boore 1978; Hron and Mikhailenko 1981]. However, the usual approach to the double-couple point source when applied to actual problems requires a massive amount of computation due to a lengthy numerical integration over the entire fault surface. In a quite different approach, Seyyedian-Choobi and Robinson [1975] developed a two-dimensional analytical seismic source model using a finite propagating dislocation instead of a point source. With the aid of a simple idea of superposition of sources starting at different time and locations, Seyyedian-Choobi and Robinson showed that the induced motions change character considerably if the front of the propagating fault in the dislocation model is suddenly stopped.

In the present paper, the studies of Seyyedian-Choobi and Robinson are extended to include three-dimensional effects. A simplified three-dimensional geological model is used. The medium is assumed to consist of two layers: a horizontal layer of softer elastic material overlying a stiff homogeneous half-space (bedrock). The interface between these layers is taken parallel to the free surface. For the purpose of simulating fault-rupture processes, an elliptical dislocation source is considered to expand intermittently in the bedrock on a plane parallel to both the interface and the free surface. In order to solve this problem analytically, the intermittent propagation problem is decomposed into a series of problems starting at different time, which when added

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together in sequence represent the activation, the stopping, and the restarting of the dislocation propagation, respectively. Each problem is then solved by employing the methods of self-similar potentials and rotational superposition [Lai and Robinson 1996, 1997]. In addition, the solution for the problem of an elliptical shear dislocation that propagates on a plane in an unbounded homogeneous medium is found first. These results for a full space are then used as the incident waves for studies of the same dislocation source in the layered simplified geological model.

SHEAR-DISLOCATION SEISMIC SOURCE MODEL

In this study, a simplified geological model has been used. The medium is assumed to consist of two layers: a horizontal layer of softer elastic material overlying a stiff homogeneous half-space (bedrock), as shown in Fig. 1. The interface between these layers is taken parallel to the free surface. The elliptical shear-dislocation source is considered to originate at time t = 0 and then propagate intermittently on the $y = y_0$ plane in the bedrock. The boundary conditions on the dislocation plane are given by the following equations:

$$u_\rho(x,z,t)\big|_{y=y_0^-} = -u_\rho(x,z,t)\big|_{y=y_0^+} = u_x(\rho_b,t)\cos\omega$$

$$u_\omega(x,z,t)\big|_{y=y_0^-} = -u_\omega(x,z,t)\big|_{y=y_0^+} = -u_x(\rho_b,t)\sin\omega$$

$$\sigma_y = 0 \quad \text{for} \quad y = y_0 \tag{1}$$

where (ρ,ω,y) are cylindrical coordinates corresponding to the Cartesian coordinates (x, z, y) and $u_x(\rho_b,t)$ is the distribution function of the tangential relative displacements in the x-direction. At the time the dislocation

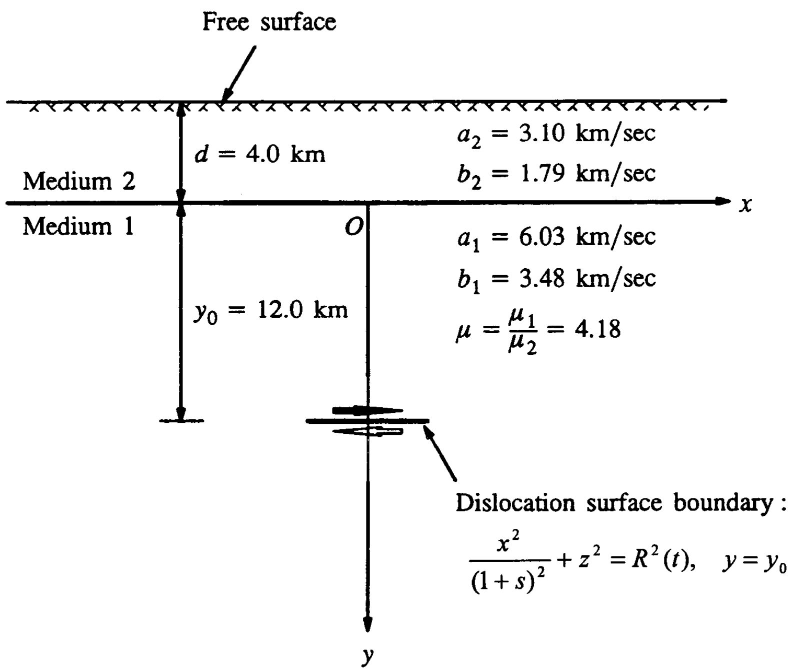

Figure 1 Elliptical shear-dislocation model in a layered elastic half-space

just starts expanding, the relative displacement function is taken as

$$u_x(\rho_b,t) = D\sqrt{\alpha_1{}^2 t^2 - \rho_b{}^2} \quad \text{for} \quad \rho_b \le \alpha_1 t$$
$$= 0 \quad \text{for} \quad \rho_b > \alpha_1 t \tag{2}$$

in which D is a dimensionless constant, α_1 is the initial propagating speed of the elliptical dislocation boundary along the minor axis, the parameter

$$\rho_b = \sqrt{\frac{x^2}{(1+s)^2} + z^2}\ , \tag{3}$$

and $1+s$ is the aspect ratio of the ellipse. After propagating steadily for a short period, the dislocation is assumed to stop expanding suddenly at time $t = t_1$. The relative displacements on the dislocation plane after the stopping occurs are assumed to remain unchanged, which gives

$$u_x(\rho_b,t) = D\sqrt{R_1{}^2 - \rho_b{}^2} \quad \text{for} \quad \rho_b \le R_1$$
$$= 0 \quad \text{for} \quad \rho_b > R_1 \tag{4}$$

where $R_1 = \alpha_1 t_1$. If the dislocation resumes spreading at time $t = t_{r1}$ and propagates steadily again at a speed of α_2, the relative displacement distribution on the fault surface becomes

$$u_x(\rho_b,t) = D\sqrt{\alpha_2{}^2 (t-t_1^*)^2 - \rho_b{}^2} \quad \text{for} \quad \rho_b \le \alpha_2 (t-t_1^*)$$
$$= 0 \quad \text{for} \quad \rho_b > \alpha_2 (t-t_1^*) \tag{5}$$

in which the time shift

$$t_1^* = t_{r1} - R_1/\alpha_2 \tag{6}$$

As the expanding dislocation suddenly stops once more at time $t = t_2$, the relative displacement distribution thereafter is again assumed to be fixed at the value it had just before stopping, which is

$$u_x(\rho_b,t) = D\sqrt{R_2{}^2 - \rho_b{}^2} \quad \text{for} \quad \rho_b \le R_2$$
$$= 0 \quad \text{for} \quad \rho_b > R_2 \tag{7}$$

where $R_2 = R_1 + \alpha_2 (t_2 - t_{r1})$. In a similar manner, the restarting and stopping processes of the dislocation propagation repeat over and over. Then the intermittent propagation finally comes to rest at time $t = t_e$ and the minor-axis radius of the final elliptical dislocation boundary is equal to R_e.

SOLUTION TECHNIQUES

Based on the methods of self-similar potentials and rotational superposition, two analytical approaches have been developed by the author and Robinson for the study of three-dimensional shear-dislocation problems [1996, 1997]. The first approach deals with the sudden stopping of a spreading circular shear dislocation. The second approach is an extension of the usual rotational superposition by including a stretching effect, which can be used to express a three-dimensional problem with elliptical distribution of boundary values in terms of certain fictitious two-dimensional problems. The combined use of these two techniques permits an analytical solution for the problem of an elliptical shear dislocation that expands on a plane in an unbounded medium and then suddenly stops. In the present paper, these works are extended to find the solution of an intermittently propagating elliptical shear dislocation in an elastic unbounded medium. The results for a full space are then used as the incident waves for studies of the same dislocation source in a layered half-space.

Dealing with the Intermittent Propagation Processes

In a similar way as done in Lai and Robinson [1997], the intermittent propagation problem is decomposed into a series of problems starting at different time, which when added together in sequence represent the activation, the stopping, and the restarting of the dislocation propagation, respectively. The solution to the basic problem, defined by Eqs. (1) and (2), describes the response of the medium before the first stopping occurs. Starting at time $t = t_1$, a correcting problem S1 that will be superposed with the basic problem is determined by Eq. (1) and the following relative displacement function:

$$
\begin{aligned}
u_x(\rho_b, t) &= D\left(\sqrt{R_1{}^2 - \rho_b{}^2} - \sqrt{\alpha_1{}^2 t^2 - \rho_b{}^2}\right) \quad \text{for} \quad \rho_\mathrm{b} < R_1 \\
&= -D\sqrt{\alpha_1{}^2 t^2 - \rho_b{}^2} \quad \text{for} \quad R_1 \le \rho_\mathrm{b} \le \alpha_1 t \\
&= 0 \quad \text{for} \quad \rho_\mathrm{b} > \alpha_1 t
\end{aligned}
\tag{8}
$$

This correcting solution when added to the basic problem gives the field satisfying the boundary conditions of the intermittent problem for $t_1 \le t \le t_{r1}$, defined by Eqs. (1) and (4). When the dislocation resumes propagating at time $t = t_{r1}$, another correcting problem R1, defined by Eq. (1) and the relative displacement function

$$
\begin{aligned}
u_x(\rho_b, t) &= D\left(\sqrt{\alpha_2{}^2 (t - t_1^*)^2 - \rho_b{}^2} - \sqrt{R_1{}^2 - \rho_b{}^2}\right) \quad \text{for} \quad \rho_\mathrm{b} < R_1 \\
&= D\sqrt{\alpha_2{}^2 (t - t_1^*)^2 - \rho_b{}^2} \quad \text{for} \quad R_1 \le \rho_\mathrm{b} \le \alpha_2 (t - t_1^*) \quad , \\
&= 0 \quad \text{for} \quad \rho_\mathrm{b} > \alpha_2 (t - t_1^*)
\end{aligned}
\tag{9}
$$

is superposed with the basic problem and the correcting problem S1. The summation of the solutions for the basic problem and the correcting problems S1 and R1 then gives the field response of the intermittent problem for $t_{r1} \le t \le t_2$. The succeeded stopping and restarting of the dislocation propagation are treated in the same manner.

Rotational Superposition

For some special three-dimensional problems, there exists a one-to-one correspondence between a three-dimensional problem and certain two-dimensional problems that can be constructed. Rotational superposition is an effective approach to expressing a three-dimensional problem in terms of the corresponding fictitious plane problems [Alexandrov 1968]. To extend the applicability of the usual rotational superposition to problems with boundary values given over an elliptical region, Lai and Robinson [1996] introduced an additional weighting function

$$
w_e(\beta) = \frac{1+s}{(1+s)^2 \cos^2 \beta + \sin^2 \beta} \tag{10}
$$

for the superposition from geometrical considerations and modified the rotational superposition rule. This new approach relates the solution of any dynamic or static elasticity problem, which corresponds to boundary values on a circular area to the solution of the problem in which the same boundary values are "stretched" in one direction. From the two-dimensional problems that correspond by rotational superposition to the circular case, new two-dimensional problems are formulated which, when superposed in accordance with the modified superposition rule, result in the solution for the elliptical boundary distribution. The detailed expressions of the modified rotational superposition for elliptical shear-dislocation problems can be found in Lai and Robinson [1996].

Method of Self-Similar Potentials

The method of self-similar potentials is a very effective analytical tool, first developed by V. I. Smirov and S. L. Sobolev in the 1930s, for solving self-similar two-dimensional wave propagation problems, where the boundary and initial conditions are homogeneous functions of the spatial variables and time. This technique when applied together with rotational superposition is capable of dealing with certain three-dimensional problems as well (Eringen and Suhubi 1975). The intermittently propagating dislocation problem, apparently, is no longer a self-similar problem since the first stopping occurs; and thus all of the related correcting problems are not self-similar problems, either. The fictitious plane problems that correspond by rotational superposition to the correcting problems, however, can be decomposed into self-similar sub-problems that have different degrees of homogeneity. For the case being considered, each fictitious plane problem can be expressed as a superposition of four self-similar sub-problems, which can then be solved readily by employing the method of self-similar potentials.

Calculation of Reflection and Refraction at the Discontinuity Surface

Since the dislocation distribution is taken parallel to both the free surface and the interface, it is fairly direct to obtain the corresponding fictitious plane problems using the method of rotational superposition. The effect of the layering on the surface motion is taken into account on the basis of fictitious plane problems. This makes it possible to deal with the reflection and refraction of uncoupled P-wave and S-wave components separately by using the method of self-similar potentials.

In finding the induced motions on the free surface, it is only necessary to take into account the transmitted waves at the interface and the multiple reflections in the upper layer. That is, any reflections or refractions back into the bedrock are of no interest. Three kinds of computations for the fictitious plane problems are therefore needed: (1) calculation of the transmitted waves generated at the interface due to an incident disturbance from the lower medium; (2) computation of the reflection of the waves at the free surface in the upper layer; and (3) determination of the reflected waves due to a disturbance travelling in the upper medium and then reaching the interface of two media. Employing the method of self-similar potentials, all of these calculations can be readily done by applying the boundary conditions on the free surface or the continuity conditions at the interface.

NUMERICAL RESULTS

The computation for the dislocation problem in a layered half-space is considerably tedious due to the multiple reflections in the upper stratum. The "bookkeeping" of the reflected and refracted waves has to be incorporated into the analysis. The fictitious plane problems corresponding to the shear dislocation problem are constructed and solved using the solution techniques described above. The three-dimensional solution is then obtained by superposing the two-dimensional fields in accordance with the rotational superposition for elliptical shear-dislocation problems.

A numerical example with properties shown in Fig. 1 has been examined. The soft upper layer has a constant thickness of 4.0 km with P-wave speed a_2 = 3.10 km/sec and S-wave speed b_2 = 1.79 km/sec. The underlying bedrock, idealized as an elastic half-space, has wave speeds of a_1 = 6.03 km/sec and b_1 = 3.48 km/sec. The ratio of the shear moduli of the two media is taken to be $\mu = \mu_1/\mu_2 = 4.18$. An elliptical shear dislocation of aspect ratio $1+s = 1.5$ is assumed to propagate on a plane parallel to the free surface 12 km deeper than the interface of two media. The dislocation is assumed to expand steadily with an initial speed α_1 = 1.28 km/sec from a point of nucleation at time $t = 0$ and then suddenly stops at time $t = 1.04$ sec. The propagation restarts at a constant speed α_2 = 1.213 km/sec at time $t = 1.6$ sec. The dislocation source finally comes to rest at time $t = 2.15$ sec. The surface motions at points (ρ = 14 km, ω) due to the assumed intermittent seismic source have been computed and are shown in Fig. 3. For the purpose of comparison, the ground motions due to a steadily expanding dislocation source of speed α_1 = 1.28 km/sec are also shown in Fig. 2.

From the results, we can see that if intermittent spreading does not occur, all motions in the three directions are very smooth curves and the effect of a layered structure of the medium is not apparent. By contrast, the inclusion of intermittent propagation changes the response significantly and involves slope discontinuities. The layered structure of the medium then also becomes an important factor and influences the surface response considerably. The effect of multiple reflections in the upper layer can be easily seen from the oscillatory character of the calculated ground motions in Fig. 3. Because energy is drained slowly from the upper layer by the (imperfect) reflection at the interface, the displacements on the surface gradually approach a constant static value.

Although the responses at only a few points on the free surface are shown in Fig. 3 for the simplified seismic source model, the problem of an elliptical shear-dislocation source of any aspect ratio at any depth in a layered half-space can be treated in the same manner. The response at any location can be found without any difficulty if it is of interest.

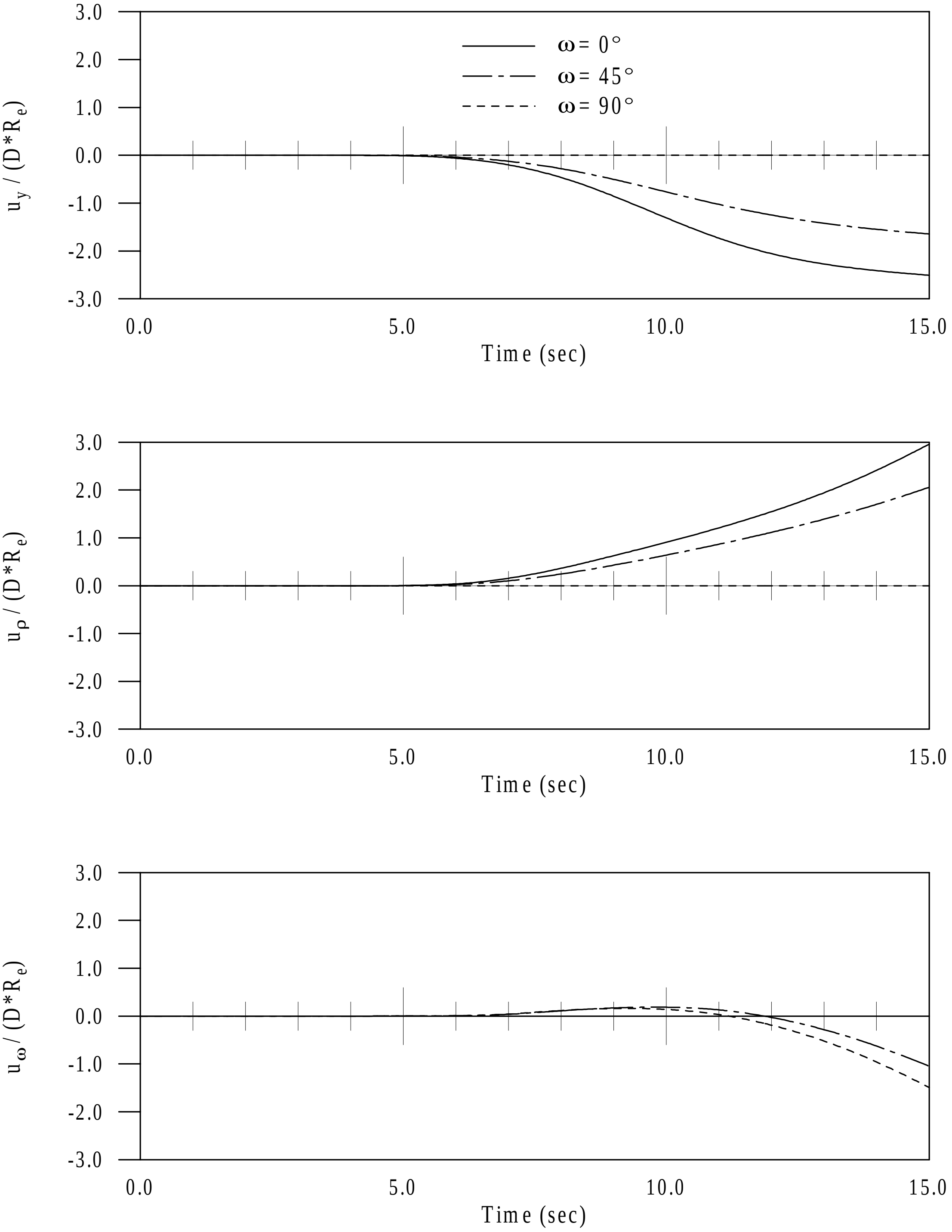

Figure 2 Response at ρ = 14 km on the free surface due to a steadily expanding dislocation

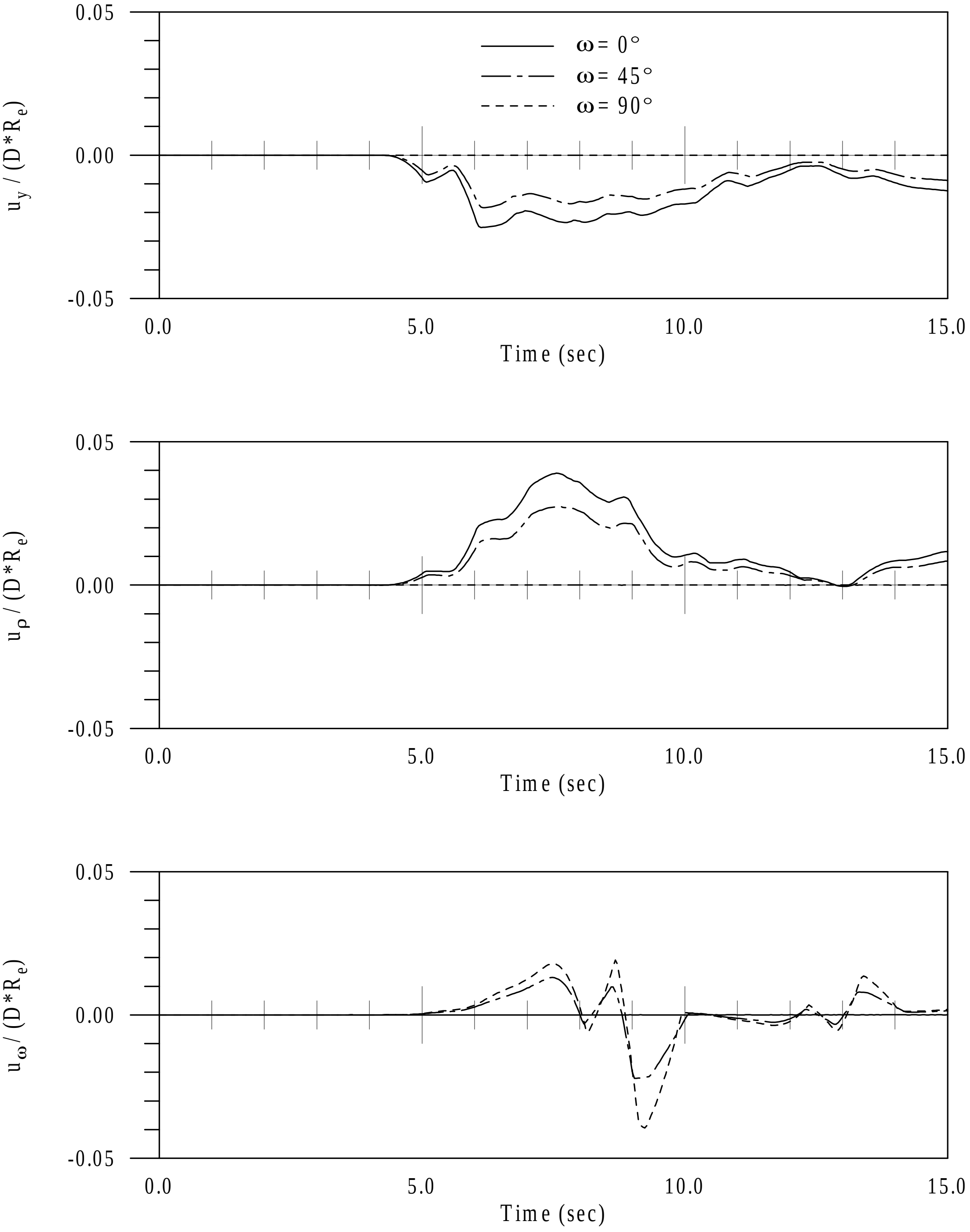

Figure 3 Response at ρ = 14 km on the free surface due to an intermittently propagating dislocation

CONCLUSIONS

An intermittently propagating shear-dislocation seismic source model has been examined. This sophisticated problem is solved analytically by decomposing the intermittent processes into a series of problems starting at different time, which when added together in sequence represent the activation, the stopping, and the restarting of the dislocation propagation, respectively. As expected, the induced ground motions are influenced considerably by the intermittent propagation of the dislocation and the layered structure of the medium. The results suggest that intermittent faulting processes as well as local geology must be taken into account in the development of a meaningful earthquake source model.

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